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Review

Finite element method with energy minimization (FEMEM) in structural analysis: State-of-the-art

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ABSTRACT

The Finite Element Method (FEM) is a powerful and widely adopted numerical technique in structural engineering for the analysis of structures and systems. Although FEM is highly effective for linear problems and can address certain classes of nonlinear problems through appropriate iterative schemes, there exist structural analysis cases that cannot be satisfactorily solved using conventional FEM formulations. Such cases include problems with non-unique equilibrium states, nonlinear or missing boundary conditions, and structures with ill-conditioned flexibility matrices, for example truss systems following the failure of one or more members. To overcome these limitations, an emerging approach—the Finite Element Method with Energy Minimization (FEMEM)—has been introduced. FEMEM is based on the same fundamental principles of structural mechanics as FEM; however, it differs in its solution strategy. Instead of solving matrix equations, FEMEM formulates the problem as the minimization of a functional, namely the total potential energy of the structure. In this paper, the fundamental concepts of FEMEM are first introduced, followed by a comparative discussion with classical FEM to highlight the advantages and limitations of the method. Subsequently, representative applications of FEMEM are presented for various structural systems, ranging from trusses and truss-like structures to planar and three-dimensional structural models. Finally, directions for future research are discussed, with emphasis on extending the applicability of FEMEM to more complex structural systems and a wider range of nonlinearities, including material, geometric, and constraint-related effects.

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1. Introduction

The Finite Element Method (FEM) has been, for more than half a century, an indispensable technique in the field of structural mechanics and is widely regarded by structural engineers as a principal tool for solving structural analysis problems. In engineering practice, FEM enables virtual structural models to incorporate loads, constraints, boundary conditions, material definitions, and, when required, dynamic loading scenarios such as ground motion records, allowing stress distribution, deformation, and overall structural response to be evaluated before or alongside experimental testing (Zardari et al. 2024; Alshabrawi and Uysal 2025). Following the emergence of FEM (Gupta and Meek 1996; Clough 2004; Sabat and Kundu 2020), most previously developed methods have largely been abandoned in practical applications or retained mainly for their historical and theoretical significance. The theoretical foundations and numerical formulation of FEM have been extensively mentioned in the literature. (Zienkiewicz et al. 2005; Reddy 2019; Chandrupatla and Belegundu 2021).

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Classical FEM is particularly effective for solving linear structural analysis problems. However, nonlinear finite element formulations and iterative schemes have been developed to address material and geometric nonlinearities, including problems involving complex stress redistribution, cracking, and failure mechanisms (De Borst et al. 2012; Reddy 2014; Belytschko et al. 2014; Polat and Karaman 2025). In such applications, experimentally determined material properties can be incorporated into nonlinear finite element models to represent tensile and compressive responses, stiffness degradation, and post-peak behavior more realistically (Tarhan and Tarhan 2025). Similarly, experimentally validated finite element models are frequently used to simulate dynamic structural responses involving impact loads, large displacements, contact interaction, plastic deformation, and energy absorption (Kandil et al. 2025). Together, these applications show that nonlinear FEM is powerful, but it often requires careful material idealization, boundary condition definition, contact/interface definition, and incremental solution control, especially for systems involving material nonlinearity, large displacements, sliding, and strength degradation (Ozdemir 2025).

Despite these significant advances, it has become evident that not all structural statical analysis problems can be solved using even the most sophisticated variants of FEM. A simple illustrative example is that of a simply supported horizontal bar subjected to vertical loads, where one of the supports fails. From the perspective of energy minimization, the resulting equilibrium configuration is evident and trivial. However, if the problem is approached through the derivation and solution of matrix equilibrium equations, as is customary in FEM, no solution can be obtained due to the loss of stiffness and the resulting singularity of the system.

A second example illustrating the limitations of the FEM can be drawn from progressive failure analysis. In certain cases, changes or increases in loading may lead to partial failures of structural members such that the flexibility matrix becomes ill-conditioned, even in the absence of total structural collapse. Under these conditions, the FEM may fail to determine the updated structural configuration. Additional limitations arise in the presence of unilateral boundary conditions, as inequality constraints cannot be directly incorporated into standard FEM formulations. Similarly, structures containing members with nonlinear stress–strain relationships cannot be solved directly using conventional FEM approaches; instead, such problems require enhanced solution strategies involving iterative procedures and repeated trial-and-error analyses.

The Finite Element Method with Energy Minimization (FEMEM), first introduced in 2004, builds upon the fundamental concepts of FEM by discretizing the structural system into finite elements, but departs from the classical approach by focusing directly on the potential energy stored in the elements rather than on the formulation of equilibrium equations. By adopting this energy-based perspective, FEMEM enables the analysis of certain classes of problems that are difficult or impossible to address using classical FEM or nonlinear FEM formulations. To date, numerous studies have demonstrated the applicability of FEMEM to such problems; nevertheless, further developments are required to extend its applicability to a wider range of structural systems and nonlinear behaviors.

It should be emphasized that, although classical FEM is also rooted in variational principles and the minimum potential energy concept, the minimization is usually performed implicitly through the derivation of Euler–Lagrange equations, which ultimately leads to a system of algebraic equilibrium equations in matrix form. The solution process therefore relies on the existence and numerical stability of the global stiffness matrix. In contrast, FEMEM adopts a fundamentally different computational strategy by performing the minimization of the total potential energy explicitly and directly, without deriving equilibrium equations or assembling stiffness matrices. The nodal degrees of freedom are obtained as the solution of an optimization problem, rather than a system of linear or nonlinear algebraic equations. This distinction is particularly important for structural problems involving non-unique equilibrium states, singular or ill-conditioned stiffness matrices, or unconventional boundary conditions, where classical variational FEM formulations may fail.

The remainder of this paper is organized as follows. Section 2 presents the fundamental principles of FEMEM, highlighting its similarities to and differences from classical FEM. Section 3 provides a comparative discussion of the advantages and disadvantages of FEM and FEMEM. Section 4 reviews applications to trusses and truss-like structures, while Section 5 focuses on plane stress and plane strain problems. Applications to three-dimensional volumetric structures are discussed in Section 6. Finally, conclusions and directions for future research are presented.

This state-of-the-art article aims to review and present research conducted to date that adheres to the fundamental principles of the Finite Element Method while being formulated through the minimization of total potential energy rather than through conventional matrix-based equations. Representative studies are discussed, ranging from trusses and truss-like systems to plate elements and fully three-dimensional structures. It is demonstrated that, in cases where both FEM and FEMEM are applicable, the two approaches yield identical results. However, there exist classes of problems that cannot be solved by FEM under standard formulations, or only with significant algorithmic modifications, whereas such problems can be addressed naturally and efficiently using FEMEM without additional computational effort. As an emerging numerical technique, FEMEM is expected to broaden its range of applications, particularly in the analysis of nonlinear systems for which FEM does not provide straightforward or robust solutions.

2. Comparison of FEM and FEMEM

The initial stages of FEM and FEMEM are identical. Both methods begin with modeling of the structure, discretizing it into finite elements, selecting the nodes at which the displacements are defined, and applying external loads. The unknowns of the problem are the nodal displacements associated with the chosen discretization.

These steps correspond to the first four stages summarized in Table 1. The fifth step is also common to both methods and involves the selection of appropriate shape functions consistent with the number of unknowns and the assumed linear or nonlinear behavior of each element (Zhu 2018).

The difference between FEM and FEMEM starts at stage 6 (Table 1). In applications for FEM, equilibrium equations are derived for each finite element and expressed in matrix form using local coordinates, which are subsequently transformed into the global coordinate system. In contrast, FEMEM does not formulate equilibrium equations at the element level; instead, total potential energy stored in each finite element is expressed to be minimized. In both cases the unknowns are the field variables, i.e. the unknown displacements at the nodes.

In the assembly stage (at stage 7), the element equations are combined to represent the entire structure. In FEM, the element equilibrium equations written with local coordinates are rewritten using global coordinates and then combined to give an equation of the form:

$$[\mathrm{K}]\{\mathrm{u}\} = \{\mathrm{p}\} \tag{1}$$

where [K] is the global stiffness matrix of the system to be analyzed, {u} is the vector of nodal displacement as unknowns, and {p} denotes vector of applied nodal forces. In FEMEM, the combination yields a functional to be minimized:

$$\min \Pi(\mathrm{u},\mathrm{p}), \{\mathrm{u}\} = ? \tag{2}$$

where {u} and {p} are as defined above, and Π represents the total potential energy (TPE) functional of the system including the energy stored in the structure and the work done by the external forces acting on it. In other words, Π defined as the difference between the strain energy stored in the elements and the work performed by external forces.

Table 1. Comparison between FEM and FEMEM approaches.

#	FEM	FEMEM
1	Modeling of the structure	
2	Dividing the structure into finite elements	
3	Nodes are selected at the common ends of elements	
4	Defining the displacements of the structure as the displacements at nodes, and considering that the loads are applied at related nodes	
5	Choosing shape functions on the elements depending on the number of unknowns such that displacements on every point on the element can be calculated using these functions	
6	Equilibrium equations on the elements are written in matrix form in local coordinates which are then converted to global coordinates	Total potential energies on elements are written
7	A global matrix is constructed by combining the equilibrium matrices of members [K]{u}={P}→{u}=?.	A global total potential energy function is constructed by combining the potential energies of the members min U(u,p), u=?
8	Displacements are calculated via solving the global matrix-based equation.	Displacements are calculated via minimizing the global total potential energy function.
9	Other unknown responses (e.g., stresses, support reactions) are evaluated by using the nodal displacements	

For linear elastic systems with constant stiffness matrix, Eq. (1) provides an efficient and robust solution framework. If K is a function of displacements and/or stress levels, it is evident that such an equation cannot represent the behavior of the related structure. Fortunately, there are iterative FEM methods to deal with these nonlinear problems, at least with some of them, which are more complicated than the linear ones. Eq. (2) can be considered as valid for all systems; this can already be seen by noting that U is given as a function of u and p in this equation. To make that expression more general, one can rewrite it in the form:

$$\min \Pi\,(\mathrm{u},\mathrm{p}),\ \mathrm{C}\{\mathrm{u}\} \leq 0, \{\mathrm{u}\} = ? \tag{3}$$

where C{u}represents kinematic or physical constraints on the displacements (Chong and Żak 2013). One can give the following examples for such constraints $u_i < 0.1$, $-.2 \leqslant u_i < 0.1$, $u_i = 4\,u_j + 2$ where, u_i and u_j are two arbitrary field variables, i.e., displacements.

