Research Article

Optimization of cylindrical wall domes via metaheuristic algorithms

Aylin Ece Kayabekir a, *

a Department of Civil Engineering, Istanbul Gelisim University, 34310 Istanbul, Turkey

ABSTRACT

Optimization is a widely used phenomenon in various problems and fields. Because time and resources are very limited in today’s world, it can be said that the usage area of the optimization process will be expanded and spread in all areas of life. Although different methods are used in the realization of the optimization process, the performance of metaheuristic algorithms in solving problems has led to an increase in research on these methods. As in other fields, the application examples of these algorithms are diversifying and increasing in the field of structural engineering. In this study, the performance comparison of five different algorithms for the optimum design of an axisymmetric cylindrical wall with a dome is investigated. These algorithms are Jaya (JA), Flower pollination (FPA), teaching-learning-based optimization (TLBO) algorithms and two hybrid versions of these algorithms. ACI 318 regulation was used in reinforced concrete design with a flexibility method-based approach in the analyses. In the analyzes with five different situations of the wall height, some statistical values, and data of analysis numbers were obtained by running the algorithms a large number of times. According to the analysis results, Jaya algorithm is slightly better in terms of the speed of reaching the optimum result, but also all algorithms are quite effective and reliable in solving the problem.

1. Introduction

In almost all of the natural events that take place in the world, it is seen that the events operate in the most appropriate order under various conditions such as time, situation, and environmental conditions, and the operation is updated to maintain the most appropriate order in the face of changing situations.

These events, which take place for various reasons such as migration, hunting, movement in a hierarchical order, usually take place to survive in the balance of nature. For this reason, the process must be in the most appropriate way according to environmental factors. The most appropriate order seen in the behavior of other living things can be observed in the design and processes of civilizations established by humans, such as the locations of cities, road routes, tools used, city infrastructure systems, structural designs, defense systems. However, the main difference between humans and other living things is that while other creatures manage the processes with their intuition, people’s intuitions and their minds, which include various processes such as observation, experience, and thinking, evaluate the situation and act. Today, many engineering designs have been developed as a result of these observations of nature by scientists, and have been presented to humanity in many different ways, directly or indirectly. Metaheuristic algorithms can be given as an example of this situation.

Metaheuristic algorithms are methods developed as a result of mathematical equations expressing the heuristic behaviors of living things to maintain processes such as movement, migration, feeding in the most appropriate way possible as a result of nature observations, and are generally used for optimization purposes. Examples of metaheuristic algorithms, the variety and number of which have increased with the development of computer processor technology, are presented in Table 1.

* Corresponding author. E-mail address: aekayabekir@gelisim.edu.tr (A. E. Kayabekir)
ISSN: 2149-8024 / DOI: https://doi.org/10.20528/cjsme.2021.04.003
Today, metaheuristic algorithms of optimization problems are frequently used in many engineering fields. One of the field of structural engineering, which is one of the sub-branches of civil engineering, where studies are carried out within the scope of structural design. Examples of optimization problems in which metaheuristic algorithms are used in structural engineering are beams (Coello et al., 1997; Rafiq and Southcombe, 1998; Govindaraj and Ramasamy, 2005; Akin and Saka, 2010; Fedghouche and Tilouine, 2012; Bekdaş and Nigdeli, 2013; Kayabekir et al., 2019; Zhao et al., 2021), columns (Bekdaş and Nigdeli, 2014; 2016a; 2016b; de Medeiros and Kripta, 2014; Nigdeli et al., 2015; Çakıroğlu et al., 2021), frames (Bekdaş and Nigdeli, 2017; Nigdeli and Bekdaş, 2016; Ghatte, 2021), foundations (Nigdeli et al., 2018; Kashani et al., 2021), retaining walls (Ceranic et al., 2001; Yepes et al., 2008; Camp and Akin, 2011; Kayabekir et al., 2020; Yöcel et al., 2021; Martinez-Muñoz et al., 2021; Shalchi Tousi et al., 2021), Carbon Fiber Reinforced Polymer retrofit (Kayabekir et al., 2017; 2018) and cylindrical walls (Bekdaş, 2014; 2015; 2018; 2019; Kayabekir, 2021).

