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Research Article

Seismic performance and strain localization in a historic masonry minaret: A comparative evaluation of internal steel strengthening strategies

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ABSTRACT

In this study, the seismic performance of a historic masonry minaret was investigated through nonlinear time-history analyses, and the relationship between global stiffness enhancement and local strain localization was evaluated. Four different internal steel strengthening scenarios, developed within a finite element model constructed by jointly considering experimental studies conducted on similar stone specimens and parameter ranges proposed in the literature, were comparatively analyzed under three distinct ground motion records. The modal analysis results indicated that the natural frequencies increased significantly in all strengthening scenarios, demonstrating a substantial improvement in overall structural stiffness. However, nonlinear dynamic analyses revealed that an increase in stiffness does not necessarily lead to an improvement in seismic performance. Displacement-time histories showed that the onset of instability was delayed in certain strengthening configurations; nevertheless, none of the interventions was able to completely prevent global collapse. The maximum principal strain distributions demonstrated that the collapse behavior was governed by tensile-dominated strain localization developing at the shaft-substructure transition region. In particular, strengthening configurations exhibiting partial continuity were found to induce stiffness discontinuities that could trigger early instability. The results indicate that the seismic performance of masonry minarets is closely related not only to global stiffness enhancement but also to the tensile strength, damage-softening characteristics, and energy dissipation capacity of the material. The study provides significant implications for performance-based strengthening design of masonry structures with cultural heritage value.

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1. Introduction

Türkiye is located within one of the world's most active seismic regions due to the interaction of the Anatolian plate with the Eurasian, African, and Arabian plates, which has generated major tectonic structures such as the North Anatolian Fault, the East Anatolian

Fault, and the Western Anatolian graben system (Bozkurt 2001). As a result, destructive earthquakes occur periodically across the country, causing significant loss of life and property as well as extensive damage to cultural heritage structures. The 6 February 2023 Kahramanmaraş earthquakes once again highlighted the seismic vulnerability of both the existing

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building stock and historic masonry structures in Türkiye (Kocaman 2023).

Anatolia hosts a rich inventory of historic structures dating from the Roman, Byzantine, Seljuk, and Ottoman periods, including mosques, madrasas, churches, bridges, towers, and monumental buildings. Among these structures, masonry minarets represent a distinctive architectural and structural typology. Their slender geometries, masonry construction with limited tensile capacity, and often partially independent structural interaction with the main mosque building make them particularly vulnerable to seismic actions (Yılmaz et al. 2025), consistent with broader risk assessments identifying tower-type historic masonry structures among the highest-priority seismic risk categories (Kahraman et al. 2026).

Historic masonry minarets are generally characterized by a cylindrical shaft supported on a square or polygonal base and terminated by a balcony and conical spire. Structural discontinuities such as transition zones,

door openings, and internal staircases introduce localized weaknesses in the load-bearing system. Under seismic loading, these features lead to high displacement demands, particularly in the upper shaft, balcony, and spire regions, where damage frequently concentrates (Usta 2021).

Recent earthquakes have repeatedly confirmed the seismic vulnerability of such structures. Field observations following the 6 February 2023 events showed that, although many mosque bodies remained largely intact, their minarets either collapsed or experienced severe structural damage. The observed damage patterns closely correspond to collapse mechanisms reported in the literature, with critical damage zones typically occurring at the base-shaft transition, beneath the balcony, along the upper shaft, and within the spire segment (Kocaman et al. 2025). Fig. 1 illustrates typical damage mechanisms observed in historic masonry minarets after earthquakes in Türkiye.



Fig. 1. Examples of historic masonry minarets damaged following earthquakes in Türkiye (Kocaman et al. 2025).

Various strengthening techniques have been proposed to improve the seismic performance of historic masonry minarets. These interventions generally aim to reduce displacement demands associated with their slender geometries and to control damage mechanisms developing in critical regions of the structure.

FRP- and CFRP-based wrapping systems represent one of the commonly investigated approaches. More broadly, fiber- and polymer-based composite systems have been investigated across structural applications for improving mechanical performance (Çelik et al. 2024; Mohd Nazri et al. 2025; Ünal et al. 2025; Khalbous et al.

2026). Jadallah et al. (2025) demonstrated through nonlinear time-history analyses of the Şirvani Minaret that FRP confinement effectively limited tensile crack propagation, particularly in the upper shaft and spire regions, and reduced displacement demands under repeated earthquake excitations. Although FRP strengthening provided only a limited increase in global stiffness, it proved effective in controlling damage progression.

Alternative strategies focus on improving axial continuity and global load transfer along the shaft. Bayraktar and Hökelekli (2021) reported that vertically oriented FRCM-based reinforcement elements acting as longitudinal strengthening components significantly reduced tensile stresses and displacement demands when soil-structure interaction was considered. Such approaches directly influence the global structural response of the minaret.

Circumferential strengthening techniques based on steel bands or tension rings aim to enhance sectional integrity in regions characterized by geometric discontinuities. Kocaman et al. (2025) showed through nonlinear dynamic analyses of the Gaziantep Kabasakal Mosque minaret that circumferential steel rings reduced displacement demands in the upper regions and prevented the development of global collapse mechanisms by improving circumferential stiffness.