The possibility of being able to put constraints on displacements is an obvious advantage for FEMEM, indeed FEM does not have such a possibility.

The determination of the nodal displacement vector u corresponds to stage 8 in both approaches of FEM and FEMEM. In FEM, this involves solving a system of algebraic equations, either directly or iteratively, provided that the stiffness matrix (K) is well-conditioned. In fact, in real life, there may be structural problems with ill-conditioned stiffness matrices; it is evident that FEM will not be able to find equilibrium configurations in those cases. Such situations may happen, for instance in progressive failure cases after failure of a member or a support condition.

Another disadvantage of FEM as compared to FEMEM in this case is that if the equilibrium position is non-unique, as in stability problems after bifurcation point, FEM will not be able to solve the problem as suggested by theory of matrices.

For these cases where FEM is not applicable, FEMEM can be used successfully to arrive at the equilibrium configurations.

In FEM, the solution is obtained by solving a linear matrix equation for a linear problem, or by solving linear matrix equations for a nonlinear problem many times to deal with the nonlinearity. The matrices for all these cases may be solved directly or by iterative methods based on the well-established matrix theory (see for instance Zhang 2017).

In FEMEM, the solution is obtained through numerical optimization algorithms applied to the total potential energy functional. Early and subsequent FEMEM studies demonstrate this optimization-based solution strategy (Toklu et al. 2013a; Toklu et al. 2021a; Istianto et al. 2022), and hybrid approaches combining optimization algorithms with neural networks have also been proposed within this energy-minimization framework (Mai et al. 2022). More broadly, recent data-driven civil engineering applications demonstrate that evolutionary feature generation and neural network learning can be combined to capture complex nonlinear input-output relationships while improving prediction accuracy (Kulkarni et al. 2025). Similarly, reinforced concrete design optimization studies, where Jaya-based procedures have been used to search for economical beam cross-sections and reinforcement layouts under combined shear, bending, and torsional constraints, illustrate the wider relevance of optimization-based search strategies in structural engineering (Duysak et al. 2024). These latter studies should be treated as contextual support for optimization-based structural computation rather than as direct FEMEM applications.

Metaheuristic optimization techniques have generally been employed as the primary tools for the formulation and development of the FEMEM framework. It is well established that a large number of such techniques are available and that this number continues to grow rapidly (Rajwar et al. 2023). In the development of FEMEM, a wide range of these methods have been utilized, including Genetic Algorithms, Differential Evolution, Particle Swarm Optimization, Artificial Bee Colony Optimization, Flower Pollination Algorithm, Teaching–Learning-Based Optimization, Jaya Algorithm, Grey Wolf Optimization, Harmony Search, and Ant Colony Optimization, either in their original forms, with modifications, or as hybrid approaches combining multiple techniques (see, for example, Toklu et al. 2013d; Temür et al. 2017; Bekdaş et al. 2019; Toklu et al. 2021a).

As is well known, the performance of these algorithms, which are stochastic in nature, strongly depends on factors such as parameter selection, initial conditions, and related settings. Consequently, it is difficult to establish a general hierarchy among them in terms of effectiveness. The applications presented in this study demonstrate that all of the problems considered can be successfully solved using these algorithms, provided that appropriate configurations are adopted. One significant advantage of employing optimization techniques instead of traditional matrix-based methods in such problems is the flexibility they offer in handling various types of constraints. A simple yet illustrative example is the treatment of unilateral boundary conditions. While imposing a constraint on a displacement variable (e.g., $x \leq a$) is generally impractical within a matrix-based formulation, optimization algorithms — particularly metaheuristic approaches—allow such constraints, as well as more complex ones, to be incorporated in a straightforward manner (Toklu et al. 2021a).

The final step (stage 9) in both FEM and FEMEM consists of evaluating secondary response quantities, such as stresses, strains, member internal forces, and support reactions, using the nodal displacements obtained from the solution process. The similarities and differences between FEM and FEMEM are summarized in Table 1 and illustrated schematically in Fig. 1.

3. Advantages and Disadvantages of FEM and FEMEM

For linear structural problems in which the equilibrium configuration is governed by a well-conditioned system of linear algebraic equations as in Eq. (1), classical FEM is generally more computationally efficient than FEMEM. In such cases, highly optimized direct and iterative linear solvers can be employed, resulting in low computational cost and excellent numerical accuracy. In contrast, FEMEM formulations typically rely on iterative optimization algorithms, which may require a larger number of function evaluations to converge. For symmetric linear systems, FEM naturally preserves symmetry properties through the underlying matrix formulation, provided that symmetric discretization and solvers are employed. In FEMEM, exact numerical symmetry cannot always be guaranteed due to the characteristics of the optimization algorithms; however, reported solutions are typically symmetric within acceptable numerical tolerances.

From a computational perspective, a fundamental difference between FEM and FEMEM lies in the nature of their numerical solvers. In standard FEM, especially for linear and mildly nonlinear problems, the dominant computational cost is associated with the assembly and solution of global stiffness matrices, for which highly optimized direct and iterative solvers are available. In contrast, FEMEM formulations rely on iterative optimization algorithms, where the computational cost scales primarily with the number of objective-function and, when applicable, gradient evaluations. For large-scale systems with many degrees of freedom, this may result in a significantly higher computational burden for FEMEM compared to FEM, particularly when metaheuristic algorithms are employed. Several studies report that the number of function evaluations required for convergence in FEMEM can be orders of magnitude larger than the number of linear system solves typically needed in FEM, which constitutes a major practical limitation for large linear systems.

Another point where FEM is more advantageous than FEMEM can be seen in elements with small contributions to the TPE of the system. FEMEM arises in linear problems when certain degrees of freedom contribute only marginally to the total potential energy. In such cases, optimization-based solvers may exhibit reduced sensitivity to these variables, potentially affecting the accuracy of the corresponding displacement components.

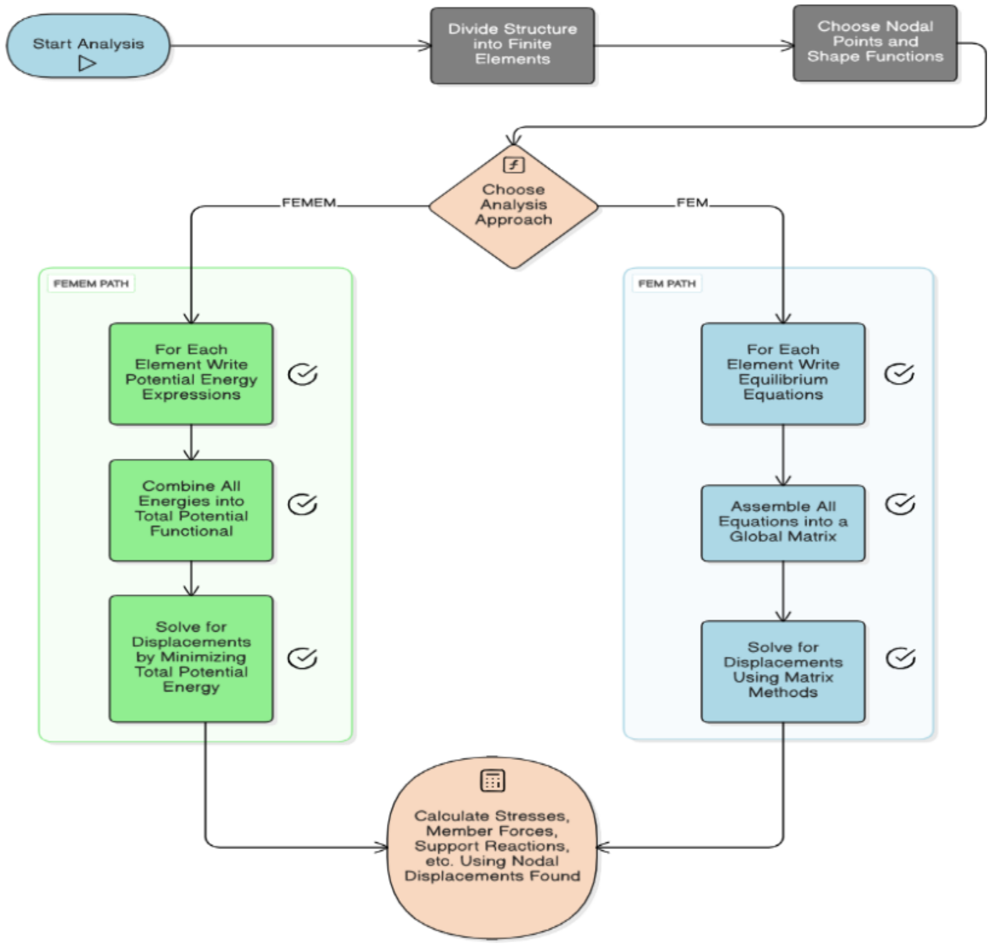

Fig. 1. Flow charts for structural analyses through FEM and FEMEM.

Despite these limitations, FEMEM has been reported to offer advantages over classical FEM for specific classes of structural problems, particularly those involving nonlinearities, degeneracies, or unconventional constraints. These cases are summarized below:

- The system is geometrically nonlinear;
- The system has materials that have nonlinear properties;
- The system has more than one equilibrium configuration;
- There are constraints on support conditions.

Problems involving material and/or geometric nonlinearities constitute an important application area. In classical FEM, nonlinear problems are typically addressed using incremental–iterative strategies, such as Newton–Raphson or arc-length methods, which require repeated assembly and solution of tangent stiffness matrices. The convergence and accuracy of these methods may depend on step size selection, initial guesses, and numerical conditioning. In FEMEM, the same energy minimization framework can be applied to both linear and nonlinear problems by appropriately defining the strain energy function, without altering the overall solution structure. Several studies have reported that this leads to a more uniform formulation across different problem classes.

A systematic comparison of the advantages and limitations of FEM and FEMEM for different problem cases is summarized in 2. It is expressed in this table that, for linear systems, i.e. for cases 1 and 2, FEM is much more advantageous in analysis time. For large linear systems, FEMEM has an additional disadvantage on the accuracy of variables that make small contribution to TPE of the system.

Case 3 is related to systems which have material or geometric nonlinearities or both. For geometric nonlinearity, i.e. for large deformations, there is absolutely no attention paid to FEMEM solutions. If there is nonlinearity coming from materials, the only adjustment to do in FEMEM applications is to include information about materials in the system. For both cases the same program can be used for linear and nonlinear problems in FEMEM. In FEM, this is not the case, a program prepared for linear problems cannot be used for nonlinear problems. The easiest solution is to insert into the procedure iterations that will solve the problem by increasing loads in steps. In this case step size may play a role in accuracy of the results.