In this study, the optimum design of axially symmetrical cylindrical reinforced concrete walls with a dome on top has been investigated. In the optimization problem, the objective function is defined as the minimization of the total cost of the cylindrical wall consisting of steel and concrete. Long cylindrical walls were assumed in the analyzes and the flexibility method was used to calculate the internal forces. Hybrid algorithms of Flower Pollination Algorithm, Jaya Algorithm and Teaching Learning-Based Optimization have been proposed for optimization. The performance of the hybrid algorithm was tested under various conditions of the cylindrical wall. Then, these analysis results were also compared with the Flower pollination algorithm, teaching learning-based algorithm, and Jaya, which are known to be effective in the literature in optimization.

2. Optimum Design Methodology

In this section, the methodology including the details of the application of metaheuristic algorithms to the optimization problem is presented. In the literature, in optimization problems in which metaheuristic algorithms are used in general, the methodology is collected in five main steps. The operations performed in steps 1, 2, 3, and 5 of these five steps are similar in almost all algorithms. Only the operation performed in step 4 includes some algorithm-specific mathematical expressions. These five steps and the operations performed in each step are summarized below.

In the first step, the data entry of the problem is done. These data are design constants, ranges of design variables and special parameters of the optimization algorithm, number of solution vectors (vn), and stopping criteria (sc). Information about the design constants and design variables used in optimization are given in Tables 2 and 3, respectively.

Some of the algorithms, especially those developed in recent years, are parameter-free, that is, they do not have algorithm-specific parameters. Except for FPA, this study is parameter-free in the algorithms used. FPA algorithm has a parameter called switch probability (sp).

The stopping criterion, which stops the optimization process, is used in different ways in the literature. One of these, perhaps the most common, is to set a maximum value for iterations. This approach was also applied in this study. In this study, vn, sp, and sc values were taken as 20, 0.5, and 20000, respectively.
In the second step, the initial solution matrix is constructed. This matrix contains randomly generated design variables within the range for each solution vector (Table 3). Each design variable is generated between the lower and upper limits as in Eq. (1) and stored in the corresponding row of the solution vector.

\[ X_i = X_{i(\text{min})} + \text{rand}(X_{i(\text{max})} - X_{i(\text{min})}) \]  

(1)

In the equation, \( X_{i(\text{min})} \) and \( X_{i(\text{max})} \) represent the lower and upper limits of the \( i \)th design variable, respectively, and the rand is a function that produces a single uniformly distributed random number in the interval \((0,1)\).

The third step includes the analysis and design phases. In this step, analysis and design are done for each solution vector, and then the objective function is calculated. In this study, the sum of the steel and concrete costs of the cylindrical wall design is defined as the objective function (Eq. (2)).

\[ f(x) = C_c V_c + C_s W_s \]  

(2)

In the equation, \( V_c \) and \( W_s \) represent the total concrete volume and steel weight, respectively.

Analyzes were done using the flexibility theory (see in Section 3) according to the assumption of a long cylindrical wall, and the design was done by the requirements of ACI 318-Building code requirements for structural concrete. Accordingly, the constraints controlled during the design process are presented in Table 4.

If these constraints are not satisfied in any solution vector, the process called punishment is applied. It is known that there are different types of punishment in the literature. In this study, punishment is done by equating the objective function to a very large value.

Since the target in optimization is to obtain the lowest cost design, this penalty in the objective functions ensures that these solutions are eliminated in the following steps, thus convergence to the lowest solutions. In the study, this penalty value was taken as \(10^6\).

The fourth step is the step in which new solution vectors are obtained according to the algorithm rules. As mentioned before, this step is unique in algorithms because each algorithm applies its own rules.