More recently, mechanically anchored strengthening techniques have also been proposed. Cavuşlu and Ülger (2025) demonstrated that iron clamp and dowel systems applied to a nineteenth-century stone minaret improved the continuity between masonry units and reduced stress concentrations, particularly in transition regions. Their results highlighted the advantages of mechanically anchored interventions in terms of minimal intervention and reversibility.

Overall, previous studies show that different strengthening techniques improve seismic behavior through distinct structural mechanisms. However, most investigations focus on a single intervention applied to a specific minaret. Comparative evaluations of different strengthening strategies on the same structure and systematic assessments of their influence on damage mechanisms remain limited. Consequently, the selection of an optimal strengthening strategy depends on multiple factors including the geometric characteristics of the minaret, existing damage conditions, target performance levels, conservation constraints, and implementation feasibility.

Within this framework, the present study investigates the historic masonry minaret of Yakutiye Madrasa located in Erzurum, a region characterized by significant seismic hazard. A detailed finite element model was developed to represent the geometric and structural characteristics of the minaret, with material parameters defined through a combination of field observations, experimental data obtained from stone specimens, and information reported in the literature. Three representative ground motion records were scaled to match the target design spectrum, and nonlinear time-history analyses were conducted to evaluate the existing seismic response and potential damage mechanisms.

Based on the identified vulnerabilities, four strengthening scenarios were developed in accordance with the

conservation principle of minimum intervention and evaluated under identical seismic inputs. These scenarios include (i) a stepped internal plate system integrated along the spiral staircase, (ii) localized strengthening elements applied in critical transition regions, (iii) a distributed internal steel plate system along the shaft, and (iv) a hybrid system combining internal steel plates with longitudinal reinforcement elements. All strengthening configurations were analyzed using the same nonlinear dynamic framework to allow direct comparison of their structural performance.

Through this approach, the study systematically evaluates the relative effectiveness of different strengthening strategies on the seismic response of a historic masonry minaret under identical structural and seismic conditions. Rather than proposing a single strengthening solution, the objective is to provide a mechanism-based comparative framework that can support the selection of appropriate intervention strategies for similar historic masonry towers.

2. Yakutiye Madrasa and Minaret

Yakutiye Madrasa was constructed in the early 14th century during the Anatolian Seljuk–Ilkhanid period and represents one of the most significant monumental structures located in the city center of Erzurum. The building is considered an authentic example of the Anatolian madrasa tradition and reflects the architectural, artistic, and structural characteristics of Seljuk-era educational complexes. Due to their outstanding cultural and historical significance, Anatolian madrasas are recognized within the UNESCO World Heritage Tentative List framework, and Yakutiye Madrasa is considered one of the most prominent representatives of this architectural tradition (UNESCO 2014). Therefore, the structure possesses not only architectural importance but also significant cultural heritage value requiring careful preservation.

The minaret, constructed adjacent to the madrasa, constitutes one of the most visually distinctive and structurally sensitive components of the complex. It consists of a cylindrical shaft seated on a square base and terminated by a conical spire. According to the geometric dimensions illustrated in Fig. 2, the total height of the minaret is approximately 24.7 m, the external diameter of the shaft is about 3.18 m, and the base dimensions are approximately 3.18 m × 3.18 m. The internal spiral staircase core has a diameter of approximately 0.50 m. Abrupt cross-sectional transitions at the base–shaft and shaft–spire interfaces create geometric discontinuities that represent critical regions where damage may concentrate under seismic loading.

The load-bearing system of the minaret consists of irregularly arranged masonry stone units. Over time, the mortar layers between the stone blocks have partially lost their mechanical properties due to environmental effects and material aging, which is particularly relevant because mortar bond and physical properties strongly influence the integrity of masonry systems (Kara et al. 2025). The negligible tensile strength of masonry and the discontinuous nature of stone–mortar interfaces

make such systems highly susceptible to crack initiation and brittle behavior under seismic loading (Lourenço 2006). In addition, the internal spiral staircase extending

along the shaft introduces localized cross-sectional weaknesses at certain elevations, reducing the continuity of the load-bearing system.

