
Nonlinear Time-Dependent Behaviour of Concrete Insulated Panel

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Abstract

The insulated structural sandwich panel is a composite laminated structure, made of two stiff skin faces and lightweight thick core. The low-density low-shear rigidity laminated thick core (foam, honeycomb) is structurally connected to the laminated skin faces by structural adhesive. The rigid skin (concrete, steel, wood) faces provide high bending stiffness for such lightweight structure, while the core resists the local buckling of the skin faces. The time-dependent deflection behaviour of the concrete insulated panel (CIP) varies with the change of the span-to-depth ratio for the instantaneous deflection, and the cyclic change of temperature and relative humidity over time.

This paper discusses the nonlinear theoretical analysis for long-term creep deflection under sustained load that accounts for loads, time, temperature and relative humidity.

Keywords: Sandwich Structure; Concrete Insulated Panel, Time-Dependent; Nonlinear Modelling.

1. Introduction

The Insulated Panel Systems (IPS) is an engineered composite product composed of an insulating core i.e. foam core sandwiched to provide the insulation and rigidity, and two face-skin materials to provide durability and strength. The skin material may take the form of metal, concrete, or wood products [1]. The change of skin-faces types for the Insulated Metal Panel (IMP) leads to different commercial products like: Steel Insulated Panel (SIP) using galvanized steel sheets, or aluminium sheets. IPS may use concrete to form Concrete Insulated Panel (CIP), Insulated Precast Concrete (IPC), or use wood products to form Structural Insulated Panel (SIP), and Permanent Wood Foundation (PWF). All IPS are structurally compared to I-Beam where the foam core acts the web, while the skins act as the flanges. In case of flexural loading, all elements of the IPS are stressed; the skins are in tension and compression, while the core resists shear and buckling. Under in-plane axial loading, the facing of an IPS acts as slender

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columns, and the core stabilizes the facing and resists forces that may causes local buckling of the facings. This Insulated Panel Systems (IPS) can be used in industrial, commercial and residential applications. They have merits over the conventional construction due to energy saving insulation, design capabilities, cost effectiveness, speed of construction and exceptional strength make IPS the future material for high performance buildings [2]. The insulated core usually made of expandable polystyrene (EPS) foam that are intended to be used as thermal insulation for temperature -53.9 to $+73.9$ °C [3]. The ASTM C578-12 provided the material specifications for the insulated core materials [4].

Concrete Insulated Panel (CIP) which its skin faces are made of reinforced concrete or fibre reinforced concrete (FRC) materials laminated to flexible insulated foam core increases the challenge to predict the structural behaviour. The rate of compressive strength of the concrete increases with the increase of time [5], however this doesn't allow the occurrence of the creep. The surrounding environment of cyclic relative humidity and temperature influences the rate of creep over time. ACI 209R defined the creep (time-dependent) as a constant stress under condition of steady relative humidity and temperature, assuming the strain at loading (nominal elastic strain) as the instantaneous strain at any time [6]. Both ASTM D2990-09 [7] and ASTM D6112-10 [8] divided the creep deflection into three phases; the primary creep which occurs directly after applying the sustained load and has high strain rate; the secondary creep is increasingly slow over time, with minimum strain rate due to thermal softening; the third phase is the tertiary creep where strain rate exponentially increase to failure.

Sandwich panels have flexible cores [9], therefore their behaviour is more complex than that of the plain plates and it is important to understand the numerous failure modes of sandwich panels so that appropriate design criteria can be developed. In addition to face buckling, the other possible modes of failure are as follow: (1) failure of connection; (2) crushing of the sandwich panel at a point of support or line load; (3) yielding of metal face in tension; (4) shear of the core, including shear bond failure; (5) failure of the sandwich panel at a point of connection; and (6) blistering [10].

This paper provides a simplified theoretical analysis to calculate the time-dependent deflection that accounts for sustained load, the nonlinear viscoelastic behaviour, and the nonlinear viscoelastic behaviour that accounts for temperature and relative humidity over time. The ordinary beam theory is used to analyse the long-term flexural behaviour for the Concrete Insulated Panel.

