

On the strength prediction in concrete construction based on earlier test results: Case studies

Hossein Akbarzadeh Kasani ^a

^a Faculty Member & Chair, Department of Civil Engineering, Institute of Applied Science and Technology (ACECR), Rasht, Guilan, IRAN

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Abstract

Early prediction of strength is crucial in efficient planning for concrete construction projects. There are several empirical correlations which allow estimation of concrete strength from early age results. However these correlations have limitations in application. This study established an experimental database which comprised of 382 datasets of strength tests of ordinary Portland cement concrete. These tests were performed over a period of 8 years as part of QA/QC program on 51 construction projects in the Province of Guilan, Northern Iran. From the data, strength ratios between ages (28 and 7 days), (42 and 7 days), (42 and 14 days), and (42 and 28 days) were analysed. New linear and power relations were proposed for estimating 28- and 42-day strength values. Analyses of relative errors along with cumulative probability approach revealed that three well-known models from literature tend to over- or under-predict strength for the current database. It was found that a correlation by Slater (1926) over-predicted 28-day strength from 7-day test data. Furthermore, the ACI committee 209 (1997) and CEB-FIP (1990) models under-predicted 42-day strength using 28-day strength results. This research should assist in a worldwide, yet simple, understanding of strength development of ordinary Portland cement concrete with age.

Keywords: Concrete; compressive strength; strength prediction; age; cumulative probability.

1. Introduction

Compressive strength of standard concrete cylinder (diameter 150 mm; height 300 mm) at 28 days has been adopted by various standards as a strength basis for structural design in concrete construction [1-2]. Fast construction has necessitated early estimation of this strength. The urge is, in particular, due to the need for early stripping off the formworks and preventing non-working days. ACI committee 209 [3] recommended Equation 1 for prediction of compressive strength of the standard concrete cylinder made with ordinary (ASTM Type I) Portland cement which is moist-cured and tested in a standard condition (as for example, in accordance with ASTM C 192-00 [4] and ASTM C 39-99 [5]). In Equation 1, compressive strength at an age of t days (f_{ct}) is determined as a function of the compressive strength at age of 28 days (f_{c28}).

$$f_{ct} = f_{c28} \left(\frac{t}{4+0.85t} \right) \quad (1)$$

^a Present Address: Department of Civil and Environmental Engineering, University of Alberta, Edmonton, Alberta, Canada, T6G 1H9, Email: akbarzad@ualberta.ca
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The CEB-FIP Model Code [6] proposed Equation 2 for concrete specimens made with normal hardening cement and cured at 20 °C. In this equation $t_1 = 1$ day.

$$f_{ct} = f_{c28} \exp \left[0.25 \left(1 - \sqrt{\frac{28}{t/t_1}} \right) \right] \quad (2)$$

Historically, the 28-day strength was correlated to earlier age strengths such as those of 7 and 14 days that resulted in different relationships in literature. Some of those relationships, particularly for concrete specimens made with ordinary Portland cement, can be found in [7-10]. Equation 3, developed by Slater [7], is one of the earliest correlations in this regard. In Equation 3, strengths are in MPa. Ranges of 7- and 28-day strength of the standard concrete cylinders in the Slater's database were 2.5~22 MPa and 5.5~31 MPa, respectively.

$$f_{c28} = f_{c7} + 2.49\sqrt{f_{c7}} \quad (3)$$

Pineiro (1963) proposed a power relation between f_{c28} and f_{c7} shown in Equation 4 (as mentioned by Neville [11]).

$$f_{c28} = k_2 f_{c7}^{k_1} \quad (4)$$

where k_1 and k_2 are coefficients dependent on cement type and curing conditions. The value of k_1 varies from about 0.3 to 0.8 while k_2 ranges from 3 to 6 [11].

Neville (1996) stated that f_{c28} varies between 1.3 to 1.7 times of f_{c7} with average of 1.5 [11]. The corresponding ratios from Equations 1 and 2 for a moist-cured specimen are 1.42 and 1.28, respectively. Numerous research studies were conducted to predict f_{c28} based on concrete mix properties [12-14]. Others utilized advanced computational techniques such as Neural Network, Genetics Algorithm, and Fuzzy Logic to correlate f_{c28} strength to those of earlier ages and/or mix properties [15-17].