In this study, JA, TLBO, FPA, and two hybrid algorithms were applied to the problem and the performances of the algorithms were compared. These algorithms were proved as effective ones in structural engineering problems and include unique features. The use
of Levy distribution, being a user-defined specific parameter-free algorithm and being a single-phase method using both of best and worst solutions in an equation are the unique features of FPA, TLBO and JA, respectively. These unique features provide differentiation of these algorithms. To reach a better solution sensitively, the classical algorithms need to be improved and hybrid algorithms are constructed via these algorithms. The equations used to generate a new solution matrix for these algorithms are given below.

The Jaya algorithm is a parameter-free algorithm using a single equation. In the algorithm, each solution (for example, \( X_{ij} \)) is tried to be improved by using the best (\( g^* \)) and worst (\( g^w \)) solutions in terms of the objective function. Accordingly, a new solution (\( X^{i+1}_{ij} \)) can be found as follows:

\[
X^{i+1}_{ij} = X^*_j + \text{rand}(g^* - X^w_j) \tag{3}
\]

In the TLBO algorithm, two different equations such as FPA are used in new solutions. These are called the teacher and student phases, respectively. However, unlike the choice done depending on the \( sp \) value in FPA, these two equations are applied sequentially in TLBO.

The teacher role is the person who has the best knowledge in the classroom and teaches the others. In the teacher phase of the algorithm, this corresponds to the solution with the best objective function among the solution vectors. According to this, the teacher phase can be written as

\[
X^{t+1}_{ij} = X^*_j + \text{rand}(g^* - TF \times X^\text{mean}_j) \tag{4}
\]

In this equation, \( X^\text{mean}_j \) is the mean value of the design variables, and TF, called the teaching factor, is a coefficient calculated according to Eq. (5).

\[
TF = \text{round} \left( 1 + \text{rand} () \right) \tag{5}
\]

The student phase, on the other hand, stimulates the learning of students as a result of their interaction. This situation, which shows the interaction of randomly selected solutions (\( X^s_{mi} \) and \( X^s_{ni} \)) is seen in Eq. (6).

\[
X^{s+1}_{ij} = \begin{cases} f(X)_n < f(X)_m, & X^s_{ij} + \text{rand}(X^s_{mj} - X^s_{ij}) \\ f(X)_n > f(X)_m, & X^s_{ij} + \text{rand}(X^s_{nj} - X^s_{ij}) \end{cases} \tag{6}
\]

According to the FPA, new solutions can be produced according to two different equations. These equations simulate global and local pollination, respectively. The equation to be used in generating a new value is decided according to the switch probability (\( sp \)) value. Accordingly, a random value is generated with the rand function and compared with \( sp \). If the rand value is less than \( sp \), global pollination is applied (Eq. (7)), otherwise, local pollination is applied (Eq. (8)). Levy flight, which is the value simulating the flight of pollen in the global pollination equation, is defined by the letter \( L \).

\[
X^{s+1}_{ij} = X^s_{ij} + L (g^* - X^s_{ij}) \tag{7}
\]

\[
X^{s+1}_{ij} = X^s_{ij} + \text{rand}(X^s_{mj} - X^s_{nj}) \tag{8}
\]

The hybrid algorithms used in the study consist of the student phase/local pollination phase with Jaya. In addition, a modification was done to the Jaya equation by using \( L \) instead of the rand function as given in Eq. (9).

\[
X^{s+1}_{ij} = X^s_{ij} + L (g^* - X^s_{ij}) - L (g^w - X^s_{ij}) \tag{9}
\]

During the optimization, for the first hybrid algorithm called JALS, one of the Eqs. (8) and (9) is used to generate new solutions according to the switch probability value defined in FPA.

The second hybrid algorithm, JALS2 is used to two phases consecutively like TLBO. In these phases, both Eq. (8) and Eq. (9) in the generation of new solutions. In these phases, the production of new solutions is done according to Eq. (8) and Eq. (9), respectively.

In the last step, new solutions are compared with existing solutions. If objective function value of new solutions better than existing ones, existing solution matrix is updated with new solutions. Otherwise no change is done. The last step and the fourth step are continued as satisfied stopping criteria of the optimization. The optimization process defined in five steps is also summarized with the pseudo code in Fig. 1.