2. Sandwich Beam Application of Ordinary Beam Theory

The sandwich beam is illustrated in Fig. 1 and consists of two thin skins each of thickness t , separated by low density core of thickness c . The overall depth of the sandwich beam is h and width is b . All the three laminated layers are firmly bonded by structural adhesive, where this behaves as isotropic section [11].

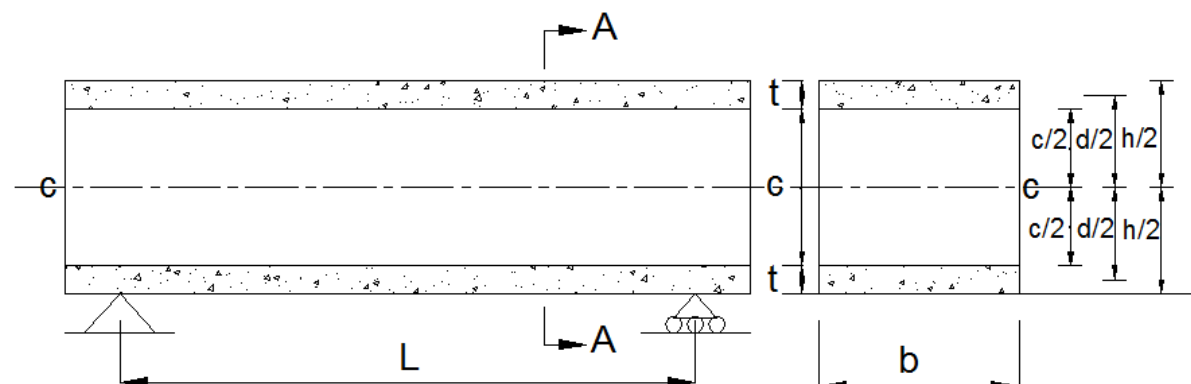


Fig. 1: SANDWICH BEAM. SECTION AA ON RIGHT [11]

The stresses and flexural deflection of such a beam is approximated by the using of the ordinary beam theory that assumes that the flexural rigidity is commonly referred to as D and can be defined as the sum

of the flexural rigidities of the faces and the core measured about the neutral axis of the sandwich cross-section. Allen (1969) has defined the flexural rigidity for a narrow sandwich beam (transverse stresses in the y direction are assumed to be zero) as follows [11].

$$D = E_f \frac{bf^3}{6} + E_f \frac{bfd^2}{2} + E_c \frac{bc^3}{12} \quad (\text{EQ.1})$$

Where: E_f = modulus of elasticity of the concrete facing material follows the CSA A23.4, *Design of Concrete Structures* [12]; E_c = modulus of elasticity of core material follows CAN/ULC-S701 [13] and ASTM C578 [3]; D = sandwich flexural rigidity ($D = EI$); b = specimen width; c = core thickness; f = facing thickness; and d = distance between neutral axis of faces ($c + f$ for equal facing thicknesses)

3. Short-Term Deflection

3.1. Deflection limitation

Table 1 provides the load combination for the ultimate limit state and allowable span-to-deflection ratio as permitted in the National Building Code of Canada (NBC 2012) [14]. The instantaneous deflection under the serviceability limit state (SLS) must be less than the permitted span/360 ratio for the beam action.

Table 1: Code Regulations [14]

| Line | Element Type | Allowable SLS Span-to-deflection ratio | Load Combination for Ultimate Limit State (ULS) |
|------|--------------|---|--|
| 1 | Beam | Span / 360 | 1.25D + 1.5L |
| 2 | Wall | Height/700* | 1.25D+1.5L+0.4W |

(*) Common practice

3.2. Instantaneous Deflection

The instantaneous deflection is the summation of deflection due bending and shear deflection as described into EQ. 2. Where Δ_B = deflection at mid-span of the sandwich panel due to bending, and Δ_S = deflection at mid-span of the sandwich panel due to shear and limited to the allowable deflection per the serviceability limit deflection (SLS) in design codes.

$$\Delta_o = \Delta_B + \Delta_S < \frac{\text{Span}}{360} \quad (\text{EQ. 2})$$