General view of Yakutiye Madrasah

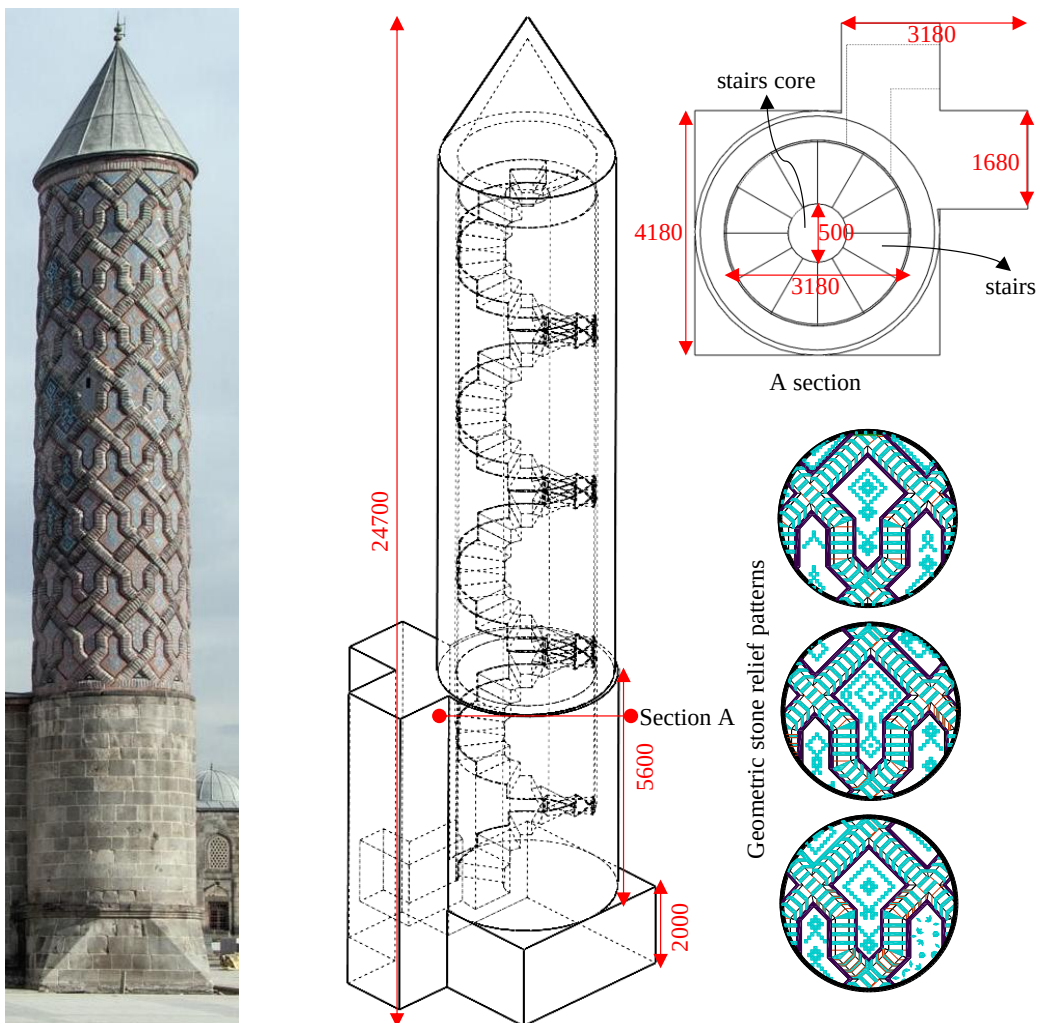


Fig. 2. Details of Erzurum Yakutiye Madrasa.

The exterior surface of the minaret is decorated with interlaced geometric stone reliefs, representing one of the most refined examples of Seljuk stone craftsmanship. Although these decorative elements enhance the cultural and artistic significance of the structure, they also create local protrusions and recesses on the surface, leading to geometric irregularities. Previous studies indicate that such relief-type geometries may influence local stiffness distribution and generate stress concentrations in certain regions (Binda et al. 2000). Therefore, these geometric features should be considered when evaluating the seismic behavior of the structure.

The Yakutiye Madrasa minaret has largely preserved its original materials and construction techniques and has not undergone any major reconstruction interventions. However, the harsh climatic conditions of Erzurum, particularly freeze–thaw cycles and large temperature fluctuations, have contributed to the gradual deterioration of both stone and mortar materials. Such environmental effects may lead to progressive weakening of material properties and consequently influence the seismic performance of the structure (ICOMOS 2003).

Considering these characteristics collectively, the Yakutiye Madrasa minaret represents a significant case study for seismic performance assessment and strengthening strategies. As an authentic Seljuk-period cultural heritage structure that has preserved its original construction features while exhibiting geometric and material discontinuities, the minaret provides a valuable opportunity to investigate the seismic behavior of historic masonry towers.

3. Numerical Modeling Approach and Finite Element Model

In recent years, the seismic resilience of historic structures such as churches, bridges, mosques, towers, and minarets has received increasing attention in earthquake engineering research (Kocaman et al 2025; Ferrante et al 2021; Orlando 2023; Colak Demirel 2024; Requena-Garcia-Cruz 2023). The preservation of these structures requires reliable prediction of their behavior under seismic loading and accurate identification of potential damage mechanisms. However, assessing historic masonry structures remains challenging due to their complex geometries, traditional construction techniques, heterogeneous material properties, and the limited availability of experimental data. Additional uncertainties related to boundary conditions, transition regions, connection details, and past repair interventions further increase the complexity of numerical analyses. Consequently, the reliable evaluation of their seismic performance requires the use of advanced numerical modeling approaches (Kocaman and Gürbüz 2023; Çelik et al. 2025; Kiral et al. 2024; Romero-Sanchez et al. 2024; Ergün and Tayfur 2024).