Among different models mentioned above, those with simple forms have particular significance. They are simple in application; furthermore, they require limited information to provide a reasonable estimation of the strength. However, the above correlations may result in different predictions of the strength in locations other than where they were originally developed, even using ASTM Type I Portland cement. This discrepancy could be a consequence of using aggregates having different mineralogy as well as difference in preparation of concrete. A previous study [18] showed that some of the well-known correlations from literature such as [3, 6-7] may not fit the data from Iran. The objective of this study was to develop relations between compressive strengths of concrete utilized in construction projects, as well as to examine accuracy of the aforementioned correlations.

2. Experimental database

The data acquired in current research consisted of concrete tests conducted by the Concrete Technology & Soil Mechanics Laboratory of the Institute of Applied Science and Technology (ACECR) where the author collaborated with them for several years. The database included samples which were taken for QA/QC purposes on 51 construction projects in the Province of Guilan, Northern Iran, between 1998 and 2006. The construction projects included residential buildings, government buildings, bridges, roads/highways medians and curbs, and so on. For current study, only ordinary concrete specimens made of ASTM Type I Portland cement (with no admixture or air-entrainment agent) were selected. It included both ready-mixed concrete, supplied by various suppliers in the province, and site-mixed concrete, prepared on job sites. Therefore, the database contained possible effects of heterogeneity of aggregates in terms of maximum size effect, differences

in mineralogy, as well as variations of on-site preparations. However, no specific information about the mix was provided to this laboratory.

In total, 382 experimental sets of ordinary concrete were tested in the time period mentioned. Each dataset included 5 concrete cubes. All specimens were compacted by hand rodding in steel molds and were kept moist on the job sites for about 24 hours. Thereafter they were transported to the laboratory, demolded and kept moist in the laboratory until the day of testing. Compressive strength of two specimens from each sets were tested after 7 days, another two after 28 days, and one cube tested at an age of 42 or 90 days. In 100 cases, as per clients' specifications, two cubes were tested at an age of 14 days and another two at 42 days, and one cube at 56 or 90 days. Using the local standard [2], average strengths of the 15 cm × 15 cm × 15 cm cubes were determined at each age and converted to strength of the standard cylinder; hence, all the analyses presented herein are based on the standard cylinder.

3. Results and discussions

3.1. Strength ratios

Table 1 presents statistical analyses for strength ratios between various ages such as (27 and 8), (42 and 7), (42 and 14), and (42 and 28) days. In the database f_{c28}/f_{c7} varied from 1.20 to 2.15 with a mean value equal to 1.56. The range was broader than those reported by Neville [11], however, the mean value was very close to the value of 1.5 reported by [11]. The ratio f_{c42}/f_{c7} was observed between 1.25 and 2.60 while having a mean value equal to 1.86. The ratio f_{c42}/f_{c14} has not commonly been reported in literature. In the database, this ratio was observed between 1.08 and 1.86 for 100 datasets. The mean value was 1.29. Ratio of 42-day to 28-day strength varied from 1.02 to 1.58, with a mean value of 1.20. The observed mean value for f_{c42}/f_{c28} was slightly larger than the corresponding values predicted by Equations 1 and 2 (which are 1.06 and 1.05, respectively).

Table 1: Statistical analyses for strength ratio between various ages in the database.

Strength ratio	No. of data points	Min	Max	Mean	Median	Standard deviation
f_{c28}/f_{c7}	270	1.20	2.15	1.56	1.54	0.18
f_{c42}/f_{c7}	82	1.25	2.60	1.86	1.84	0.26
f_{c42}/f_{c14}	100	1.08	1.86	1.29	1.27	0.12
f_{c42}/f_{c28}	80	1.02	1.58	1.20	1.19	0.11

3.2. Proposed linear and power relations

In this study, two groups of regression equations were investigated; for estimation of f_{c28} and f_{c42} . Linear and power relations were developed for each group. The data points for compressive strengths at 7 and 28 days are shown in Figure 1. The 7- and 28-days strength data were in the range of 2.1~27.6 MPa and 3.9~41.6 MPa, respectively.