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**Fig. 1.** Pseudo code of optimization process.
3. Analysis of Axially Symmetrical Cylindrical Shells

A small piece of the shell shown in Fig. 2 can be taken to obtain the general solution for cylindrical shells. Considering the equilibrium condition by taking into account symmetry and some constant forces, Eq. (10) can be written.

\[
\frac{dN_x}{dx} r dx d\phi = 0
\]

\[
\frac{dQ_x}{dx} r dx d\phi + N_x dx d\phi + q r dx d\phi = 0
\]

\[
\frac{dM_x}{dx} r dx d\phi - Q_x r dx d\phi = 0
\]  

(10)

From the solution of these equations to the axially symmetrical cylindrical shell conditions with appropriate assumptions, the general solution can be defined as

\[
w = e^{\beta x} (C_1 \cos \beta x + C_2 \sin \beta x) + e^{-\beta x} (C_3 \cos \beta x + C_4 \sin \beta x) + f(x)
\]

where \(C_1:4\) are integration constants that depend on support conditions, \(f(x)\) is a particular solution. \(\beta\) is a notation used in differential equation solutions and is given in Eq. (12).

\[
\beta^4 = \frac{Eh}{4r^2D} = \frac{3(1-\nu^2)}{r^2h^2}
\]

(12)

In the equation, \(E, h, r, D, \) and \(\nu\) are elasticity modulus, thickness, radius, rigidity (Eq. (13)), and Poisson’s ratio, respectively.

\[
D = \frac{Eh^3}{12(1-\nu^2)}
\]

(13)

When the structure of this equation is examined, it is seen that the effects on the wall decrease rapidly as you move away from the point where the load affects, and converge to zero after a certain point. In this case, it means that the cylindrical wall of a certain length and the wall of infinite length give approximately the same results. This approach greatly simplifies the general equation whose solution includes assumptions and complex mathematical operations. The solution equation of this situation, which is called the long wall, can be written as

\[
w = e^{-\beta x} (C_3 \cos \beta x + C_4 \sin \beta x)
\]

(14)

Detailed information about this situation can be found in several books (Billington, 1965; Timoshenko and Woinowsky-Krieger, 1984).

![Fig. 2. A small element of circular cylindrical shell.](image)

4. Analysis of Cylindrical Wall with Flexibility Theory

In Fig. 3, a cylindrical wall under the water loading with a fixed base supported and the unknown forces in the equivalent isostatic system can be seen.

The compatibility equations for the structure can be written as

\[
D_{10} + F_{11} X_1 + F_{12} X_2 = 0
\]

\[
D_{20} + F_{22} X_2 + F_{21} X_2 = 0
\]

(15)

In the equation, \(D_{10}, D_{20}\) are displacement terms; \(F_{11}, F_{12}, F_{21},\) and \(F_{22}\) are flexibility coefficients. The equations of these terms are as follows

\[
D_{10} = \frac{yr^2H}{Eh} \quad \text{and} \quad D_{20} = \frac{yr^2}{Eh}
\]

(16)

\[
F_{11} = \frac{1}{2\beta^2D}, \quad F_{22} = \frac{1}{\beta D} \quad \text{and} \quad F_{12} = F_{21} = -\frac{1}{2\beta^2D}
\]

(17)

where \(\gamma\) is liquid density and \(H\) is height of the wall. By implementing Eqs. (16-17) to Eq. (15), redundant forces can be obtained as
These two forces are those that only occur in the case of a cylindrical wall. In the case of a dome on the wall, the displacement (Eq. (19)) and flexibility (Eq. (20)) terms of the dome can be added to the wall terms and the result can be obtained easily.

\[
\begin{align*}
[ X_1 ] &= F^{-1} \left[ \frac{\gamma r^2 H}{E \cdot h} \right] \quad (18) \\
D_{10d} &= \frac{r_q^2 q}{E_d h_d} \left( \frac{1 + \nu_d}{1 + \cos \alpha} - \cos \alpha \right) \sin \alpha \\
D_{20d} &= \frac{r_q^2 q}{E_d h_d} (2 + \nu_d) \sin \alpha & (19)
\end{align*}
\]