Two main numerical modeling strategies are commonly adopted in the analysis of historic masonry structures: micro-modeling and macro-modeling. In the micro-modeling approach, masonry constituents such as stone units and mortar layers are modeled separately, allowing detailed investigation of local damage

mechanisms. However, the high computational cost and long solution times limit its applicability in large-scale nonlinear dynamic analyses. In contrast, the macro-modeling approach represents masonry as an equivalent homogenized continuum, providing a computationally efficient framework for structural-scale simulations. Particularly in nonlinear time-history analyses under seismic loading, macro-modeling significantly reduces computational demand while maintaining reliable predictive capability. For this reason, nonlinear dynamic analyses based on macro-modeling are widely used in the seismic assessment of historic masonry structures (Valente 2021; Valente 2022; Kocaman et al. 2024).

Within the scope of this study, the three-dimensional geometric model of the Yakutiye Madrasa minaret was developed using SolidWorks software based on detailed architectural measurements. During the modeling process, superficial stone ornaments and decorative details with negligible influence on the structural behavior were excluded. The resulting three-dimensional geometric model was subsequently transferred to the ANSYS APDL environment, where the finite element analyses were conducted (Fig. 3). Rigid ground conditions were assumed at the foundation level, and the base was fully restrained in all degrees of freedom.

The finite element mesh was generated using the eight-node SOLID65 solid-element formulation available in ANSYS APDL (ANSYS 2024). Owing to the geometric complexity of the minaret, the tetrahedral degeneration available for SOLID65 was employed by assigning coincident or repeated nodal positions at the collapsed vertices. Accordingly, each tetrahedral-shaped mesh cell contains four distinct geometric vertices while retaining the eight-node SOLID65 connectivity convention. A nominal element size ranging from 15 to 20 cm was adopted. The final finite element model comprised approximately 258,209 elements and 54,563 nodes and provided sufficient geometric resolution for the global nonlinear analyses. However, the numerical convergence of the adopted discretization was not independently verified through a formal mesh-sensitivity analysis.

Consistent with the mesh definition provided above, SOLID65 is an eight-node, three-dimensional solid element with three translational degrees of freedom at each nodal position. It is widely used in the numerical analysis of historic masonry structures because of its capability to simulate nonlinear damage mechanisms, including tensile cracking and compressive crushing. The element formulation allows the representation of crack initiation under tensile stresses, crushing under compressive stresses, and limited plastic deformation. The standard element geometry, node-numbering convention, integration points, and tetrahedral degeneration adopted in the finite element mesh are illustrated in Fig. 4a.

Material behavior in SOLID65 is initially defined as isotropic; however, during the analysis independent crack formation can occur in three principal directions at each integration point. Once cracking initiates, the stiffness matrix is updated according to crack orientation, enabling the numerical representation of multiaxial cracking and separation behavior typical of masonry materials.