The problem of multiple solutions is considered as Case 4 in Table 2. Structural systems exhibiting multiple equilibrium configurations, such as post-buckling or bifurcation problems, represent another class where FEMEM may be advantageous. Standard matrix-based solution procedures typically converge to a single equilibrium path unless specialized continuation techniques are employed. In contrast, optimization-based formulations allow the identification of multiple local minima of the energy functional, which correspond to different equilibrium states, provided that suitable search strategies are used (Das et al. 2011).

Table 2. Advantages and disadvantages of FEM and FEMEM.

Case	Problem type	FEM	FEMEM
1	Linear systems (small)	Very efficient	Effective but takes more time
2	Linear systems (large)	Very efficient	Accuracy may not be at the desired degree for displacements with small contribution to TPE.
3	Nonlinear systems	Expertise required for formulation and application. Probable loss of accuracy due to error accumulation.	Solving as easy as a linear problem. No special attention. No accumulated errors.
4	Multiple solutions	Impossible to solve directly. Requires continuation or path-following techniques.	Efficient
5	Under-constrained systems	Solution may be hindered by singular or ill-conditioned stiffness matrices.	Solving as easy as a well constrained system. No special attention. Equilibrium configurations reported via direct energy minimization.
6	Unilateral or more general constraints	Not possible to solve directly. Difficult application of iterations with continuous user intervention. May be possible via specialized formulations or iterative enforcement.	Solving as easy as a well constrained system. Just give necessary constraints. Constraints introduced naturally within optimization framework.
7	Degenerate problems	Impossible to solve. Matrix ill conditioned.	Solving as easy as a non-degenerate problem. No special attention.

The comparison (Table 2) reflects trends reported in the literature. In particular, FEMEM typically requires a substantially larger number of objective-function evaluations compared to the number of global linear system solves in FEM, which may lead to significantly higher computational cost for large-scale linear problems.

The under constrained problems and degenerate problems, numbered as Cases 5 and 7 are those where the system is not supported sufficiently or with missing members to make the system unstable. In both cases the flexibility matrix becomes ill-conditioned which means that no solution can be obtained by FEM. On the other hand, these defects do not form a problem for FEMEM, the procedure looks for the configuration(s) with minimal TPE's. In such cases, FEMEM formulations have been reported to successfully identify equilibrium configurations by directly minimizing the total potential energy, without requiring matrix inversion.

The final example, denoted as Case 6 in Table 2, has constraints on displacements of all types. Indeed, matrix equations are such that no such constraints can be defined for a system. This fact results in the conclusion that in FEM applications problems with constraints on supports cannot be solved. The handling of unilateral, inequality, or more general displacement constraints also differs between the two approaches. While such constraints can be incorporated in FEM through specialized formulations or iterative procedures, FEMEM allows these constraints to be introduced directly and systematically within the optimization framework, which is inherently designed to handle constrained problems.

Finally, FEMEM formulations typically avoid the assembly and storage of global stiffness matrices, relying instead on vector-based evaluations of the energy functional. As a result, several studies report lower memory requirements for FEMEM compared to classical FEM, particularly for large-scale problems where matrix storage becomes a limiting factor.

4. Applications on Trusses and Truss-Like Structures

4.1. Trusses

Trusses are structures elaborated an important number of times by FEMEM. In fact, the name used for the method at the beginning was Total Potential Optimization using Metaheuristic Algorithms (TPO/MA) and the first related applications took place in 2004 (Toklu 2004a; Toklu 2004b). In these publications 3 plane trusses with 2, 6 and 31 members are analyzed having linear and nonlinear constitutive equations. The results are compared, whenever possible, with SAP 2000 and with the results from another publication (Ohkubo et al. 1987). The nonlinear stress-strain relations are of piecewise continuous type, as shown in Fig. 2, or a function of the form:

$$\sigma = 250(80000\varepsilon + 27)^{1/2} - 750 \qquad (4)$$

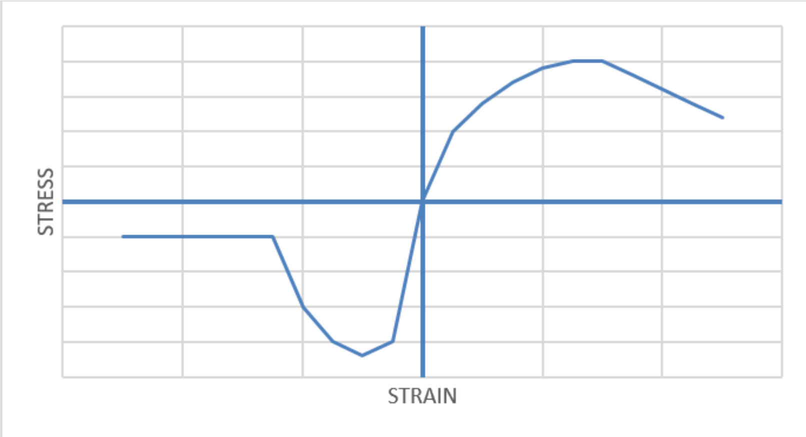

Fig. 2. General piecewise stress-strain relation for truss members.

The piecewise stress-strain relation can be arranged to represent any kind of behavior including elastic, plastic, elastoplastic, strain hardening, strain softening, and buckling ones. The ends of the lines on positive and negative sides represent the failure points, i.e. the points where strains reach the breaking point of the member. This enables the user to make progressive failure analysis of the structure. It is to be noted that in the related software prepared all members may have the same properties, or members may be assigned different stress-strain relations.

In Fig. 3 configurations adopted by a 10-bar truss with elastic-plastic members are visualized to demonstrate some cases where the solution can be obtained easily by FEMEM while FEM becomes unsuccessful (Toklu et al. 2021a). The truss investigated is supported by a hinge at the left and by a roller on the right end. The loads are acting vertically downward at the nodes at the lower chord. In Fig. 3(a), the case where the roller is failed is analyzed with loads 1 kN each. In Figs. 3(b-c), the roller is intact, but the loads are very important, 30 kN and 40 kN, respectively. This example over a simple truss shows the superiority of FEMEM over FEM in certain cases.

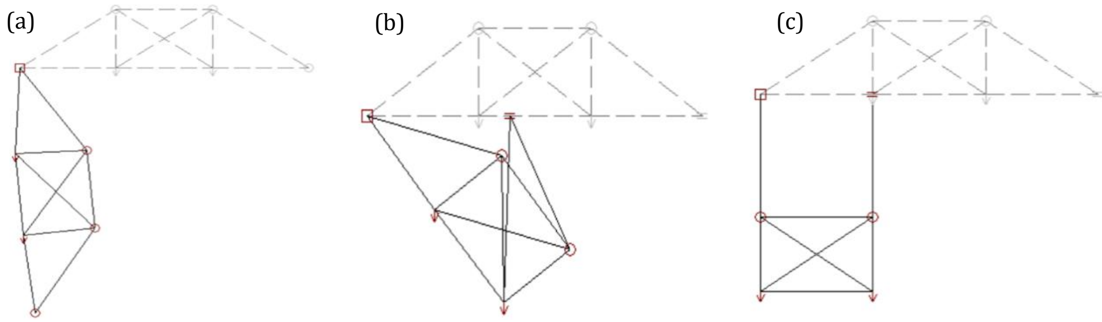

Fig. 3. Behaviors of a 10-bar truss (Toklu et al. 2021a).

In a study (Toklu et al. 2013b), a space truss with 25 members is studied with TPO/MA for different types of linear and nonlinear stress strain relations, under different loading conditions, with comparisons between different types of metaheuristic algorithms. Comparisons with linear and nonlinear FEM have shown excellent accuracy obtained by this new technique.

The case of non-unique solutions is also studied by FEMEM (Toklu et al. 2015c; Toklu 2023). One of these trusses with 24 members are shown in Fig. 4 of which the deformed shapes under the given load are given in Fig. 5. It is shown in this figure that this truss may assume 4 different deflected shapes under the given conditions. Fig. 6 gives the variation of TPE for this truss as a function of the load applied. One of the things that this figure shows is the close values obtained by FEM and FEMEM when both methods are applicable. The figure shows that when the magnitude of P is high, then the solutions are unique, corresponding to shapes S1 and S4. There is more than one solution for moderate absolute values of P, but FEM always gives one single solution which may not be the global, i.e. the best solution, the one with the minimum TPE (for instance between P=-350 kN and P=0). A close investigation of the results given by FEMEM shows that near P=0 all four deflected shapes are possible, and there are regions with 2 or 3 solutions.

Trusses with unilateral boundary conditions (Bekdaş et al. 2014b; Temür et al. 2014b), smart trusses (Toklu and Arditi 2014), trusses under thermal effects (Bekdaş et al. 2014a; Toklu et al. 2015a), under constrained and unstable trusses (Toklu et al. 2018) are among structures investigated with FEMEM. Applications of FEMEM with emphasis on different kinds of metaheuristic algorithms were also the subject for many publications (Toklu 2013; Bekdaş et al. 2019; Nigdeli et al. 2022; Toklu et al. 2015b; Bekdaş et al. 2015; Bekdaş et al. 2023) including a study where neural networks were used instead of genetic algorithms (Mai et al. 2022).

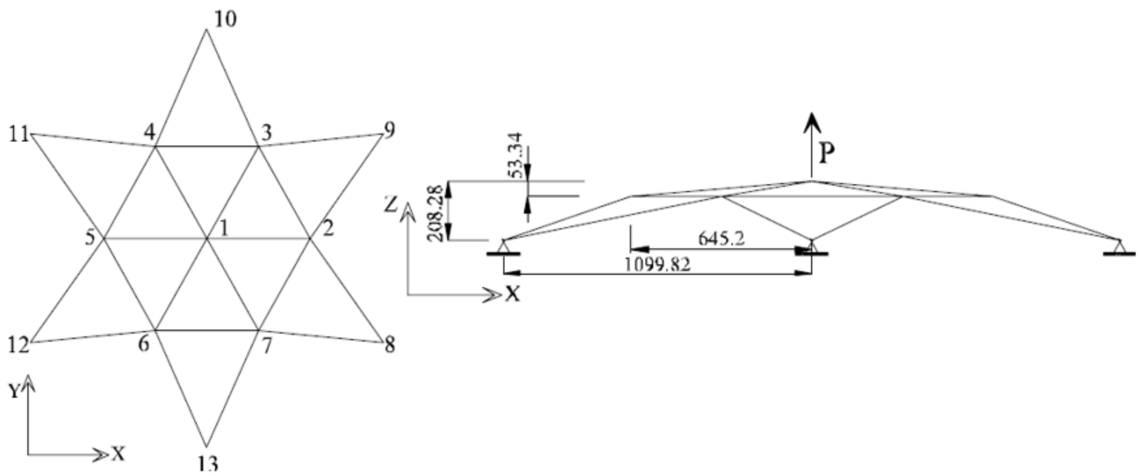

Fig. 4. 24 bar space truss investigated for multiple deflected shapes (Toklu et al. 2015c).

