Transient resonance in limited power systems

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ABSTRACT

Difficulties in obtaining – by means of known methods - the proper estimation of maximum amplitudes in the transient resonance of technical systems, were indicated in the paper. It was proved that models of 1 degree of freedom cannot be used for such systems. The energy method of estimating maximum amplitudes in the transient resonance - useful also for systems of several degrees of freedom and multiple drives - was proposed. Taking into account interactions between the rotor and body motion of vibratory machines case the nomogram method for investigating resonance amplitude for coast-down phase was also presented.

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1. Introduction

Phenomenon of a transient resonance occurs in vibratory systems subjected to quasi-harmonic excitations of a continuously variable frequency, containing the natural frequency range of the system. Technical systems, in which this effect occurs, are - among others - vibro-insulation systems of unbalanced rotor machines, unbalanced elastic rotors operating above critical frequencies, over-resonance vibratory machines such as screens, conveyers, foundry vibrating grids, vibrating tables for compacting and forming concrete elements, vibratory mills, etc.

The most often met technical systems, in which the transient resonance effect occurs during the start and coasting, can be generally reduced to the schemes shown in Fig. 1.

Due to the practical importance of the problem, it was the subject of several theoretical studies. The works were performed in various directions.

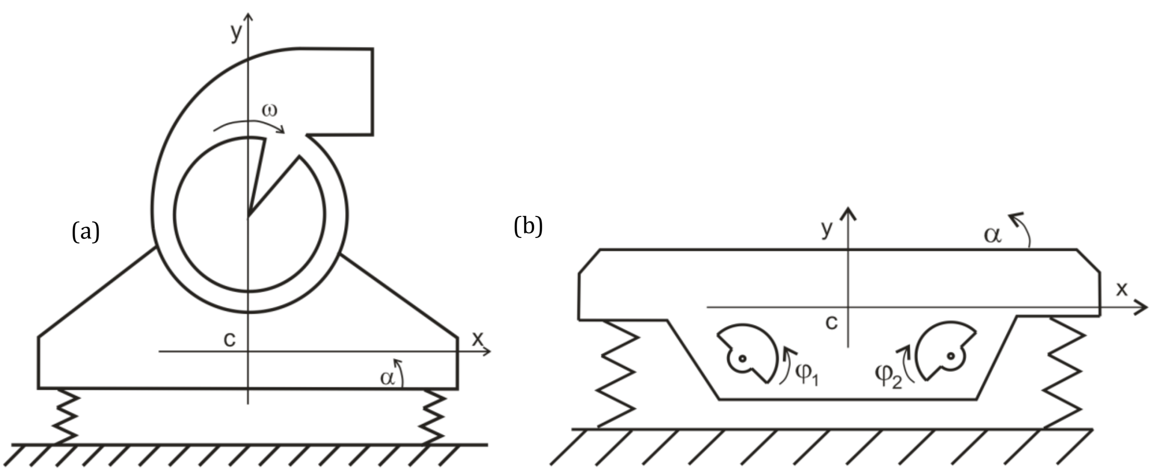

Fig. 1. Examples of technical systems, in which the transient resonance effect occurs:
a) Vibroinsulation system of an unbalanced rotor machine; b) Two-vibrator over-resonance vibratory machine.

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2. Models with the Given Time-History of the Exciting Force

The first solutions were obtained for the system shown in Fig. 2(a), for which the calculation model presented in Fig. 2(b), was assumed in papers (Lewis, 1932; Kac, 1947; Markert and Seidler, 2001) and many others, (e.g. Dorning, 1959; Ferlund, 1963; Fearn and Milsaps, 1967; Hirano, 1968; Leul 1994; Cieplok, 2009a; 2009b).