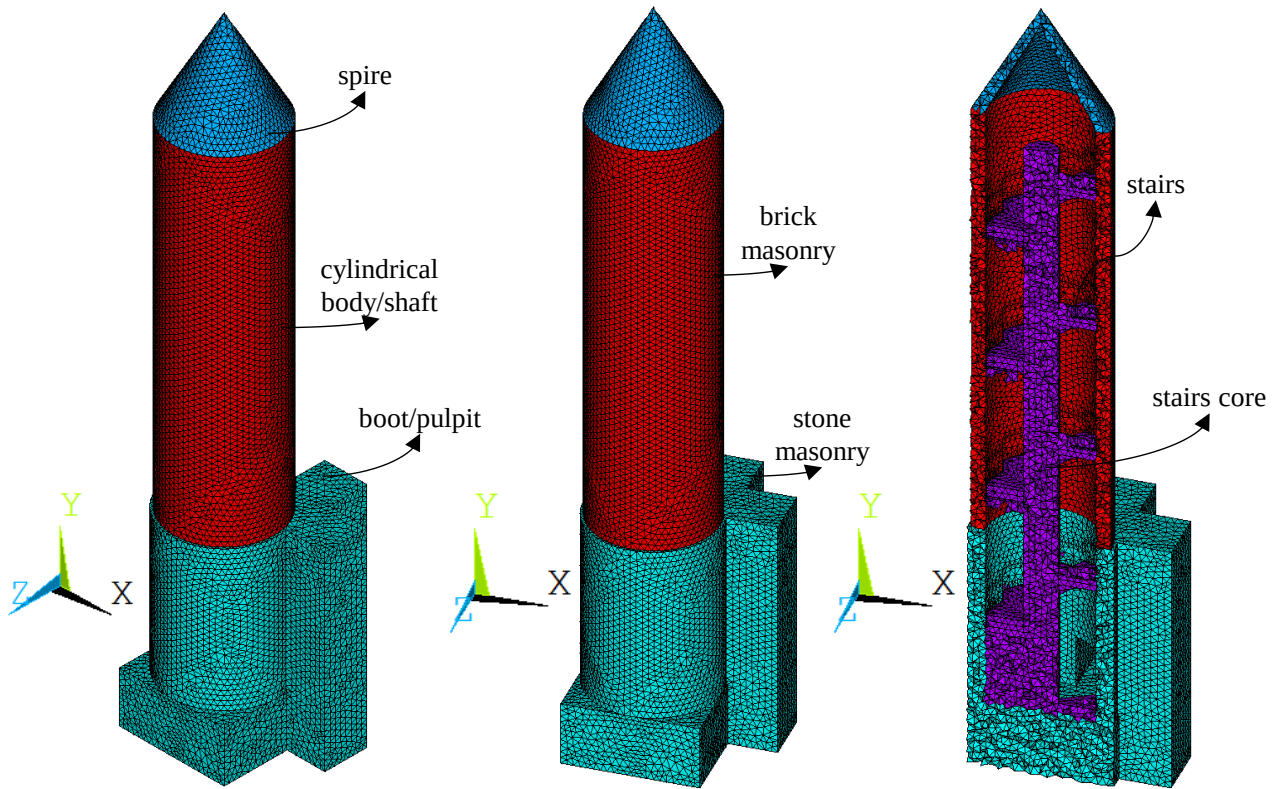


Fig. 3. Finite element model of the Yakutiye Madrasa minaret.

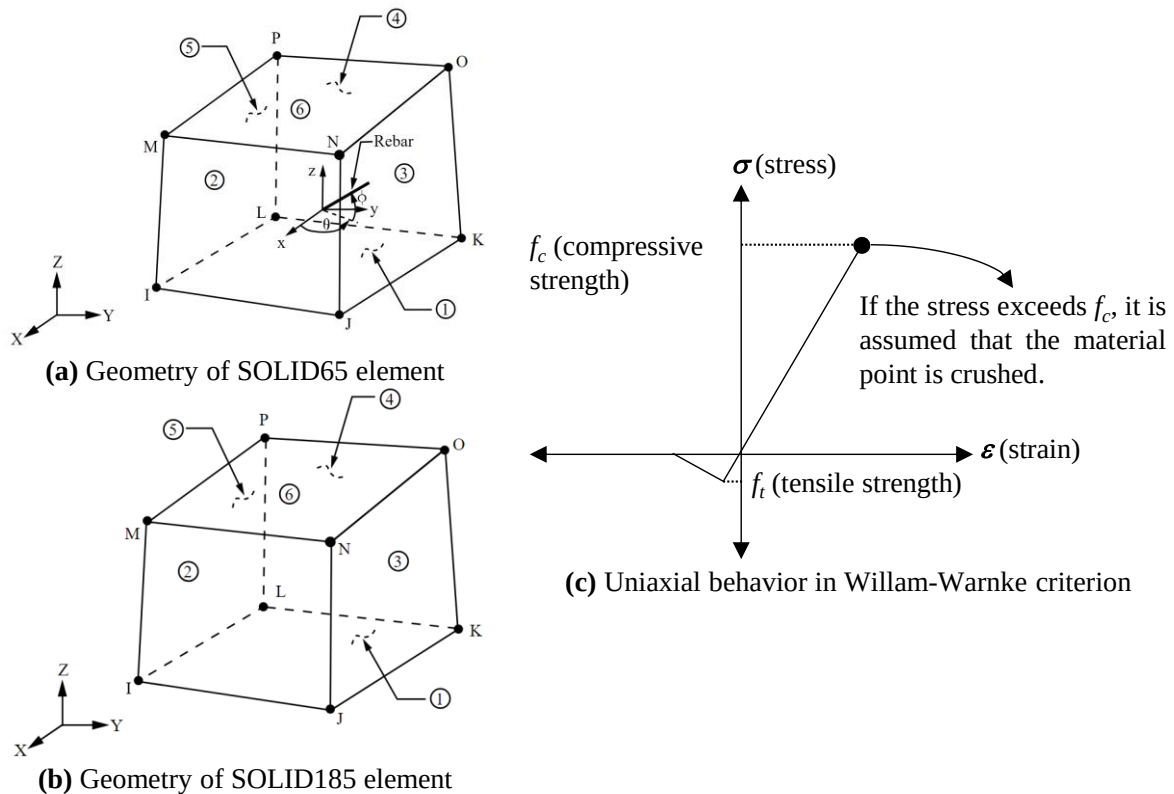


Fig. 4. Finite-element formulations and constitutive representation adopted in the numerical models: (a) Parent topology of the eight-node SOLID65 element used for masonry; (b) Parent topology of the eight-node SOLID185 element used for the S235 steel strengthening components; and (c) Schematic uniaxial tensile and compressive limits associated with the Willam–Warnke cracking–crushing criterion (Willam and Warnke 1975; ANSYS 2024).

In this study, the nonlinear behavior of the masonry material was modeled using the five-parameter Willam–

Warnke failure criterion (Karaton and Çanakçı 2020; Bartoli et al. 2015; Nebioğlu and Kocaman 2025). This

model defines a three-dimensional failure surface based on the uniaxial compressive strength (f_c) and tensile strength (f_t) of the material and accounts for crushing under uniaxial, biaxial, and triaxial stress states. When the crushing limit is exceeded, the stiffness contribution of the material at the corresponding integration points is eliminated, allowing the simulation of advanced damage and pre-collapse behavior. The corresponding failure surface and stress–strain relationships are presented in Fig. 4c.