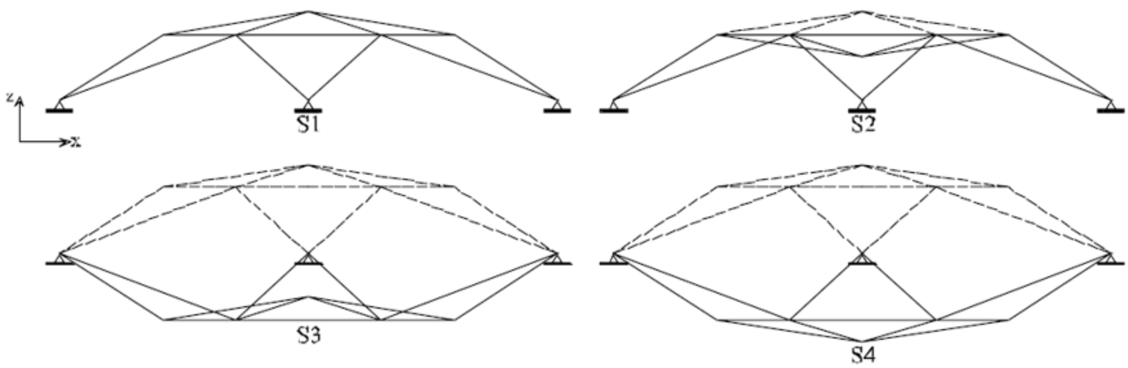

Fig. 5. Four different shapes S1 to S4 that the 24-bar space truss may assume (Toklu et al. 2015c).

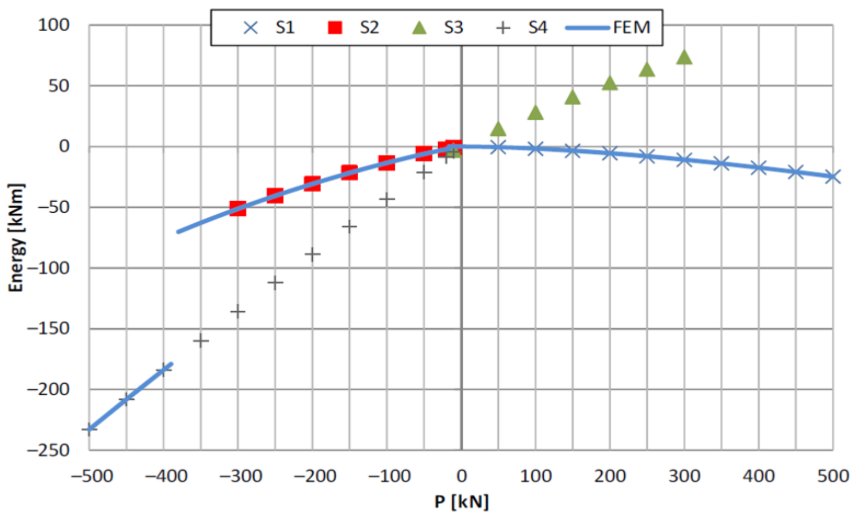

Fig. 6. TPE of the 24-bar space truss corresponding to 4 possible shapes as a function of the load applied (Toklu et al. 2015c).

4.2. Cable network structures

Cable network structures belong to another type of system that cannot be solved by FEM without special attention. These structures can be considered as special types of trusses where the cables cannot resist compressive forces, making the system nonlinear, with all the meanings of the word. They can be solved by FEM whenever it is known beforehand which elements are under compression or under tension. Otherwise, a nonlinear FEM approach is necessary based on iterations or some specially developed techniques (Toklu et al. 2017). There are several publications on their solution by FEMEM (Toklu et al. 2017; Temür et al. 2014a; Kayabekir et al. 2018; Nigdeli et al. 2019).

A demonstrative example is a spatial cable network of external dimensions 24 m x 16 m, with 38 cables and 26 nodes in which some members are prestressed

[Toklu et al. 2017). The system is shown on Fig. 7. The results obtained by FEMEM are found to be in perfect concordance with three results from the literature (Lewis 1989; Thai and Kim 2011; Hâu 2025). The results and algorithms are presented comparatively in Table 3.

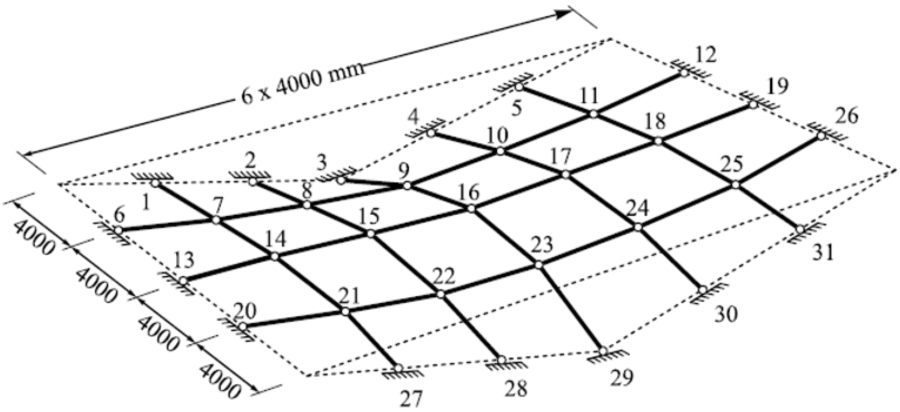

Fig. 7. Spatial cable structure (Toklu et al. 2017).

Table 3. Deflections obtained in different studies for the spatial cable structure (Hâu 2025).
Last 3 columns are obtained by Hâu (2025) using different metaheuristic algorithms (NR: Newton Raphson; TPO/MA: Total Potential Optimization using Meta-Heuristic Algorithms; BSA: Backtracking Search Optimization Algorithm; TLPO: Teaching-Learning-Based Optimization; DE: Differential Evolution Algorithm). The last row shows the TPE values Π in N·mm.

Reference	Lewis (1989)	Thai & Kim (2011)	Toklu et al. (2017)	Hâu (2025)	Hâu (2025)	Hâu (2025)
Algorithm	–	Based on NR	TPO/MA	BSA	TLBO	DE
TPE (N·mm)	6505782	6505527	6505019	6504295	6504288	6504279

4.3. Tensegric structures

Tensegric structures or tensegrities are truss-like structures made by cables and struts. In fact, they can be considered as a sub-class of cable-strut structures with an additional condition that struts do not intersect each other. For defining tensegric systems, it is cited that “a tensegrity system is established when a set of discontinuous compressive components interacts with a set of continuous tensile components to define a stable volume in space” (Kahla and Kebiche 2000).

Cable-strut structures may be used in the form of towers (Sonawane et al. 2023), beams (Jensen et al. 2007), domes (Kawaguchi et al. 1999; Fu 2006), closed habitats (Toklu and Uzun 2016; De Boeck 2013), and bridges (Rhode-Barbarigos et al. 2012) etc. The distinction with cable-strut structures and tensegric structures is discussed in 1998 (Wang 1998a; Wang 1998b).

In applications, it is possible to see whether the cables are prestressed or not, and whether they are continuous or not. In general struts do not touch each other but there may be cases where the struts are continuous, i.e., which are under bending effects besides axial forces. It is the case where there does not exist bending effects in struts can be analyzed by FEMEM as special types of trusses.

Because of the high nonlinearity of these structures, normal FEM methods become insufficient for analyzing them. All methods which are advanced for analysis of tensegric structures are based on iterative techniques (Juan and Tur 2008) using Newton-Raphson method or those similar to *P-δ* method (Kahla and Kebiche 2000; Fu 2006; Gaiotti and Smith 1989; Skelton et al. 2017; Tran and Lee 2011; Li et al. 2016). In this way, techniques are forwarded for the elastoplastic analysis of tensegric structures taking into account both geometric and material nonlinearities including yielding and buckling of struts (Kahla and Kebiche 2000; Nuhoglu and Korkmaz 2011).

It has been shown that FEMEM can be used successively for solving these structures with no special precautions (Toklu and Uzun 2016; Toklu et al. 2013c; Toklu et al. 2014a; Bekdaş et al. 2024). Fig 8(a) shows a tensegric structure with 8 struts, 24 cables and 16 nodes which are solved by FEMEM in the mentioned studies. Figs. 8(b-c) show two configurations of this system under two different loadings. In both loadings, the loads applied at two opposite joints, 11 and 15, outward in case (a) and inward in case (b).

Another 3-dimensional tensegric structure analyzed is shown in Fig. 9 (Bekdaş et al. 2024). This structure is obtained by attaching two tensegrities one on top of the other. Another tensegrity analyzed in this publication using FEMEM is a plane structure as shown in Fig. 10 which is a cantilever beam with 8 struts and 28 cables. Figs 10(a-b) shows the system analyzed; Figs 10(c-d) show the deformed system under two different loadings. In case c a downward load of 10 kN is applied at node 15, in case d a load of 100 kN is applied at node 14 in x-direction.