Thus, the total sandwich beam deflection reflecting the bending and shear component is defined in by EQ. 3. Where P = total applied load; L = beam span; E = modulus of elasticity of the beam material; I = moment of inertia of the uniform cross-section; EI = flexural rigidity; $A = bd^2 / c$ and AG is referred to as the shear stiffness; and G = core shear modulus

$$\Delta_o = \frac{11 P L^3}{384 D} + \frac{PL}{8 AG} < \frac{\text{Span}}{360} \quad (\text{EQ.3})$$

4. Total Deflection

4.1. Viscoelastic creep model

Creep-strain response for CIP is viscoelastic, where represented by elastic spring and viscous dashpot. Viscous flow to ideal fluid requires rate of strain with respect to time be proportional to the applied stress, obeying Newton's law, while plastic deformation is due irreversible changes of position, where strain does not change when the stress is removed.

$$\sigma \propto \frac{d\varepsilon}{dt} = \eta \frac{d\varepsilon}{dt} \quad (\text{EQ.4})$$

$$\varepsilon = \frac{\sigma t}{\eta} \quad (\text{EQ.5})$$

Where E is the modulus of elasticity, σ is the stress, ε is the strain, η is viscosity and t is the time. The rheological models are illustrated in Fig.2 by Kelvin-Voigt (solid) and Maxwell (fluid). Maxwell body is a dashpot and spring in series, while Kelvin-Voigt body is a dashpot and spring in parallel. Maxwell and Kelvin-Voigt are special cases of Kelvin. Kelvin body is determined by Inverse Laplace Transform through the following relation, the two parameters K_e and η_v can be determined by the using of Marquart-Levenberg algorithm (Nonlinear Least Squares Regression) in connection with experimental data [15]. Table 2 shows the simplified long-term deflection Δ_f equation forms for the mathematical models represented by the Power and the Logarithmic models, and mechanical model as shown in Fig. 2 representing Maxwell, Kelvin, Zener and Burgers models.

Table 2: Mathematical and Viscoelastic Creep Models [16] and [17]

| Model | Equation |
|--------------------------------------|---|
| Power | $\Delta_f = \Delta_o + A_1 t^{A_2}$ |
| Logarithmic | $\Delta_f = \Delta_o + A_1 \ln(t + 1)$ |
| Maxwell | $\Delta_f = \frac{L}{A_1} \left(1 + \frac{t A_1}{A_2} \right)$ |
| Kelvin | $\Delta_f = A_1 [1 - \exp(-A_2 t)]$ |
| Standard Linear Solid (SLS) or Zener | $\Delta_f = \Delta_o + A_1 (1 - \exp(-\frac{t}{A_2}))$ |
| Burgers | $\Delta_f = \Delta_o + A_1 [1 - \exp(-A_2 t)] + A_3 t$ |
| Refined Burgers model | $\Delta_f = \Delta_o + A_1 (1 - \exp(-A_2 t)) + A_3 t^{A_4}$ |

Where; $\Delta(t)$ = total time dependent deflection; Δ_o = initial deflection; A_i = creep parameters associated with creep deflection equations; L: sustained load.

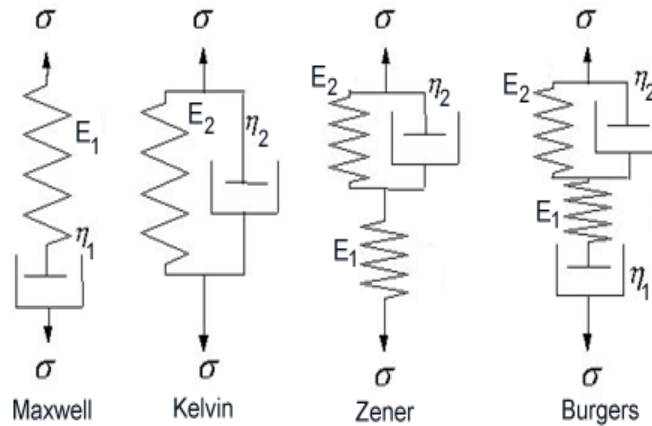


Fig. 2: COMMONLY USED CREEP MODELS FOR A VISCOELASTIC MATERIAL [18]
(σ is stress; E_1, E_2 are the elastic modulus (E) represented as spring; η_1, η_2 are the viscosity represented as dashpot)