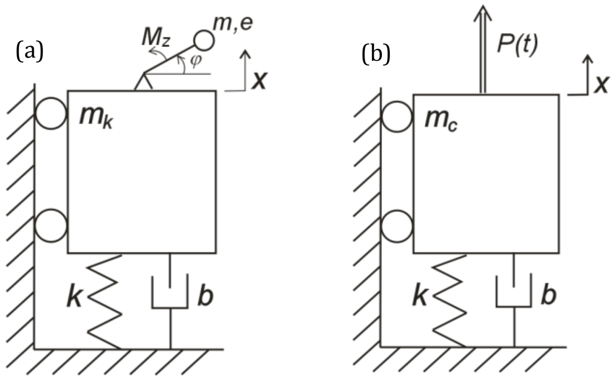

Fig. 2. Model of the vibroinsulation system of an unbalanced rotor machine of a linear vibration trajectory: a) Structural diagram; b) Calculation model.

Classic solutions of this problem and the majority of further attempts, were based on the simplified version of Eq. (1) of motion, assuming *a priori* the knowledge of the rotor angular velocity and its linear-variable over time form: $\dot{\varphi}(t) = \omega_0 \pm \varepsilon t$:

$$m_c\ddot{x} + b\dot{x} + kx = P\sin\left(\phi_0 + \omega_0 t \pm \frac{\varepsilon \cdot t^2}{2}\right), \tag{1}$$

where ε = const (+ for start-up, - for coasting).

In classic solutions described by Eq. (1) the component of the exciting force related to the rotor angular acceleration ε was neglected, since in the resonance zone it is usually much smaller than the centrifugal force.

In the first, historically speaking, solution of the transient resonance problem (Lewis, 1932), the independence of an excitation force P amplitude of a rotor angular velocity was assumed. The dependence of an excitation force amplitude on a rotor angular velocity square $P=me\omega^2$, being much closer to the reality, was introduced some years later by (Kac, 1947). In order to obtain the solution the impulse transfer function method was usually used, followed by numerical calculations of certain integrals.

Later formulations were rather similar (except for the paper of (Markert and Seidler, 2001), who wrote the transient resonance equation in a way allowing a slightly more general description of the excitation, e.g. taking into account the tangential component of an excitation force) and in a similar fashion, as the previous solutions, led to solutions in which the main component was:

$$A_{\max} \approx c\omega_n/\sqrt{\varepsilon}\,. \tag{2}$$

Solutions obtained by the analysis of the model shown in Fig. 2(b) were used for constructing nomograms (Harris, 1957), universally applied at present, for determinations of maximum amplitudes in the transient resonance.

The fundamental fault of all these elaborations is the fact, that the angular acceleration ε in the circum-resonant zone is unknown for the real systems. It cannot be determined on the basis of known driving and anti-torque moments of the rotor, since – in reality – the system being modelled (Fig. 2(a)) has two degrees of freedom and is described by the system of equations of motion:

$$m_c\ddot{x} + b\dot{x} + kx = \dot{\varphi}^2 me\sin\varphi - \ddot{\varphi}me\cos\varphi\,, \tag{3a}$$

$$J_{zr}\ddot{\varphi} = M_z - M_w\,, \tag{3b}$$

$$M_w = -\ddot{x}me\cos\varphi\,. \tag{3c}$$

Component M_w (Eq. 3c), called "the vibratory moment" (Blechman, 1971), appearing in the rotor equation of angular motion (Eq. 3b) and originated from the rotor axis vibrations (increasing in the resonance) has a decisive influence on this acceleration value. This component acts in principle as a strong braking torque and causes, that the real value of the angular acceleration at coasting is much higher than it would result from the resistance to motion alone (Fig. 3).

Thus the amplitude values, determined on the basis of nomograms (Harris, 1957), which were prepared on the grounds of the solution of Eq. (1), are several times too large for the coasting (Michalczyk, 1995) due to essential errors in ε evaluation.