Although the Willam–Warnke model was originally developed for concrete and geomaterials, it has been widely adopted in the numerical modeling of masonry systems. While the model does not explicitly represent post-peak softening, the collapse behavior of historic masonry minarets is largely governed by cracking, separation, and block-type mechanisms associated with low tensile strength. Therefore, the standalone use of the Willam–Warnke model was considered sufficient for the purposes of this study (Kocaman 2024).

The key parameters defining the damage surface include the shear transfer coefficients for open and closed cracks (β_t and β_c), as well as the compressive and tensile strengths of the material. In accordance with literature recommendations, $\beta_t=0.05$ was adopted for open cracks and $\beta_c=0.8$ for closed cracks (Kocaman and Gürbüz 2023). While SOLID65 was used to model the nonlinear behavior of masonry, the steel strengthening components were represented using the SOLID185 element, which is more suitable for modeling elastic–plastic material behavior and does not require crack–crushing capabilities. The geometric configuration and main characteristics of the SOLID185 element are illustrated in Fig. 4b.

All steel plate and bar components used in the strengthening scenarios were modeled using SOLID185 elements and assigned the MAT-5 material definition. MAT-5 represents isotropic S235 structural steel with an elastic modulus of 200,000 MPa, a Poisson's ratio of 0.30, a nominal yield strength of 235 MPa, and a mass density of 7850 kg/m³. The post-yield response was represented by a rate-independent elastic-perfectly plastic von Mises model; therefore, no strain hardening was considered and the post-yield tangent modulus was taken as zero.

The interfaces between the steel plates and masonry, between the longitudinal steel bars and masonry, and between connected steel components were idealized as fully bonded. Numerically, displacement compatibility was imposed in all three translational directions at the corresponding interface nodes. Consequently, relative normal separation and tangential slip were prevented throughout the analyses. No discrete anchor bolts, adhesive layers, frictional contact elements, or cohesive interface elements were explicitly modeled. This idealization represents the collective action of sufficient distributed anchorage and provides an upper-bound representation of composite load transfer between the steel strengthening system and the existing masonry.

4. Material Properties

In the numerical modeling of historic masonry structures, the definition of material properties represents

one of the most critical and uncertain stages of the analysis process. In masonry systems composed of stone and brick units, such as the Yakutiye Madrasa minaret, a single representative material set cannot be precisely defined due to the heterogeneity of masonry units, aging of mortar layers, and variability of stone–mortar interfaces. Therefore, homogenized (macro-scale) material parameters were adopted based on ranges reported in the literature for similar masonry systems, together with field observations and the existing structural characteristics. The material model was defined consistently with the cracking–crushing-based nonlinear modeling approach using compressive strength (f_c), tensile strength (f_t), modulus of elasticity (E), and density (ρ).

Five different material definitions were used in the finite element model. MAT-1 represents the primary load-bearing stone masonry walls, MAT-2 corresponds to the core and stair system, MAT-3 represents brick wall components, MAT-4 corresponds to the dome element, and MAT-5 represents the steel elements used in the strengthening scenarios. The strengthening components were modeled as S235 structural steel (MAT-5).

Due to the preserved condition and cultural heritage status of the structure, direct sampling from the Yakutiye Madrasa minaret was not possible. Based on field observations, material texture, and the geological characteristics of the Erzurum region, the stone used in the structure was identified as andesitic volcanic stone. To represent its mechanical behavior, sixteen cubic specimens (5 × 5 × 5 cm) were obtained from a local quarry supplying similar building stones for the region. Uniaxial compression tests were performed on these specimens, and the modulus of elasticity values were derived from the resulting stress–strain relationships.

The experimental results indicated that the compressive strengths of the stone specimens ranged between 33.8 MPa and 89.8 MPa, with an average value of approximately 72 MPa. Elastic modulus values were calculated from the stress–strain curves at 30% and 60% stress levels, yielding E_{30} values between 1893 MPa and 7832 MPa and E_{60} values between 2356 MPa and 9123 MPa. The observed failure modes and representative stress–strain curves of the tested specimens are presented in Fig. 5.

The experimental results indicate that the andesitic stones exhibit relatively high compressive strength. However, in historic masonry structures the effective mechanical properties at the structural scale are typically much lower than those of individual stone units due to weak mortar layers, discontinuities at stone–mortar interfaces, internal infill materials, and irregularities associated with traditional construction techniques. Therefore, the values obtained directly from stone unit tests were not used in the finite element model. Instead, homogenized masonry parameters were adopted to represent the composite behavior of the stone–mortar–infill system.

In defining these parameters, the methodology reported in a previous investigation on the structural behavior of the Yakutiye Madrasa was taken as a reference (Kocaman et al. 2018). That study demonstrated that, although individual stone blocks possess relatively high compressive strength, the effective strength of masonry walls decreases considerably due to mortar layers, inter-

nal infill materials, and multi-leaf wall configurations. Based on empirical relationships reported in the literature, the compressive strength of cut stone masonry walls was estimated to have an upper bound of approximately 13.45 MPa (Kocaman et al. 2018).

However, these empirical relationships were primarily developed for planar masonry walls with relatively

limited discontinuities. Masonry minarets exhibit a more complex structural behavior due to their cylindrical geometry, cross-sectional transitions, internal cavities, and spiral staircase systems. Consequently, directly adopting compressive strength values derived for planar walls may lead to overly optimistic estimations of the seismic performance of the minaret shaft.