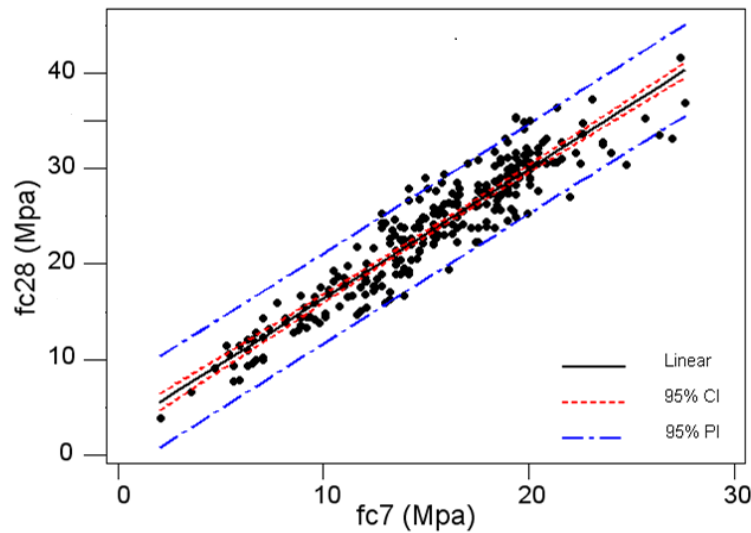


Figure 1. Fitted linear relation with 95% confidence interval and 95% prediction interval for data points of f_{c28} and f_{c7} .

Variations of f_{c42} with 7-, 14-, and 28-day strength are shown in Figure 2.a, 2.b, and 2.c, respectively. Also presented in these figures are the best fitted linear relations with their corresponding 95% Confidence Interval (CI) and 95% Prediction Interval (PI). Number of data points in each figure is the same as for the strength ratios, listed in Table 1.

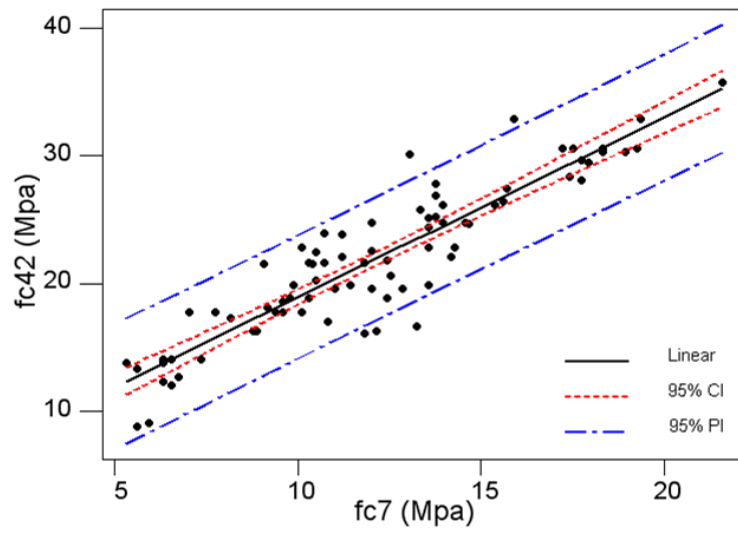
Table 2: Fitted linear and power relations for prediction of f_{c28} .

Equation	Equation No.	No. of data points	R ² *
$f_{c28} = 1.35f_{c7} + 2.90$	5	270	0.88
$f_{c28} = 2.10f_{c7}^{0.886}$	6	270	0.91