3. Models Taking Into Account an Influence of Machine Vibrations on an Unbalanced Rotor Running

Some of the works taking into account the influence of the vibratory moment are based on the system energy balance directly before the resonance and in the moment when maximum vibrations occur, obtaining estimations "from the top" of the maximum amplitudes. Thus, e.g. in paper (Michalczyk, 1993), the kinetic energy of the set of vibrators in the moment of entering the resonance zone of the i^{th} form of vibrations was equated to the machine vibration energy, performing movement in accordance with this form of vibrations, at the assumption that the main vibrations occur in a plane motion along one coordinate.

This problem will be now formulated in a more general way than in the quoted paper, by allowing the resonance vibrations corresponding to the arbitrary form of natural vibrations in the general motion of the linearised system surrounding the balance point.

The energy balance for the i^{th} resonance of the machine with n drives will be written as:

$$n \cdot \tfrac{1}{2}J_{zr}\omega_{0i}^2 = \tfrac{1}{2}\dot{\mathbf{q}}_{\max i}^T \cdot \mathbf{M} \cdot \dot{\mathbf{q}}_{\max i}\,, \tag{4}$$

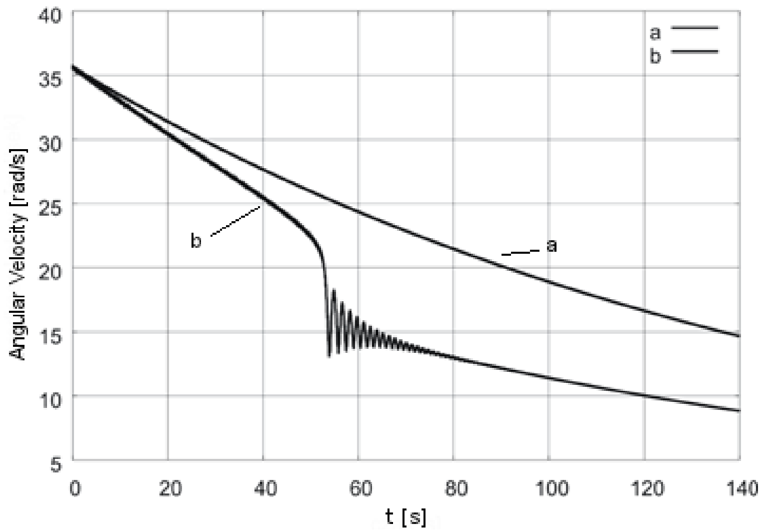

Fig. 3. Influence of the vibratory moment on the rotor angular velocity ω during the vibrator free coasting (M_z<0):
a) Coasting without taking into account the vibratory moment;
b) Coasting with taking into account the vibratory moment M_w.

where ω_{0i} is angular velocity, at which the energy exchange occurs (usually not much different from the i^{th} natural frequency of the machine body on the elastic suspension system), $\mathbf{q}$ = col{$x_c, y_c, z_c, \varphi_x, \varphi_y, \varphi_z$} – coordinates vector in the central coordinate system, describing small body vibrations versus the static balance point, J_{zr} – reduced moment of inertia of the vibrator and drive, J_{ij} – elements of the tensor of inertia – correspondingly - in the central system Cxyz, $m_c = m_k + n \cdot m$,

$$\mathbf{M} = \begin{bmatrix} m_c & 0 & 0 & 0 & 0 & 0 \\ & m_c & 0 & 0 & 0 & 0 \\ & & m_c & 0 & 0 & 0 \\ & & & J_{xx} & -J_{xy} & -J_{xz} \\ & sym. & & & J_{yy} & -J_{yz} \\ & & & & & J_{zz} \end{bmatrix} . \tag{5}$$

Maximum values of the generalised velocities $\dot{\mathbf{q}}_{\max i}$, for harmonic vibrations of ω_{0i} frequency, are related to the maximum amplitude values of the displacement vector by the dependence: $\dot{\mathbf{q}}_{\max i} = \omega_{0i} \cdot \mathbf{q}_{\max i}$, which leads to the Eq. (6):

$$n \cdot \tfrac{1}{2} J_{zr} \omega_{0i}^2 = \tfrac{1}{2} \omega_{0i}^2 \cdot \mathbf{q}_{\max i}^{\mathrm{T}} \cdot \mathbf{M} \cdot \mathbf{q}_{\max i} , \tag{6}$$

or, after the reduction:

$$n \cdot J_{zr} = \mathbf{q}_{\max i}^{\mathrm{T}} \cdot \mathbf{M} \cdot \mathbf{q}_{\max i} . \tag{6a}$$