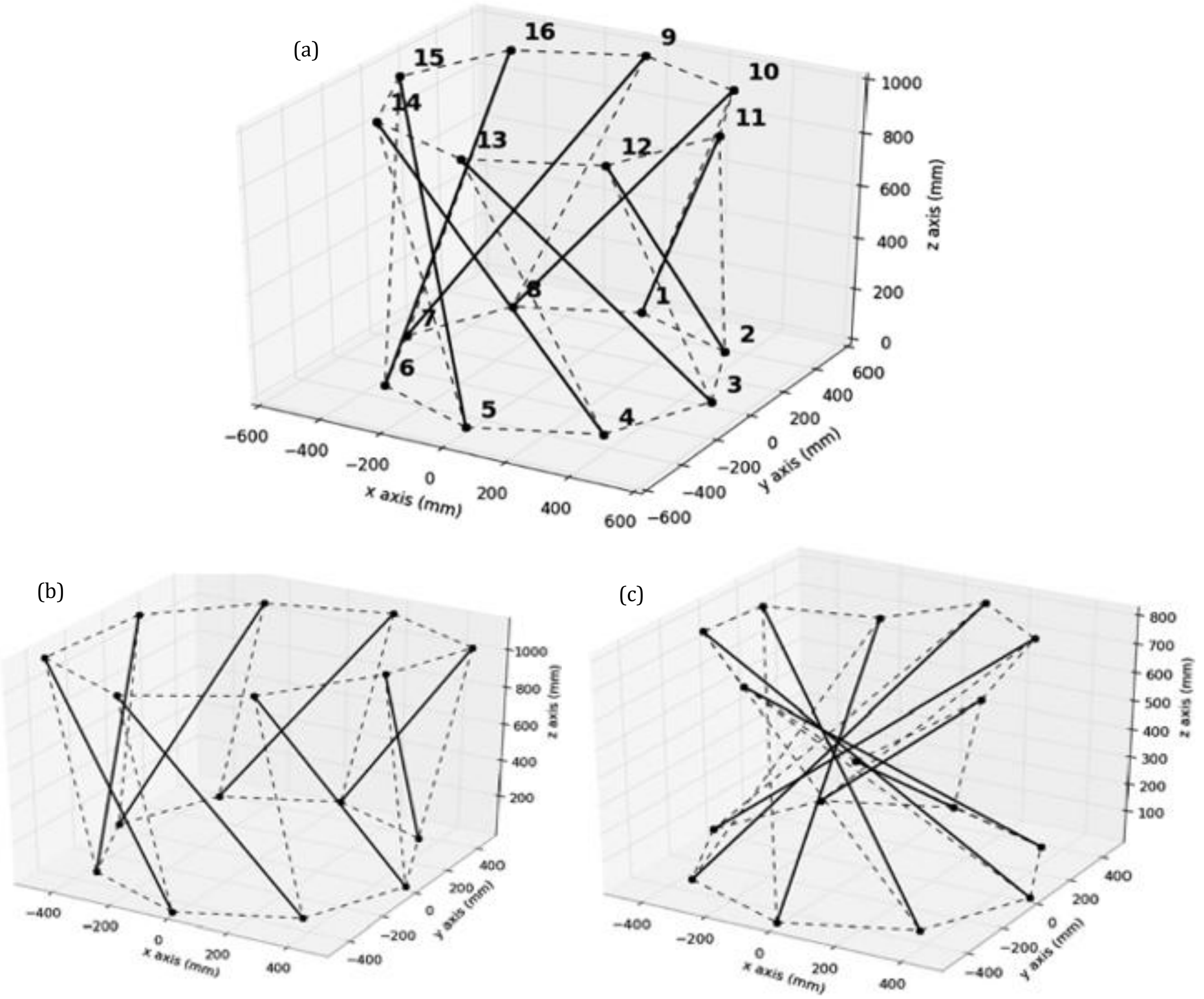

Fig. 8. A three-dimensional 8 strut tensegric structure and its deformed shapes under two different loadings.

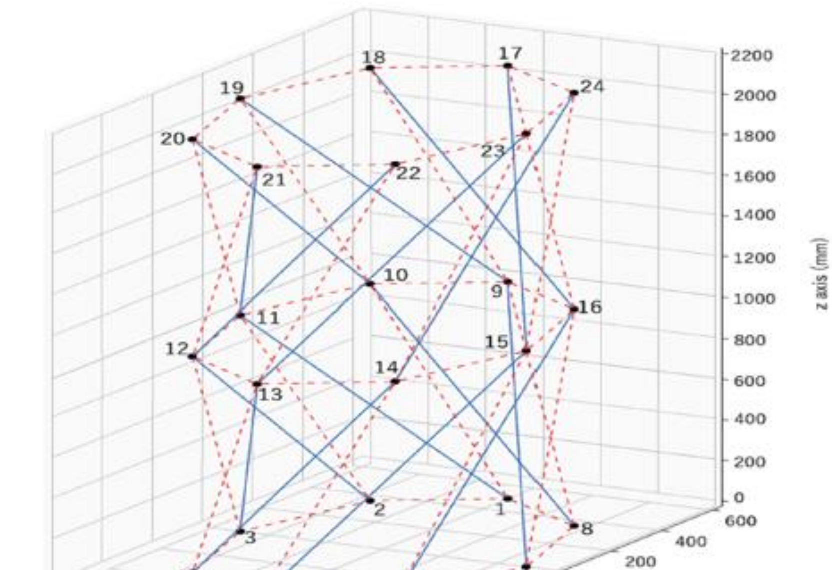

Fig. 9. A three-dimensional tensegric structure obtained by two structures put one above the other (Bekdaş et al. 2024).

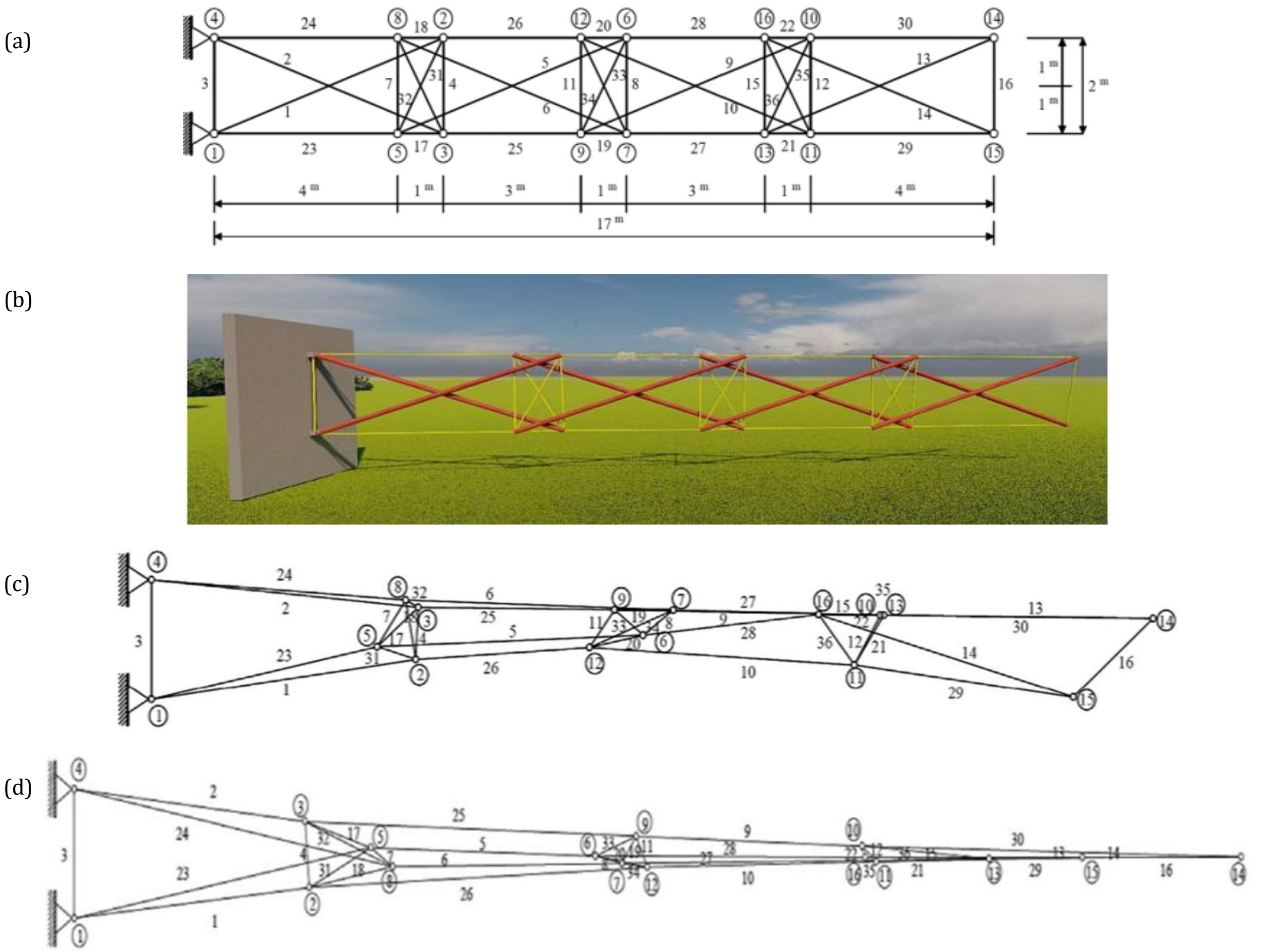

Fig. 10. A plane cantilever beam analyzed using FEMEM (Bekdaş et al. 2024).

Another type of application of FEMEM where FEM will not be successful is the progressive failure analysis of trusses (Aras 2024). The plane truss shown on Fig. 11(a), which has 42 bars and 22 nodes, is analyzed for such a case. The four nodes at both ends are fixed supports, the 6 nodes at the middle, 3 on the top chord and 3 on the lower chord, are loaded with important downward loads of magnitude 100000 N, and all the remaining loads are loaded lightly with downward forces of magnitude 250 N. The loads are applied increasingly starting from zero, at one step two members of number 24 and 27 failed and the system took the shapes shown on Figs. 11(b-c) progressively.

In Fig. 12 are shown the energy variations in the system as a function of the iterations. In this figure, red line is for work done by the external forces, blue line is for the strain energy in the system, the green line is for the sum of them, TPE.

5. Applications on Plane-Stress and Plane-Strain Structures

FEMEM applications on plates currently are restricted to plane-stress and plane-strain structures as shown in Table 4 (Toklu et al. 2014b; Toklu et al. 2020; Kayabekir et al. 2020; Toklu et al. 2021b; Temür 2021). In the application studies, comparisons with FEM were performed wherever possible, which further confirmed the robustness and effectiveness of the FEMEM approach. These comparisons were possible only for linear cases, as no nonlinear stress–strain relationships for either plane-stress or plane-strain problems are available in the literature. Thus, fictive stress-strain relations, of number three, are created and used for nonlinear cases.