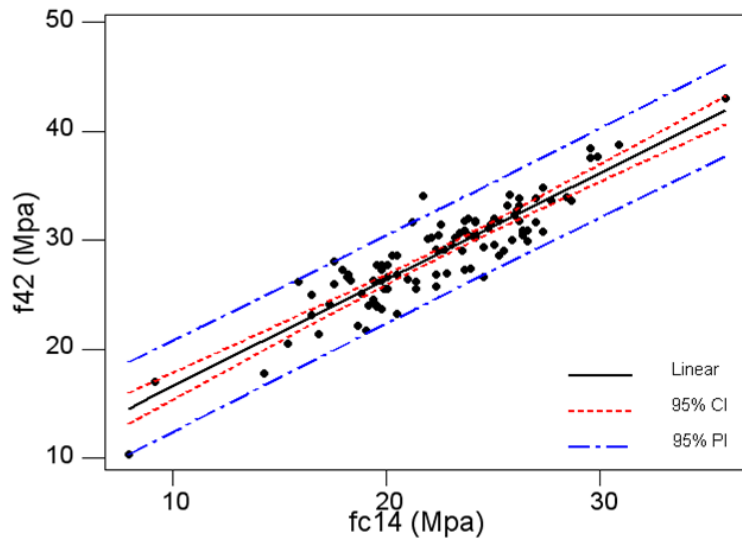
* Correlation coefficient

Linear correlations corresponding to Figures 1 and 2 are summarized in Tables 2 and 3, respectively. Table 2 contains linear and power relations for prediction of f_{c28} as a function of f_{c7} (Equations 5 and 6, respectively). Both linear and power relations exhibited high coefficients of correlation. The values of coefficient k_1 and k_2 in the power function of Equation 6 were close to those suggested by Pineiro [11].

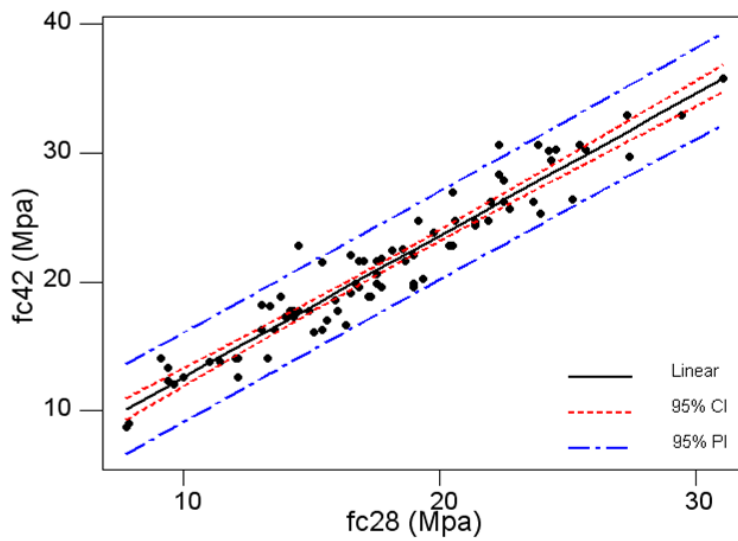
Several relations were deduced for f_{c42} prediction (Equation 7 to 13, as summarized in Table 3). Data of f_{c7} was used as independent variable in establishing linear relation (Equation 7) and power relation (Equation 8) for estimation of f_{c42} . The 100 datasets of f_{c42} and f_{c14} resulted in linear and power relations of Equations 9 and 10, respectively. Current study concluded strong linear and power relations between f_{c42} and f_{c28} (Equation 11 and 12, respectively). The multiple linear regression of Equation 13 was fitted to the 7-28-42-day strength data. Equation 13 was the strongest correlation with the highest correlation coefficient amongst the relations developed for f_{c42} .



a)



b)



c)

Figure 2. Fitted linear relation with 95% confidence interval and 95% prediction interval for data points of f_{c42} versus: a) f_{c7} , b) f_{c14} , and c) f_{c28} .

Table 3: Fitted linear and power relations for prediction of f_{c42} .

Equation	Equation No.	No. of data points	R ²
$f_{c42} = 1.41f_{c7} + 4.88$	7	82	0.84
$f_{c42} = 3.13f_{c7}^{0.781}$	8	82	0.82
$f_{c42} = 0.98f_{c14} + 6.77$	9	100	0.82
$f_{c42} = 2.65f_{c14}^{0.767}$	10	100	0.83
$f_{c42} = 1.10f_{c28} + 1.64$	11	80	0.92
$f_{c42} = 1.51f_{c28}^{0.917}$	12	80	0.92
$f_{c42} = 0.25f_{c7} + 0.93f_{c28} + 1.75$	13	77	0.92

4. On the accuracy of Slater [7], the ACI committee 209 [3] and CEB-FIP [6] models

4.1. Relative error in prediction

Application of Equation 3 from Slater [7] to the 270 data points of f_{c28} and f_{c7} resulted in -15.28% to 53.46 % relative error in estimation of the 28-day strength. The mean error was 8.23% whereas the corresponding standard deviation was 12.26%. This indicates that Slater's correlation over-predicted f_{c28} .