Let the vibrating motion of the machine body placed on the elastic suspension system be described by the equation of small natural undamped vibrations:

$$\mathbf{M} \cdot \ddot{\mathbf{q}} + \mathbf{K} \cdot \mathbf{q} = 0 , \tag{7}$$

where $\mathbf{K}$ is symmetric elasticity matrix.

For the basic – in applications - case of the machine placed on a system j=1,..,ν of parallel, identical elastic elements of constants:

- in a vertical direction, k_z ,
- in horizontal directions, k_x, $k_y = k_{xy}$

and coordinates x_j, y_j, z_j of points - where the elastic elements were mounted to the body in the static equilibrium of the machine - it is easy to prove the following:

$$\mathbf{K} = \begin{bmatrix} \nu k_{xy} & 0 & 0 & 0 & k_{xy}\Sigma z_j & -k_{xy}\Sigma y_j \\ & \nu k_{xy} & 0 & -k_{xy}\Sigma z_j & 0 & k_{xy}\Sigma z_j \\ & & \nu k_{xy} & k_z \Sigma y_j & -k_z \Sigma x_j & 0 \\ & & & k_z\Sigma x_j^2 + k_{xy}\Sigma z_j^2 & -k_z \Sigma x_j y_i & -k_{xy}\Sigma x_j z_j \\ & sym. & & & k_z\Sigma x_j^2 + k_{xy}\Sigma z_j^2 & -k_{xy}\Sigma y_j z_j \\ & & & & & k_{xy}\left(\Sigma x_j^2 + \Sigma y_j^2\right) \end{bmatrix} . \tag{8}$$

For the harmonic form of solutions $\mathbf{q} = \mathbf{q}_{\max}(\sin \omega t + \gamma)$ it leads to the matrix frequency equation:

$$[\mathbf{K} - \omega^2 \mathbf{M}] = 0 , \tag{9}$$

which allows to determine the set of natural frequencies ω_{0i}, i=1...6 and modal vectors:

$$\mathbf{\Psi}_i(\omega_i) = \mathrm{col}\{\psi_{1i}, \psi_{2i}, \psi_{3i}, \psi_{4i}, \psi_{5i}, \psi_{6i}\} . \tag{10}$$

Let it be assumed for the moment, that these frequencies are different and satisfactorily distant. This allows to present the amplitude vector for vibration of the i^{th} frequency and form, as:

$$\mathbf{q}_{\max i} = \mathrm{col}\left\{\frac{\psi_{1i}}{\psi_{ki}} \cdot q_{\max ki}, \frac{\psi_{2i}}{\psi_{ki}} \cdot q_{\max ki}, , q_{\max ki}, , \frac{\psi_{6i}}{\psi_{ki}} \cdot q_{\max ki}\right\}$$

$$= \mathbf{q}_{\max ki}\, \mathrm{col}\left\{\frac{\psi_{1i}}{\psi_{ki}}, \frac{\psi_{2i}}{\psi_{ki}} \cdot, , 1, , \frac{\psi_{6i}}{\psi_{ki}}\right\} , \tag{11}$$

where $\mathbf{q}_{\max ki}$ means the maximum amplitude of the arbitrarily selected coordinate q_k (to represent vibrations of the i^{th} form, at the assumption: $\psi_{ki} \neq 0$). Denoting:

$$\text{col}\left\{\frac{\psi_{1i}}{\psi_{ki}}, \frac{\psi_{2i}}{\psi_{ki}} \cdot,, 1,, \frac{\psi_{6i}}{\psi_{ki}}\right\} = \mathbf{a}_{ki}\,, \tag{12}$$

and substituting to Eq. (11) and Eq. (6a) the final formula for the maximum amplitude of the k^{th} coordinate during the system passage through the resonance with the i^{th} frequency of natural vibrations will be obtained:

$$\mathbf{q}_{\max ki} = \sqrt{\frac{n \cdot J_{zr}}{\mathbf{a}_{ki}^{\mathrm{T}} \cdot \mathbf{M} \cdot \mathbf{a}_{ki}}}\,. \tag{13}$$