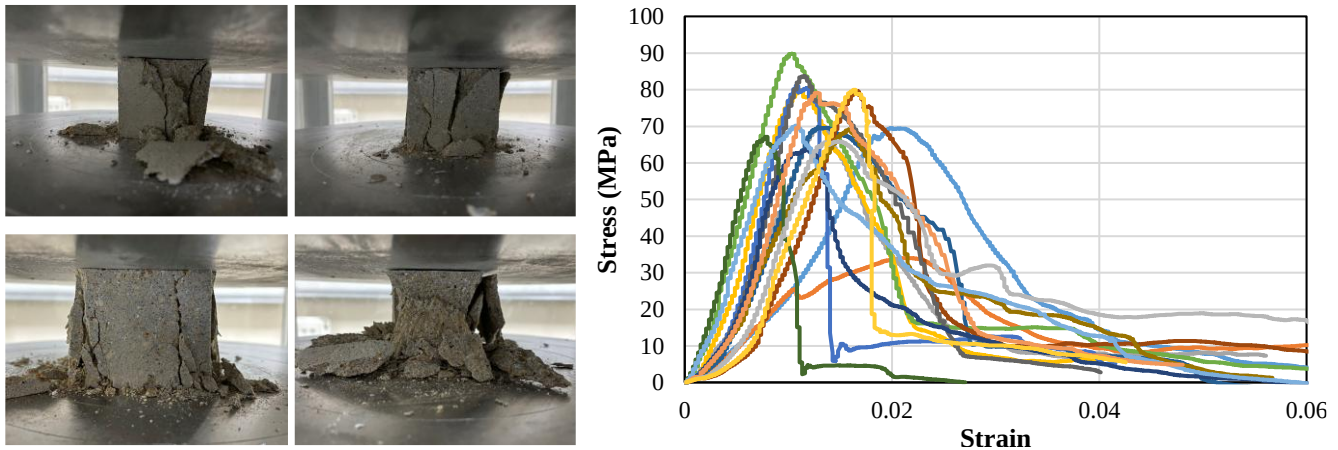


Fig. 5. Observed failure modes during compression tests of stone unit specimens.

For this reason, the homogenized masonry parameters adopted in this study were determined by considering both empirical findings and recommendations from national and international guidelines. According to the Guideline for the Management of Seismic Risks of Historic Structures in Türkiye, the compressive strength of well-cut stone masonry walls ranges between 3.0 MPa and 4.0 MPa (VGM 2017). Similarly, Eurocode 6 recommends defining the modulus of elasticity proportionally to the compressive strength using the relationship $E \approx (800-1000) \cdot f_c$ (CEN 2005).

Accordingly, a compressive strength of $f_c=3.0$ MPa and an elastic modulus of $E=3000$ MPa were adopted for the primary load-bearing stone walls of the minaret (MAT-1). The core region and spiral staircase system were assumed to exhibit a more integrated structural response; therefore, $f_c=7.0$ MPa and $E=7000$ MPa were assigned to these regions (MAT-2). This approach provides a conservative yet realistic representation of the structural behavior of the minaret.

Direct sampling of brick walls and dome elements was not feasible; therefore, their mechanical properties were defined based on experimental results reported in the literature. Long-term environmental exposure and aging may also lead historic brick units to exhibit distinct physical and mechanical characteristics (Niroula and Bhatt 2026). A study examining Ottoman-era masonry structures in the Erzincan region reported an average compressive strength of approximately 6.32 MPa for brick units obtained from $50 \times 50 \times 50$ mm specimens (Çelik et al. 2025).

Since the mechanical behavior of brick masonry depends on both brick and mortar properties, the empirical relationship proposed by Kaushik et al. (2007) was used to estimate the compressive strength of brick masonry elements:

$$f_m = 0.63 \times f_b^{0.49} \times f_j^{0.32} \quad (1)$$

where f_m is the compressive strength of brick masonry, f_b represents the compressive strength of the brick unit, and f_j denotes the mortar compressive strength. Using $f_b=6.32$ MPa and $f_j=3.1$ MPa for weak mortar conditions, the compressive strength of brick masonry elements was calculated as approximately 2.2 MPa, which is consistent with the ranges reported for historic masonry walls.

In the same study, the modulus of elasticity of brick masonry was suggested to be estimated using the relationship $E=550 \cdot f_m$ (Kaushik 2007). Based on this relationship, the elastic modulus was determined to be approximately 1225 MPa for brick walls (MAT-3) and 1500 MPa for dome elements (MAT-4). For all masonry components, the tensile strength was assumed to be approximately 10% of the compressive strength, consistent with common engineering practice. MAT-4, representing the dome masonry, was retained as a separate homogenized material definition because of the component-specific masonry arrangement. A compressive strength of $f_c=2.72$ MPa was adopted for this region. This value falls within the compressive-strength range of 2.4–4.0 MPa reported for historic solid-brick masonry constructed with lime mortar (Corradi et al. 2018). Applying the relationship $E=550 \cdot f_m$ gives $E=1496$ MPa; therefore, the elastic modulus of MAT-4 was taken as $E=1500$ MPa.