Quite a big number of metaheuristic algorithms are employed in the computations with different degrees of success. In Table 5 a simple comparison is shown for the plate with a hole problem, treated with 144 triangular members with linear stress-strain relations (Kayabekir et al. 2020). It can be seen in this table that for almost all algorithms, namely for Flower Pollination Algorithm (FPA), Jaya Algorithm (JA), Hybrid Harmony Search (HHS), and Teaching Learning-Based Optimization (TLBO), the minimum TPE values are practically the same as the result obtained by FEM. In this application the only algorithm that was not equally successful was Harmony Search (HS). The table contains also information about number of iterations that were necessary to arrive at the minimum TPE value obtained. It can be seen from the given values that the number of iterations may show great variations from algorithm to algorithm.

Fig. 13 shows original and deflected shapes of the soil under a buttress dam calculated with linear plane-stress assumptions, dividing the structure into 125 triangular finite elements, having 76 nodes and 132 degrees of freedom (Temür 2021). It has to be noted that the displacements are scaled in the diagram with a factor of 10.

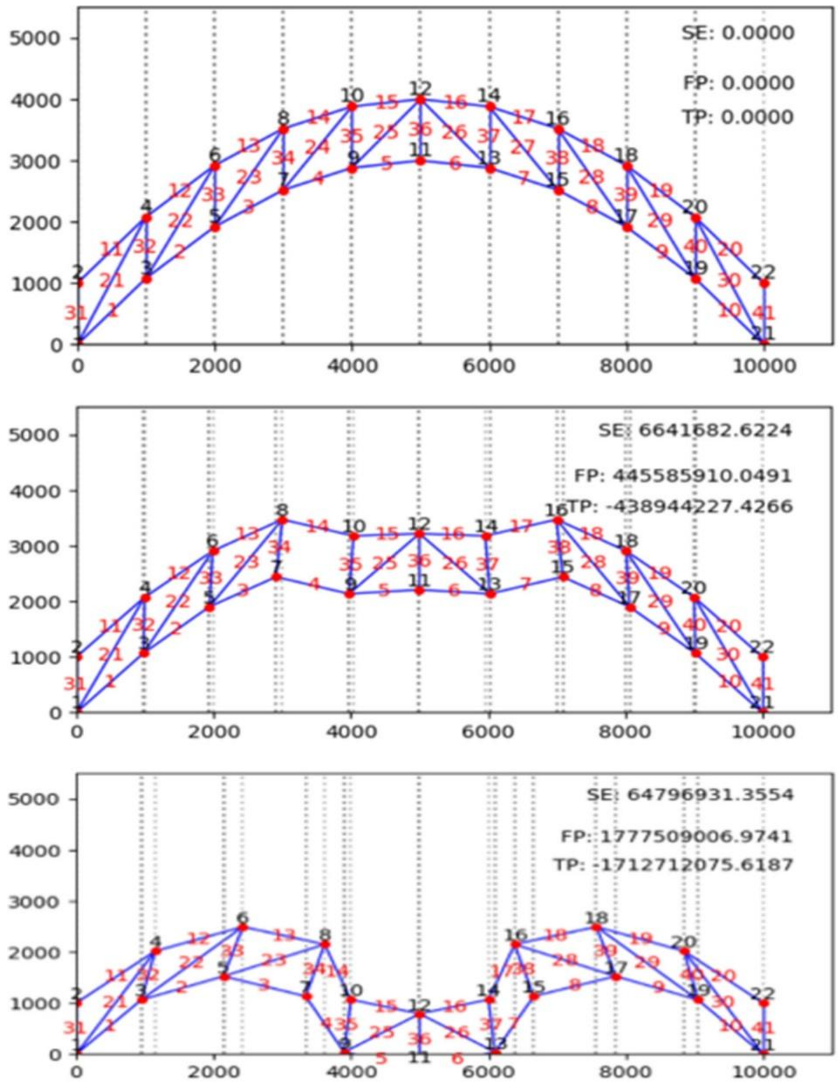

Fig. 11. 42 bar truss subject to progressive failure analysis:
(a) Original position; (b) Configuration at step 80 after failure of 2 members; (c) Final configuration (Aras 2024).

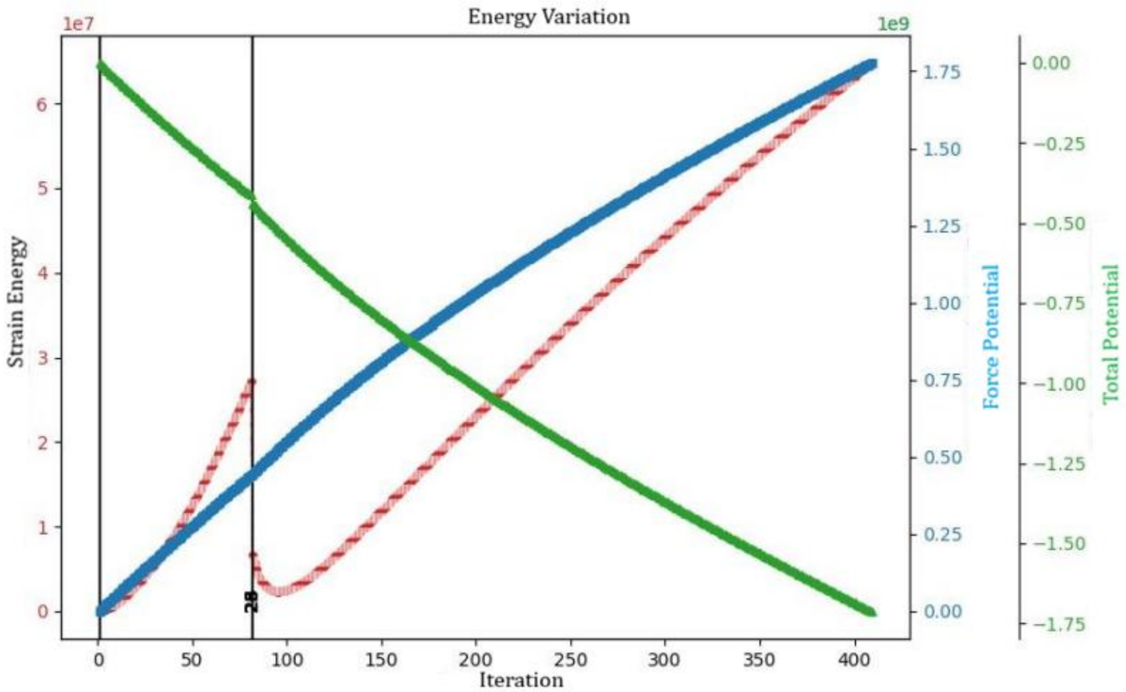

Fig. 12. Energy variation on the truss with 42 members through iterations.

Table 4. FEMEM applications on plane-stress and plane-strain structures.

Reference	Problem	Type	Stress-strain relation
Toklu et al. (2014b)	Cantilever plate	PlaneStress	Linear
Toklu et al. (2020)	Cantilever beam	PlaneStress	Linear and nonlinear
	Plate with a hole	PlaneStress	Linear and nonlinear
	Thick-walled pipe	PlaneStress	Linear and nonlinear
	Cantilever retaining wall	PlaneStress	Linear and nonlinear
Kayabekir et al. (2020)	Cantilever beam	PlaneStress	Linear
	Plate with a hole	PlaneStress	Linear
Toklu et al. (2021b)	Thick-walled pipe	PlaneStrain	Linear and nonlinear
	Cantilever retaining wall	PlaneStrain	Linear and nonlinear
Temür (2021)	Plate with a hole	PlaneStress	Linear
	The soil under a buttress dam	PlaneStrain	Linear

Table 5. Best TPE values and necessary analysis numbers for best solutions in linear plane-stress problem, plate with a hole (adopted from Kayabekir et al. 2020).

Algorithm	FPA	JA	HS	HHS	TLBO	FEM
TPE (N·mm)	-905.80	-905.80	-901.48	-905.80	-905.80	-905.80
Analysis number	999992	972853	876457	269416	947564	-

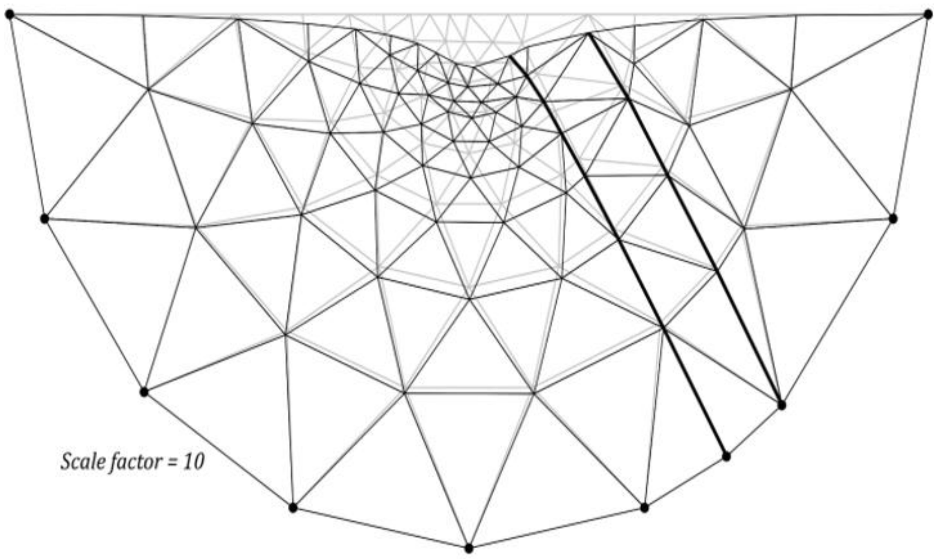

Fig. 13. Original and deflected shapes of soil under a buttress dam (Temür 2021).

6. Applications on 3-Dimensional Media

The applications of FEMEM on 3-dimensional media treats a cantilever beam, a thick plate, and a 3-dimensional prism (Can et al. 2022). In these applications cubes with 8 nodes are chosen as finite elements as shown in Fig. 14.

Figs. 15 and 16 are generated in this study based on the data reported by Can et al. (2022). Fig. 15 shows the displacements at the end point of a cantilever obtained using 3 methods, OPENSEES (Gu and Huang 2023), SAP2000 (Schueller 2008), and FEMEM. Fig. 16 represents the displacements at the mid-point of a plate loaded at that point, supported at 4 corners, again using the same methods. It can be seen from those figures that if there are enough number of finite elements in the analysis, FEMEM gives the same results as compared to the other methods.