This formula constitutes an over-estimation (usually close to the real values) of the maximum amplitude of the selected k^{th} coordinate in the i^{th} resonance, provided that the vibrator excitation force performs work at vibrations of the investigated form. This requirement is equivalent to the demand of not zeroing the respective modal force. Not satisfying this condition as a rule means that the passage through the resonance zone will be without the excitation of more intensive machine vibrations. Deviations from this rule can occur in two-vibrator system acting due to the self-synchronization principle, see (Michalczyk and Czubak, 2010).

In the case when in the spectrum of the system natural frequencies multiple frequencies occur, this procedure is usually not possible in respect of those frequencies, due to the ambiguity of the vibration form. An exception constitute symmetric systems for which forms of vibration are known, e.g. from the physical analysis of the problem (as for the machines shown in Fig. 4). An example of using this method in relation to real machines was shown in (Michalczyk, 2012a) with respect to lumped parameter systems and in (Michalczyk, 2012b) for distributed-parameter systems.

The energy balance was also applied in paper (Agranowskaja and Blechman, 1969), however, the vibrator kinetic energy was not equated to the body kinetic energy, but to the maximum potential energy of the suspension system. The difficulty of this energy method constitutes the necessity of estimation the vibrator angular velocity, at which the energy exchange occurs, while this velocity does not comply directly with the system natural vibrations frequency (Lewis, 1932) and the method does not provide the way of its determination.

4. Nomogram Method

The phenomenological model of a vibratory machine illustrated schematically in Fig. 4 is considered. A machine body, having a mass M is suspended in a flexible viscous system described by concentrated constants k and b. The system is excited to vibrations by the inertia vibrator characterized by the static unbalance me value. The vibrator moment of inertia combined with the driving system moment of inertia is reduced to the coordinate of the vibrator rotation and denoted by J_{zr}. The vibrator is exposed to the driving moment action M_{el}.

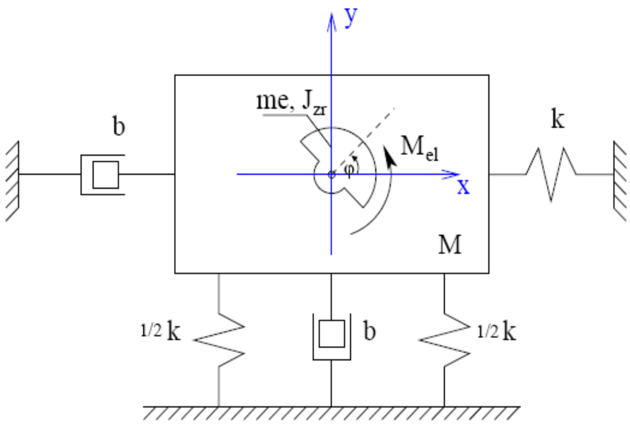

Fig. 4. Vibratory machine model.