In these equations, \( r_d, \nu_d, \alpha, E_d, \lambda_d, \) and \( h_d \) are radius, Poisson’s ratio, starting angle, elasticity modulus, a term that contains several properties of dome and thickness of the dome, respectively. After the values in the equation are calculated and added to the wall terms \( (F_s, D_{10s}, \text{and } D_{20s}) \), the unknown forces at the top of the wall can be calculated as

\[
\begin{bmatrix} X_3 \\ X_4 \end{bmatrix} = F_s^{-1} \begin{bmatrix} D_{10s} \\ D_{20s} \end{bmatrix} \quad (21)
\]

A further explanation for this application can be found in Billington (1965) and Kayabekir (2021).

5. Numerical Examples

In this section, optimization via metaheuristic-based optimization methodologies JA, JALS, JALS2, FPA, and TLBO are performed for 5 cases including different wall heights, from 5m to 7m with 0.5m increments. The optimization process of each case was done for 20 runs.

The optimum thickness of the wall \( (h_w) \) and statistical values of the objective function are given in Table 5. Table “Min. cost”, “Ave. cost”, “SD” and “Analyses num.” show minimum cost, average cost, standard deviation, and the average analysis number for which the optimum result is found respectively.

It can be seen from Table 5 that the optimum wall thickness increases as the wall height increases. Therefore, as the height increases, the cost also increases. This increase between two consecutive cases was approximately 12.43% - 16.28% for \( h_w \) and 21.88% - 26.85% for minimum cost. In addition, as the wall height increases, the differences between successive cases decrease.

Considering the \( h_w \) and minimum cost values calculated for each case, it is seen that the algorithms find approximately the same results.

According to statistical data, the standard deviation value for each algorithm is very close to zero. This situation shows that the algorithms find the nearly same minimum cost value in each run for a cylindrical wall problem having one design variable, that is, the algorithms provide stable results. For that reason, the average cost values are also similar.

Although the algorithms provide similar results, they reach optimum results at different analysis numbers. For all cases, JA reaches the optimum result very quickly with a 56.2-58.4 average analysis number. Hybrid algorithms, JALS (analyses number 177.45-206.2) and JALS2 (analyses number 178.3-201.3) have similar performance with FPA (analyses number 179.15-197.1). TLBO shows the lowest performance with 322.4-367 analyses number.
Table 5. Optimum results for 5 cases.

<table>
<thead>
<tr>
<th>Cases</th>
<th>Results</th>
<th>JA</th>
<th>JALS</th>
<th>JALS2</th>
<th>FPA</th>
<th>TLBO</th>
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<td>0.4747</td>
<td>0.4747</td>
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<td>131082.69</td>
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<td>59.7</td>
<td>59.7</td>
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6. Conclusions

In the present study, the performances of several classical and hybrid metaheuristic algorithms are compared on the optimum design of axially symmetrical cylindrical reinforced concrete walls with a dome on top. In the analysis, five different situations, three of which are classical and two of which are hybrid, of five different algorithms and different wall heights were examined. Algorithm performances were compared in terms of minimum, average, standard deviation values, and the number of analyzes in which optimum results were achieved by running the algorithms in large numbers for each case.

According to the analysis results, it is understood that all algorithms reach the optimum result. In addition, it is seen from the mean and standard deviation values of the repeated run results that the algorithms successfully perform the optimization process each time. According to these data, it can be said that all algorithms are used to give reliable and appropriate results for the optimization problem.

According to another comparison parameter, the number of analysis values, it is seen that the Jaya algorithm is the fastest in all cases. In addition, it is understood that the slowest algorithm is TLBO, although other algorithms are close to each other, the JALS algorithm is slightly faster in 3 out of 5 cases. These results show that the use of the Jaya equation for both the global equation of the FPA algorithm and the teacher phase of the TLBO algorithm contributes to the improvement of the performance of both algorithms.

In future studies, it will be useful to demonstrate the effectiveness of the methods by researching with a variety of problem and design variants.

References