Equation 1 by ACI committee 209 [3] and Equation 2 proposed by CEB-FIP [6] were employed to estimate f_{c42} from f_{c28} data. Minimum, maximum, mean value and standard deviation of relative error in predictions by using Equation 1 were -32.94%, 3.84%, -11.09%, and 7.68%, respectively. The corresponding values in predictions by using Equation 2 were -33.63%, 2.76%, -12.01%, and 7.60%, respectively. The conclusion is that both the ACI committee 209 [3] and CEB-FIP [6] models under-predicted f_{c42} .

4.2. Cumulative probability approach

In this method, comparison between the proposed models and those from literature was done based on probability distribution of their predictions. The normalized strengths, i.e. values of predicted to measured compressive strength (f_{cp}/f_{cm}) were plotted against their cumulative probabilities. For a set of n observations, $(f_{cp}/f_{cm})_1, (f_{cp}/f_{cm})_2, \dots,$ and $(f_{cp}/f_{cm})_n$ sorted in ascending order. The cumulative probability for the i -th value of f_{cp}/f_{cm} was calculated as $P_i = m/n + 1$, [19]. Eventually data points of f_{cp}/f_{cm} were plotted against their associated cumulative probability values. The value of f_{cp}/f_{cm} at $P = 50\%$ was a measure of over-prediction or under-prediction of the predictive correlation. If the $f_{cp}/f_{cm} > 1$, the model over-predicted and vice versa. Dispersion in the prediction was assessed by slope of the line through the data points. The flatter the line was, the less dispersion existed.

Cumulative probability plot for prediction of f_{c28} is presented in Figure 3. Included in Figure 3 are the proposed linear and power relations of Equations 5 and 6 as well as Equation 3 by Slater [7]. It can be concluded from Figure 3 than the Slater's correlation over-predicted f_{c28} . This finding confirmed the same observation in terms of relative error of prediction that was already discussed in 3.3.1.

Figure 4 shows cumulative probability plot for prediction of f_{c42} . Shown in Figure 4 are linear models of Equations 7, 11, and 13 along with the ACI committee 209 [3] model (Equation 1) and the CEB-FIP [6] model (Equation 2). Equation 13 exhibited the best prediction. The ACI committee 209

[3] and CEB-FIP [6] models under-predicted the f_{c42} . This complied with their negative relative errors in prediction of the strength, discussed in 3.3.1.

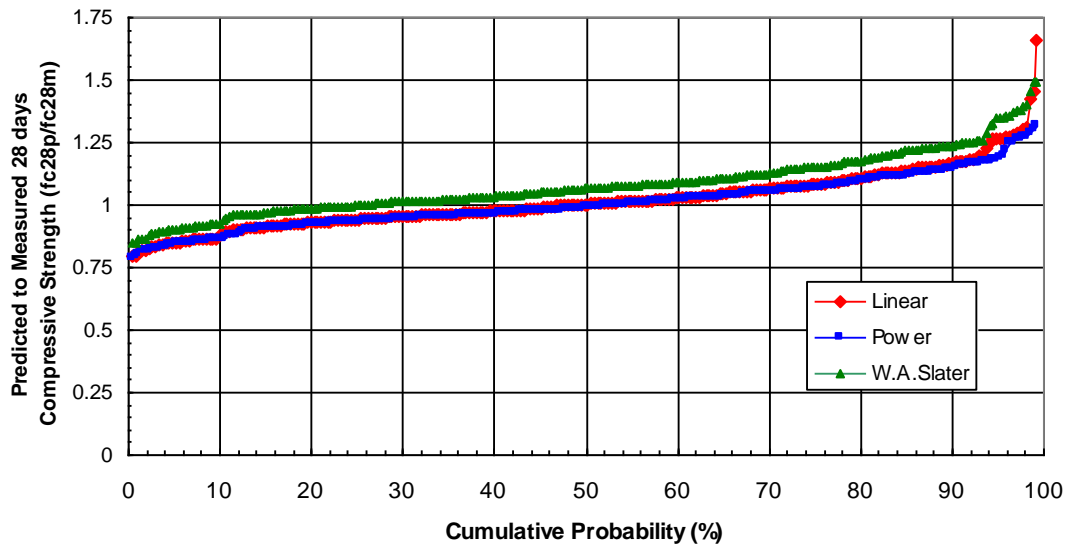


Figure 3. Comparison between proposed linear and power relations (Equations 5 and 6) and Slater’s model in prediction of f_{c28} using cumulative probability approach.