Taking into consideration three degrees of freedom (x_s, y_s, φ) the machine equations of motion can be shown in the form of Eqs. (14) (15) and (16).

$$\begin{bmatrix} m_c & 0 \\ 0 & m_c \end{bmatrix}\begin{bmatrix} \ddot{x}_s \\ \ddot{y}_s \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} \dot{x}_s \\ \dot{y}_s \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}\begin{bmatrix} x_s \\ y_s \end{bmatrix} = \begin{bmatrix} P_x \\ P_y \end{bmatrix}, \tag{14}$$

$$\begin{bmatrix} P_x \\ P_y \end{bmatrix} = \begin{bmatrix} me\sin\varphi \\ -me\sin\varphi \end{bmatrix}\ddot{\varphi} + \begin{bmatrix} me\cos\varphi \\ me\sin\varphi \end{bmatrix}\dot{\varphi}^2\,, \tag{15}$$

$$J_{zr}\ddot{\varphi} - me(\ddot{x}\sin\varphi - \ddot{y}\cos\varphi) = M_{el}\,. \tag{16}$$

Upon transformation to the rotating with the angular velocity of the vibrator system $0\xi\eta$ (Fig. 5), these equations can take the form of Eq. (17):

$$\begin{bmatrix} m_c & 0 & m_c\eta & 0 & 0 \\ 0 & m_c & m_c\xi + me & 0 & 0 \\ 0 & me & me\xi + J_{zr} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \frac{d}{dt}\begin{bmatrix} v_\xi \\ v_n \\ \omega \\ \xi \\ \eta \end{bmatrix} =$$

$$\begin{bmatrix} 2m_c\omega v_\eta - bv_\xi - (k - \omega^2 m_c)\xi + b\omega\eta + me\omega^2 \\ -2m_c\omega v_\xi - bv_\eta - (k - \omega^2 m_c)\eta - bv\omega^2 \\ -2mev_\xi\omega + me\eta\omega^2 + M_{el} \\ v_\xi \\ v_\eta \end{bmatrix}, \tag{17}$$

where:

$$m_c = M + m\,. \tag{18}$$

This transformation enables also to create a definition of relative units and parameters for the machine. Hence substituting by the following:

$$\underline{\xi} = \frac{\xi}{A_u},\ \ \underline{\eta} = \frac{\eta}{A_u},\ \ \underline{\omega} = \frac{\omega}{\omega_0},\ \ \sigma = \frac{m^2e^2}{m_cJ_{zr}}\,,$$

$$q = \frac{M_{el}}{J_{zr}}\frac{1}{\omega_0^2}\,,\ \ \gamma = \frac{b}{2\sqrt{m_ck}}\,,\ \ \ \tau = \frac{\omega_0}{2\pi}t\,,\ A_u = \frac{me}{m_c}\,. \tag{19}$$

The matrix Eq. (17) may be expressed in the following form:

$$\begin{bmatrix} \frac{1}{4\pi^2} & 0 & -\frac{1}{2\pi}\eta & 0 & 0 \\ 0 & \frac{1}{4\pi^2} & \frac{1}{2\pi}(1+\xi) & 0 & 0 \\ 0 & \frac{\sigma}{4\pi^2} & \frac{1}{2\pi}(\sigma\xi+1) & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \frac{d}{d\tau}\begin{bmatrix} \underline{v}_\xi \\ \underline{v}_\eta \\ \underline{\omega} \\ \underline{\xi} \\ \underline{\eta} \end{bmatrix} =$$

$$\begin{bmatrix} \underline{\omega}^2 - (1-\underline{\omega}^2)\underline{\xi} - \frac{\gamma}{\pi}\underline{v}_\xi + \frac{1}{\pi}\underline{\omega}\,\underline{v}_\eta + 2\gamma\underline{\omega}\,\underline{\xi} \\ -(1-\underline{\omega}^2)\underline{\eta} - \frac{\gamma}{\pi}\underline{v}_\eta + \frac{1}{\pi}\underline{\omega}\,\underline{v}_\xi + 2\gamma\underline{\omega}\,\underline{\xi} \\ -\frac{\sigma}{\pi}\underline{v}_\xi\underline{\omega} + \sigma\underline{\eta}\omega^2 + q \\ \underline{v}_\xi \\ \underline{v}_\eta \end{bmatrix}. \tag{20}$$