4.2. Influence of temperature and humidity on creep

In addition to the influence of creep the surrounding medium has a more direct “environmental” influence on creep. The relative humidity affects the moisture content presents into the concrete

influences the magnitude of the creep. Temperature is the second major environmental factor in creep; it affects the dimensions of concrete. Usually the temperature factor is neglected but at hot weather accompanied by higher rate of relative humidity it influences the creep rate [19]. Sayed-Ahmed and Sennah (2013) developed the Arrhenius model to account for the time-dependent deflection in addition to the humidex which is the combination of temperature and relative humidity over time [20].

$$\Delta_{f,t} = \Delta_0 + A_1 L e^{-A_2/A_3(T+\theta)t^{A_4}} \quad (\text{EQ.6})$$

Where Δ_0 = instantaneous deflection ($\Delta_0 = \sigma/E$), A_1, A_2, A_3, A_4 = creep constants, L = applied load, T = temperature, θ = relative humidity, t = time, n = constant, and $\Delta_f(t)$ = final deflection. Microsoft Excel has the Solver tool that utilizes the Generalized Reduced Gradient (GRG) algorithm to minimize the summation square of error (SSE) to obtain the creep constants with high value of the coefficient of determination (R^2) that falls between zero and one ($0 < R^2 < 1.0$) as a level of trust toward the one value.

4.3. Fractional Creep

The fractional creep K_f is the ratio of the final deflection to the instantaneous deflection. It should be less than 2.0 after 90 days [21]. To determine the creep rate, the change of creep deflection shall be segmented every month, where the creep rate can be expressed as $\Delta_{30} - \Delta_0 > \Delta_{60} - \Delta_{30} > \Delta_{90} - \Delta_{60}$ where Δ_0 is the instantaneous (initial deflection), and $\Delta_{30}, \Delta_{60}, \Delta_{90}$ are the deflection measured on 30th, 60th, and 90th days respectively. On the long-term of serviceability the fractional creep should be less than 4.0 at any time otherwise the sandwich beam will fail under flexural load. However to obtain the failure fractional creep, the sandwich panels should be tested to-collapse under incremental step flexural loading.

$$K_f = \frac{\Delta_{f,t}}{\Delta_0} \quad (\text{EQ.7})$$

4.4. Creep Constant

Flexural creep is defined as deflection under constant load over period of time beyond the initial deformation due to the application of load. ASTM C480-08, *Standard Test Methods of Flexural Creep of Sandwich Construction* (2008) covers the determination of the creep rate of sandwich panel under constant flexural load [22]. In case of the floors typical test consists of a simply-supported panel loaded uniformly distributed load along the panel. Thus, the CIP is subjected to one-dimensional flexure thereby minimizing the influence of transverse stiffness as specified in ASTM E72-02, *Standard Test Methods for Conducting Strength Tests of Panels for Building Construction* [23], as well as, ICC-ES AC04, *Acceptance Criteria for Sandwich Panels* [24].

$$K = (\Delta_{f,t}/\Delta_0) - 1 \quad (\text{EQ.8})$$

4.5. Total Long-Term Deflection

The total deflection accounts for the creep constant is multiplied by the long-term deflection and then summed to the instantaneous deflection.

$$\Delta_{total} = K (\Delta_{f,t}) + \Delta_0 \quad (\text{EQ.9})$$

5. Conclusions and Discussions

The application of Concrete Insulated Panels (CIP) has good future however the evaluation of a full-scale structure by loading should be carried over with simulation to the cyclic environmental changes of

temperature and relative humidity in addition to the sustained load. This paper provided simplified procedure for the nonlinear time-dependent analysis of structural sandwich panel, adding more simplified equation to determine the humidex influence of creep by using EQ. 6. It is to note that the concrete mix, raw materials and contents rapidly change every day, the creep parameters will vary and will need more investigation.

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