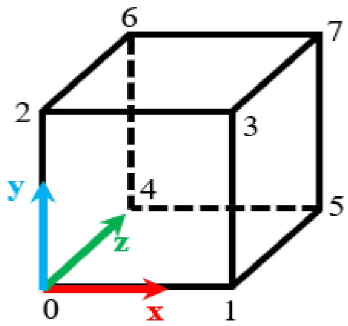

Fig. 14. Cubic finite element for analysis of a 3-dimensional media (Can et al. 2022).

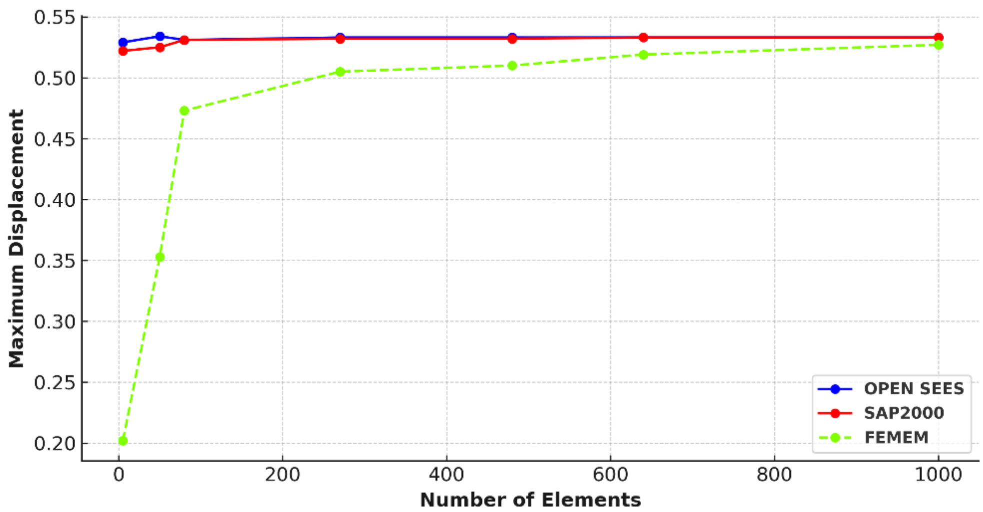

Fig. 15. Displacement of a cantilever beam obtained by three methods as a function of number of finite elements (adopted from Can et al. 2022).

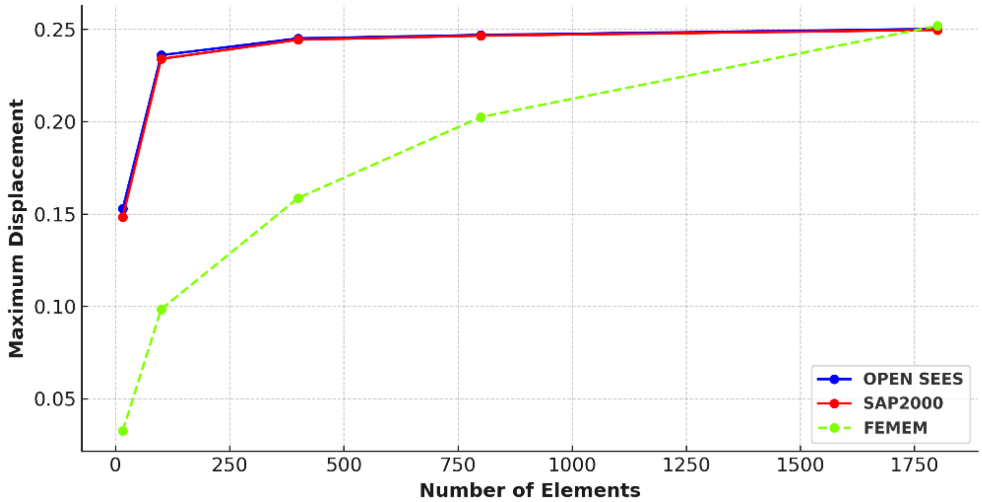

Fig. 16. Central displacement of a plate obtained by three methods as a function of number of finite elements (adopted from Can et al. 2022).

7. Conclusions

This state-of-the-art review has synthesized existing studies that formulate structural analysis problems within the FEMEM framework, in which equilibrium is achieved through the minimization of total potential energy rather than conventional matrix-based finite element formulations. The reviewed literature consistently demonstrates that, whenever classical FEM is applicable, FEMEM produces identical solutions, thereby confirming its theoretical soundness and numerical consistency.

More importantly, the review highlights that FEMEM extends beyond a mere alternative formulation of FEM. By embedding the governing equations into an optimization framework, FEMEM enables the natural treatment of inequality constraints, multiple equilibrium states, progressive failure mechanisms, and nonlinear material behavior—problem classes that are inherently difficult or impractical to address using standard linear or nonlinear FEM formulations. This capability represents the principal contribution and added value of FEMEM within the broader landscape of computational structural mechanics.

From a critical standpoint, the survey reveals that FEMEM applications to date have been largely confined to relatively simple structural systems, including trusses, plane-stress and plane-strain problems, and selected volumetric elements such as beams and plates. The method's potential for more complex and realistic engineering systems—such as frames, shells, large-scale three-dimensional structures, and integrated systems including buildings and bridges—remains largely unexplored. Furthermore, the widespread reliance on metaheuristic optimization techniques introduces challenges related to parameter sensitivity, convergence behavior, and computational efficiency, which have not yet been systematically investigated.

Future research should therefore focus on extending FEMEM to complex, highly nonlinear structural systems and on establishing rigorous comparative frameworks involving different optimization and hybrid solution strategies. Systematic studies addressing robustness, scalability, and computational cost are essential for assessing the practical viability of FEMEM in large-scale engineering applications. Such efforts are expected to further clarify the role of FEMEM as a complementary—and in certain problem classes, superior—alternative to traditional finite element formulations.

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Conflict of Interest

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Data Availability

The datasets generated and/or analyzed during the current study are not publicly available but are available from the corresponding author upon reasonable request.

AI Assistance

No AI-based tools were used in the preparation of this manuscript.

Author Contributions

All authors made substantial contributions to the conception and design of the study, acquisition of data, analysis and interpretation of data; drafted or critically revised the manuscript for important intellectual content; and approved the final version to be published.

References

Alshabrawi HAM, Uysal H (2025). Enhancing the mechanical performance of perforated steel plates through fiber-reinforced composite reinforcement: A finite element analysis study. *Challenge Journal of Structural Mechanics*, 11(4), 174-183.

Aras D (2024). Progressive Collapse of Truss Systems. *M.Sc. thesis*, Eskişehir Osmangazi University, Eskişehir, Türkiye.

Bekdaş G, Temür R, Toklu YC (2014a). Analyses of truss structures under thermal effects. *Proceedings of the 5th European Conference of Civil Engineering (Eccie'14)*, Florence, Italy, 22-24.

Bekdaş G, Temür R, Toklu YC (2014b). Analysis of structures with unilateral boundary conditions. *Proceedings of the 15th EU/ME Workshop Metaheuristics and Engineering*, Istanbul, Türkiye, 113-116.

Bekdaş G, Temür R, Toklu YC (2015). Analyses of truss structures via total potential optimization implemented with teaching learning based optimization algorithm (TLBO). *Proceedings of the 3rd International Conference on Optimization Techniques in Engineering*, Roma, Italy, 99-108.

Bekdaş G, Kayabekir AE, Nigdeli SM, Toklu YC (2019). Advanced energy-based analyses of trusses employing hybrid metaheuristics. *The Structural Design of Tall and Special Buildings*, 28(9), e1609.

Bekdaş G, Nigdeli SM, Toklu YC (2023). A modified Jaya algorithm and application to structural analysis of trusses. *AIP Conference Proceedings*, Rhodes, Greece, 2849(1).

Bekdaş G, Ocak A, Nigdeli SM, Toklu YC (2024). Metaheuristic analysis of tensegrity. *The Structural Design of Tall and Special Buildings*, 33(7), e2091.

Belytschko T, Liu WK, Moran B, Elkhodary K (2014). Nonlinear Finite Elements for Continua and Structures. 2nd ed. John Wiley & Sons, Chichester, UK.

Can UC, Ozbasaran H, Toklu YC (2022). Structural analysis of solids using finite element method with optimization. *Proceedings of the 5th International Conference on Data Science and Applications (Icondata'22)*, Muğla, Türkiye, 274-277.

Chandrupatla T, Belegundu A (2021). Introduction to Finite Elements in Engineering. 5th ed. Cambridge University Press, Cambridge, UK.

Chong EK, Żak SH (2013). An Introduction to Optimization. 2nd ed. John Wiley & Sons, New York, USA.

Clough RW (2004). Early history of the finite element method from the view point of a pioneer. *International Journal for Numerical Methods in Engineering*, 60(1), 283-287.

Das S, Maity S, Qu BY, Suganthan PN (2011). Real-parameter evolutionary multimodal optimization: A survey of the state-of-the-art. *Swarm and Evolutionary Computation*, 1(2), 71-88.

De Boeck JD (2013). Tensegrity Bridges. *M.Sc. thesis*, Delft University of Technology, Delft, Netherlands.

De Borst R, Crisfield MA, Remmers JJ, Verhoosel CV (2012). Nonlinear Finite Element Analysis of Solids and Structures. 2nd ed. John Wiley & Sons, Chichester, UK.

Duysak Y, Nigdeli SM, Bekdaş G (2024). Optimum design of reinforced concrete beam sections with JAYA algorithm. *Challenge Journal of Concrete Research Letters*, 15(4), 134-141.

Fu F (2006). Non-linear static analysis and design of tensegrity domes. *Steel and Composite Structures*, 6(5), 417-433.

Gaiotti R, Smith BS (1989). P-delta analysis of buildings. *Journal of Structural Engineering*, 115(4), 755-770.

Gu Q, Huang S, editors (2023). A Practical Guide to OpenSees. Vol. 4. World Scientific, Singapore.

Gupta KK, Meek JL (1996). A brief history of the beginning of the finite element method. *International Journal of Numerical Methods in Engineering*, 39(22), 3761-3774.

Hậu MT (2025). Physics-informed differential evolution for cable nets. *Journal of Science and Technology*, 73(1).

Istianto DT, Tjong WF, Prayogo D (2022). Analisis non-linear struktur tensegrity menggunakan total potential energy optimization with metaheuristic methods (TPO/MA). *Dimensi Utama Teknik Sipil*, 9(2), 97-115.

Jensen Ø, Wroldsen AS, Lader PF, Fredheim A, Heide M (2007). FE analysis of tensegrity for aquaculture. *Aquacultural Engineering*, 36(3), 272-284.