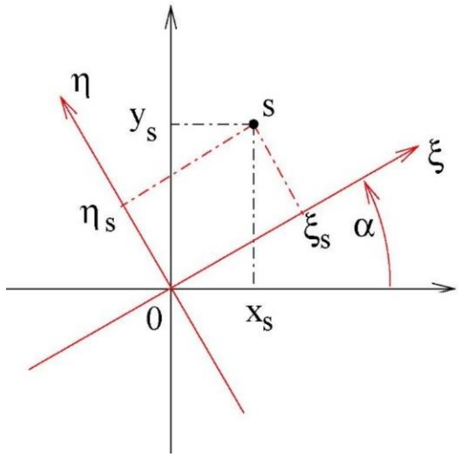

Fig. 5. Position of the machine body mass centre in the stationary coordinate systems $0xy$ and rotating system $0\xi\eta$.

Due to an application of relative parameters the set of six physical parameters m_c, me, J_{zr}, M_{el}, k, b - needed to determinate the machine dynamics in the natural coordinate system - was reduced to three parameters σ, γ, q. Based on the system above (Eq. (20)), it is possible to develop the nomogram (Fig. 6) that enables to determine the amplitude multiplication factor α for the coast-down phase based on values of parameters σ, γ. For the free coasting the parameter q is assumed to be zero.

In the case when during a coast-down phase external moments of friction play an important role, it is possible to use the contour map presented in Fig. 7. The map was prepared for parameter γ=0.01, which can represent suspensions built of steel material properties. The friction is expressed by the relative parameter q.

The data map can be approximated by the function of the following form:

$$\alpha = (a_2 - a_3 \ln\sigma)e^{-a_1 q} + a_4 \,. \tag{21}$$

where:

for	for
$\sigma \in (0.0003, 0.03)$	$\sigma \in (0.0003, 0.03)$
$q \in (0.001, 0.049)$	$q \in (0.05, 1.2)$
$a_1 = 49.51$	$a_1 = 6.75$
$a_2 = -9.51$	$a_2 = 2.57$
$a_3 = 2.68$	$a_3 = 0.31$
$a_4 = 4.42$	$a_4 = 1.33$ (22)