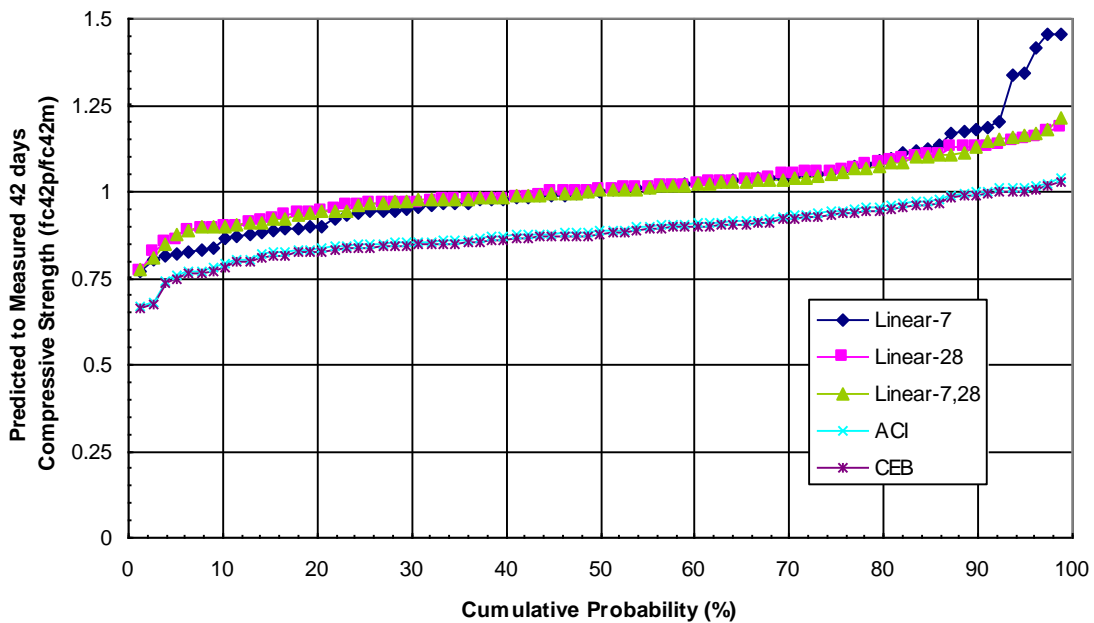


Figure 4. Comparison between proposed single and mutple linear relations (Equations 7, 11, and 13) and the ACI committee 209 and CEB-FIP models in prediction of f_{c42} using cumulative probability approach

5. Conclusions

This study analysed 382 datasets of strength test, performed over a period of 8 years on samples of ordinary concrete taken from 51 construction projects in the Province of Guilan, Northern Iran. The following conclusion can be made based on this research:

- Strength ratios between ages (28 and 7 days), (42 and 7 days), (42 and 14 days), and (42 and 28 days) were determined. The ratio f_{c28}/f_{c7} varied from 1.20 to 2.15 with a mean value of 1.56. The ratio f_{c42}/f_{c7} was between 1.25 and 2.60 with a mean value of 1.86. f_{c42}/f_{c14} ranged from 1.08 to 1.86 (mean value 1.29). The corresponding range for f_{c42}/f_{c28} was 1.02-1.58 and the corresponding mean value was 1.20.
- New linear and power relations with high correlation coefficients were proposed for $(f_{c28} - f_{c7})$, $(f_{c42} - f_{c7})$, $(f_{c42} - f_{c14})$, $(f_{c42} - f_{c28})$. A multiple linear regression function was also suggested for f_{c42} as a function of both f_{c7} and f_{c28} .
- Two statistical methods, namely relative error and cumulative probability approach, were employed for assessing accuracy of 3 models from literature in predicting compressive strength based on early age test results. It was observed from the two methods that the Slater's correlation over-predicted f_{c28} from f_{c7} test data. Furthermore, the ACI committee 209 and CEB-FIP models under-predicted f_{c42} using f_{c28} strength results.

6. Acknowledgements

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