Juan SH, Tur JMM (2008). Tensegrity frameworks: Static analysis review. *Mechanism and Machine Theory*, 43(7), 859-881.

Kahla NB, Kebiche K (2000). Nonlinear elastoplastic analysis of tensegrity systems. *Engineering Structures*, 22(11), 1552-1566.

Kandil K, El-Shami M, Hekal GM, ElGouhary OM (2025). Behavior of multi-cell steel columns under impact loading. *Challenge Journal of Concrete Research Letters*, 16(2), 95-114.

Kawaguchi M, Tatemichi I, Chen PS (1999). Optimum shapes of cable domes. *Engineering Structures*, 21(8), 719-725.

Kayabekir AE, Nigdeli SM, Bekdaş G, Toklu YC (2018). The analyses of spatial cable structures employing new generation metaheuristic methods. *Proceedings of the 9th GRACM International Congress on Computational Mechanics*, Chania, Crete, Greece.

Kayabekir AE, Toklu YC, Bekdaş G, Nigdeli SM, Yücel M, Geem ZW (2020). A novel hybrid harmony search approach for the analysis of plane-stress systems via total potential optimization. *Applied Sciences*, 10(7), 2301.

Kulkarni P, Londhe SN, Dixit PR (2025). A novel neuro genetic programming framework for modelling compressive strength of recycled aggregate concrete. *Challenge Journal of Concrete Research Letters*, 16(4), 203-214.

Lewis WJ (1989). Numerical efficiency for prestressed nets and frames. *Computers & Structures*, 33(3), 791-800.

Li H, Zhang H, Zheng Y, Zhang L (2016). Peridynamic nonlinear tensegrity analysis. *Computational Mechanics*, 57(5), 843-858.

Mai HT, Lieu QX, Kang J, Lee J (2022). A robust unsupervised neural network framework for geometrically nonlinear analysis of inelastic truss structures. *Applied Mathematical Modelling*, 107, 332-352.

Nigdeli SM, Bekdaş G, Toklu YC (2019). Total potential energy minimization for spatial cable systems. *Proceedings of the 12th International Congress on Mechanics (Hstam2019)*, Thessaloniki, Greece, 1-8.

Nigdeli SM, Bekdaş G, Yücel M, Kayabekir AE, Toklu YC (2022). Analysis of non-linear structural systems via hybrid algorithms. *Proceedings of the 4th International Conference on Intelligent Computing & Optimization (Ico2021)*, Hua Hin, Thailand, 536-545.

Nuhoglu A, Korkmaz KA (2011). Practical nonlinear analysis of tensegrity. *Engineering with Computers*, 27, 337-345.

Ohkubo S, Watada Y, Toshio F (1987). Nonlinear analysis of truss by energy minimization. *Computers & Structures*, 27(1), 129-145.

Ozdemir MA (2025). Seismic performance assessment of timber-framed (hımış) structures with different infill materials. *Challenge Journal of Structural Mechanics*, 11(3), 149-159.

Polat E, Karaman G (2025). Predicting shear strength in reinforced concrete deep beams through finite element modeling of diverse concrete materials. *Challenge Journal of Structural Mechanics*, 11(1), 24-41.

Rajwar K, Deep K, Das S (2023). An exhaustive review of the metaheuristic algorithms for search and optimization: Taxonomy, applications, and open challenges. *Artificial Intelligence Review*, 56(11), 13187-13257.

Reddy JN (2014). An Introduction to Nonlinear Finite Element Analysis. 2nd ed. OUP Oxford, Oxford, UK.

Reddy JN (2019). Introduction to the Finite Element Method. McGraw-Hill Education, New York, USA.

Rhode-Barbarigos L, Ali NBH, Motro R, Smith IF (2012). Deployable tensegrity footbridge design. *International Journal of Space Structures*, 27(2-3), 81-95.

Sabat L, Kundu CK (2020). History of finite element method: A review. In: *Recent Developments in Sustainable Infrastructure: Select Proceedings of Icrdsi 2019*, 395-404.

Schueller W (2008). Building Support Structures with Sap2000. Computers & Structures Inc., Berkeley, USA.

Skelton RE, Helton JW, Adhikari R, Pinaud JP, Chan W (2017). Mechanics of tensegrity structures. In: *Mechanical Systems Design Handbook*. CRC Press, 315-388.

Sonawane A, Hawaldar C, Ankad P, Joshi A (2023). Nonlinear geometric analysis of multilayer tensegrity tower. *Materials Today: Proceedings*, 77, 788-793.

Tarhan İH, Tarhan Y (2025). Nonlinear in-plane response of 3D-printed concrete walls with varied infill patterns: Experimental mix design and numerical structural assessment. *Challenge Journal of Structural Mechanics*, 11(3), 160-173.

Temür R, Bekdaş G, Toklu YC (2014a). Analysis of cable structures through TPO using metaheuristics algorithm. *Proceedings of the 11th International Congress on Advances in Civil Engineering (Ace2014)*, Istanbul, Türkiye.

Temür R, Bekdaş G, Türkan YS, Toklu YC (2014b). Investigating the behavior of the truss structures with unilateral boundary conditions. *Proceedings of the 5th European Conference of Civil Engineering (Eccie'14)*, Florence, Italy, 140-145.

Temür R, Bekdaş G, Toklu YC (2017). Total potential energy minimization method in structural analysis considering material nonlinearity. *Challenge Journal of Structural Mechanics*, 3, 129-133.

Temür R (2021). Plane stress & plane strain by minimization. *Structures*, 33, 728-736.

Thai HT, Kim SE (2011). Nonlinear static and dynamic analysis of cable structures. *Finite Elements in Analysis and Design*, 47(3), 237-246.

Toklu YC (2004a). Application of optimization techniques to structural analyses: Truss example. *Proceedings of Structures Congress 2004: Building on the Past, Securing the Future*, Nashville, Tennessee, USA, 1-12.

Toklu YC (2004b). Nonlinear analysis of trusses through energy minimization. *Computers & Structures*, 82(20-21), 1581-1589.

Toklu YC (2013). Structural analysis, metaheuristic algorithms and the method TPO/MA. *Proceedings of the 14th EU/ME Workshop*, Hamburg, Germany, 44-50.

Toklu YC (2023). Nonlinear analysis of structures by total potential optimization using metaheuristic algorithms (TPO/MA). *Journal of Civil Engineering and Architecture*, 17, 152-160.

Toklu YC, Arditi D (2014). Smart trusses for space applications. *Proceedings of the 14th ASCE Earth & Space Conference*, St. Louis, Missouri, USA.

Toklu YC, Uzun F (2016). TPO/MA for tensegric structures. *Journal of Aerospace Engineering*, 29(5), 04016023.

Toklu YC, Bekdaş G, Temür R (2013a). Analysis of trusses by total potential optimization method coupled with harmony search. *Structural Engineering and Mechanics*, 45(2), 183-199.

Toklu YC, Bekdaş G, Temür R, Toklu ŞÇ, Uzun F (2013b). Nonlinear analysis of structures by total potential optimization using metaheuristic algorithms (TPO/MA). *Proceedings of Eg-Ice 2013*, Vienna, Austria.

Toklu YC, Temür R, Bekdaş G, Uzun F (2013c). Space applications of tensegrity. *Proceedings of Recent Advances in Space Technologies (Rast 2013)*, Istanbul, Türkiye, 29-32.

Toklu SÇ, Aydin N, Toklu YC (2013d). Analyzing structures: Total potential optimization using meta-heuristic algorithms (TPO/MA). In: *Proceedings of the International Symposium on Computing in Science & Engineering*.

Toklu YC, Uzun F, Bekdaş G, Temür R (2014a). Tensegric structures for space applications. *Proceedings of ASCE Earth & Space 2014*, St. Louis, Missouri, USA.

Toklu YC, Yayli MO, Altan B (2014b). Plane stress as optimization. *Proceedings of Metaheuristics & Engineering*, Bilecik University, 45-47.

Toklu YC, Bekdaş G, Temür R (2015a). Investigation of thermal effects on analyses of truss structures via metaheuristic approaches. *Proceedings of the 6th European Conference of Civil Engineering*, Budapest, Hungary, 99-105.

Toklu YC, Temür R, Bekdaş G (2015b). Teaching learning based optimization algorithm for analyses of trusses. In: *Recent Advances in Mechanical Engineering Series*, 17.

Toklu YC, Temür R, Bekdaş G (2015c). Computation of nonunique solutions for trusses undergoing large deflections. *International Journal of Computational Methods*, 12(3), 1550022.

Toklu YC, Bekdaş G, Temür R (2017). Analysis of cable structures through energy minimization. *Structural Engineering and Mechanics*, 62(6), 749-758.

Toklu YC, Bekdaş G, Nigdeli SM (2018). Analysis of under-constrained and unstable structures by the method TPO/MA. *AIP Conference Proceedings*, 1978(1).

Toklu YC, Kayabekir AE, Bekdaş G, Nigdeli SM, Yücel M (2020). Nonlinear plane-stress via TPO. *Journal of Structural Engineering*, 146(11), 04020249.

Toklu YC, Bekdaş G, Nigdeli SM (2021a). Metaheuristics for Structural Design and Analysis. John Wiley & Sons, Hoboken, USA.

Toklu YC, Bekdaş G, Yücel M, Nigdeli SM, Kayabekir AE, Kim S, Geem ZW (2021b). Metaheuristic nonlinear plane-strain. *Applied Sciences*, 11(7), 3220.

Tran HC, Lee J (2011). Geometric and material nonlinear analysis of tensegrity. *Acta Mechanica Sinica*, 27, 938-949.

Wang BB (1998a). Cable-strut systems: Part I. Tensegrity. *Journal of Constructional Steel Research*, 45(3), 281-289.

Wang BB (1998b). Cable-strut systems: Part II. Cable-strut. *Journal of Constructional Steel Research*, 45(3), 291-299.

Zardari S, Kutlu İ, Nayeb A (2024). Investigation of structural performances of historical building elements made with local materials using the finite element method. *Challenge Journal of Concrete Research Letters*, 15(3), 101-111.

Zhang X (2017). Matrix Analysis and Applications. Cambridge University Press, Cambridge, UK.

Zhu B (2018). The Finite Element Method: Fundamentals and Applications in Civil, Hydraulic, Mechanical and Aeronautical Engineering. Wiley, Singapore.

Zienkiewicz OC, Taylor RL, Zhu JZ (2005). The Finite Element Method: Its Basis and Fundamentals. 6th ed. Elsevier, Oxford, UK.