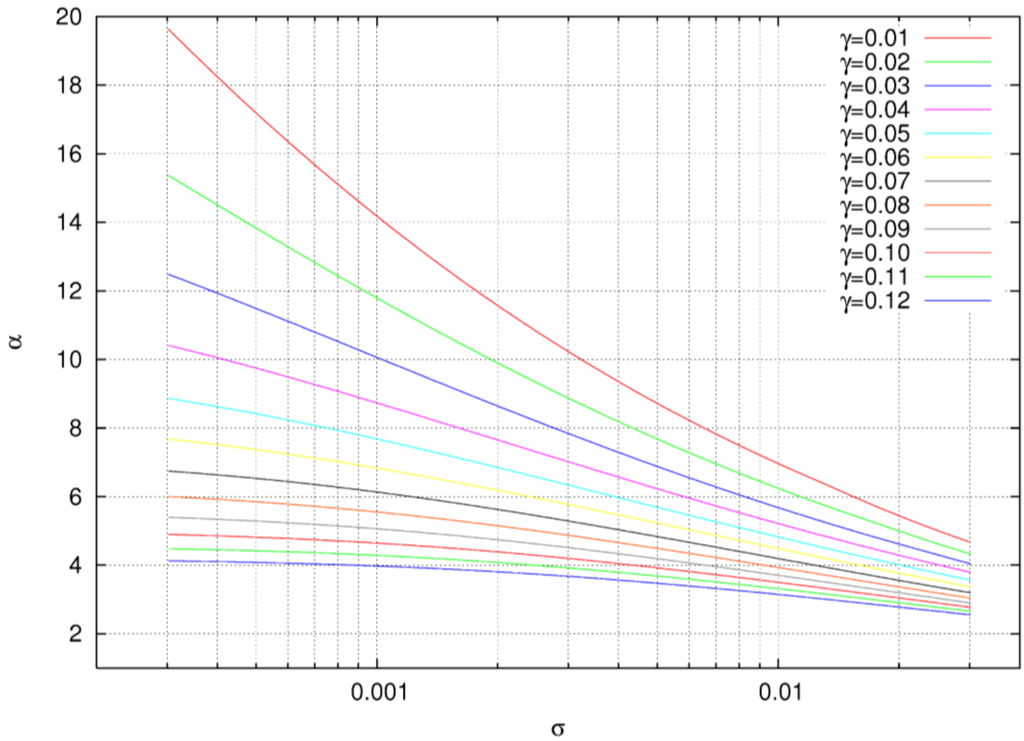

Fig. 6. Machine body vibration amplitude multiplication factor $\alpha = \frac{A_{of\ \text{resonanse}}}{A_u}$ for the coast-down phase.

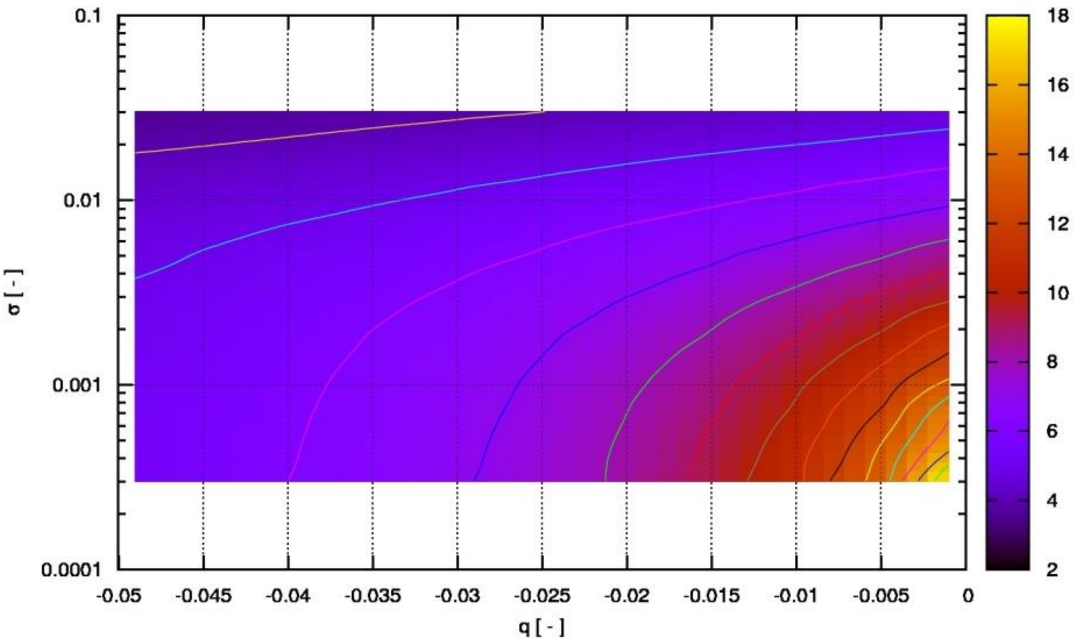

Fig. 7. Amplitude multiplication factor α taking into account friction during the coast-down phase.

5. Conclusions

The problem of the determination of the maximum amplitudes in the transient resonance during the free coasting of vibratory machines and vibroinsulating systems of rotor machines was analysed in the study. It was proved that the classic methods, based on the assumption that the time-history of the unbalanced rotor motion is known, lead to serious errors.

The reason of these errors constitutes a strong influence of the machine body vibrations on the rotor running in the resonance zone. Two methods of assessments the maximum amplitudes, in this case, were formulated:

- Method of the kinetic energy balance in the system,
- Nomogram method.

The first method has a wider range of applications encompassing general motions of elastically supported bodies and vibrations of continuous systems, while the second one is characterised by a higher accuracy and a possibility of taking into account the dissipation in the system as well as driving and anti-torque moments however, its application is limited to translatory motion of machine bodies.

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