Through this approach, experimental findings, empirical relationships, and code recommendations were jointly considered to define a conservative and representative material set for the finite element model (Table 1). Despite the relatively high compressive strength of individual stone units, the adopted homogenized parameters account for the reduced effective mechanical properties at the wall and minaret scale and allow realistic simulation of cracking, separation, and collapse mechanisms in the nonlinear analyses.

Table 1. Homogenized material properties adopted in the finite element model.

Material	Structural component	f_c (MPa)	f_t (MPa)	f_y (MPa)	E (MPa)	ρ (kg/m ³)
MAT-1	Stone masonry walls	3.00	0.30	–	3000	2500
MAT-2	Core and stair system	7.00	0.70	–	7000	2800
MAT-3	Brick masonry walls	2.20	0.22	–	1225	2350
MAT-4	Dome element	2.72	0.15	–	1500	2350
MAT-5	Steel plates and longitudinal bars	–	–	235	200000	7850

5. Existing Condition of Reference Model

Although numerous strengthening techniques have been proposed to improve the seismic behavior of historic masonry minarets, the effectiveness of such interventions strongly depends on the geometry, material properties, and governing damage mechanisms of the specific structure. Therefore, a reliable assessment of the existing structural condition is required before proposing any strengthening strategy.

In this section, the current seismic response of the Yakutiye Madrasa minaret is evaluated using the Reference Model (RM), representing the unstrengthened condition of the structure. The objective is to establish a baseline for comparison with the strengthening scenarios examined in the following sections and to determine the initial seismic vulnerability of the minaret.

First, modal analyses were performed to identify the dominant dynamic characteristics and deformation patterns of the structure. Subsequently, nonlinear time-history analyses were conducted using the selected ground motion records to evaluate displacement demands, damage initiation, and stress-strain accumulation in critical regions. Based on these results, the governing damage concentration zones and potential collapse mechanisms of the minaret were identified, providing the basis for the strengthening strategies investigated in the subsequent sections.

5.1. Modal characteristics of the reference model

Linear elastic modal analyses were performed for the Reference Model (RM) representing the existing condition of the Yakutiye Madrasa minaret. The aim of these analyses was to identify the dynamic characteristics of the structure and determine the dominant vibration modes that may influence its seismic behavior. Accordingly, natural frequencies, vibration periods, and modal mass participation ratios were evaluated.

The first three natural frequencies of the reference model were calculated as 1.90 Hz, 2.02 Hz, and 8.15 Hz, respectively. The close proximity of the first two frequencies indicates similar stiffness characteristics in the two principal horizontal directions and confirms that the dynamic behavior of the minaret is dominated by flexural response under horizontal seismic excitation. Such behavior is typical for slender masonry minarets with cylindrical shafts. The third mode, characterized by a higher frequency, reflects localized deformations primarily concentrated in the upper shaft and transition regions (Fig. 6).

The effective modal mass participation ratios indicate that the first two modes represent the principal global lateral-bending modes but do not, by themselves, dominate the overall dynamic response. In the X direction, the cumulative effective mass participation ratios of the first two and first three modes are approximately 27.36% and 30.17%, respectively. The corresponding values in the Z direction are 27.55% and 30.92%. Mode-shape assessment further indicates that the third mode is characterized predominantly by torsional deformation. The remaining effective mass is therefore distributed among the higher modes, including modes associated with higher-order global deformation and localized response mechanisms. Consequently, the nonlinear time-history response should be interpreted as the combined contribution of lower and higher vibration modes, although the first two modes remain important in describing the principal global lateral behavior of the minaret.

In addition, a notable participation in the rotational (ROTY) direction is observed starting from the first mode. This suggests that bending-torsion interaction cannot be neglected despite the approximate axial symmetry of the structure, mainly due to stiffness and mass discontinuities caused by the stair core and transition regions. Such characteristics indicate that damage accumulation and crack concentration may develop in the upper shaft and transition zones during seismic excitation.

Overall, the modal analysis results reveal that the Yakutiye Madrasa minaret exhibits a low-frequency dynamic character dominated by horizontal flexural behavior with noticeable torsional interaction. This dynamic behavior implies significant displacement demands under seismic loading and indicates potential damage concentration in the upper shaft and transition regions, highlighting the structural vulnerability of the minaret in its current condition.

5.2. Nonlinear dynamic response of the reference model

Nonlinear time-history analyses were performed to evaluate the seismic behavior of the Yakutiye Madrasa minaret. In order to represent realistic seismic conditions, three recorded earthquake ground motions characteristic of strong earthquakes in Türkiye were selected: the 1992 Erzincan earthquake (Mw=6.8), the 1999 Gölcük–İzmit earthquake (Mw=7.6), and the 1999 Düzce–Bolu earthquake (Mw=7.1). These events represent the seismic characteristics of both the East Anatolian and North Anatolian fault systems and allow the structural response of the minaret to be examined under different frequency contents and acceleration levels. The acceleration records were obtained from the AFAD TADAS database (AFAD 2026).

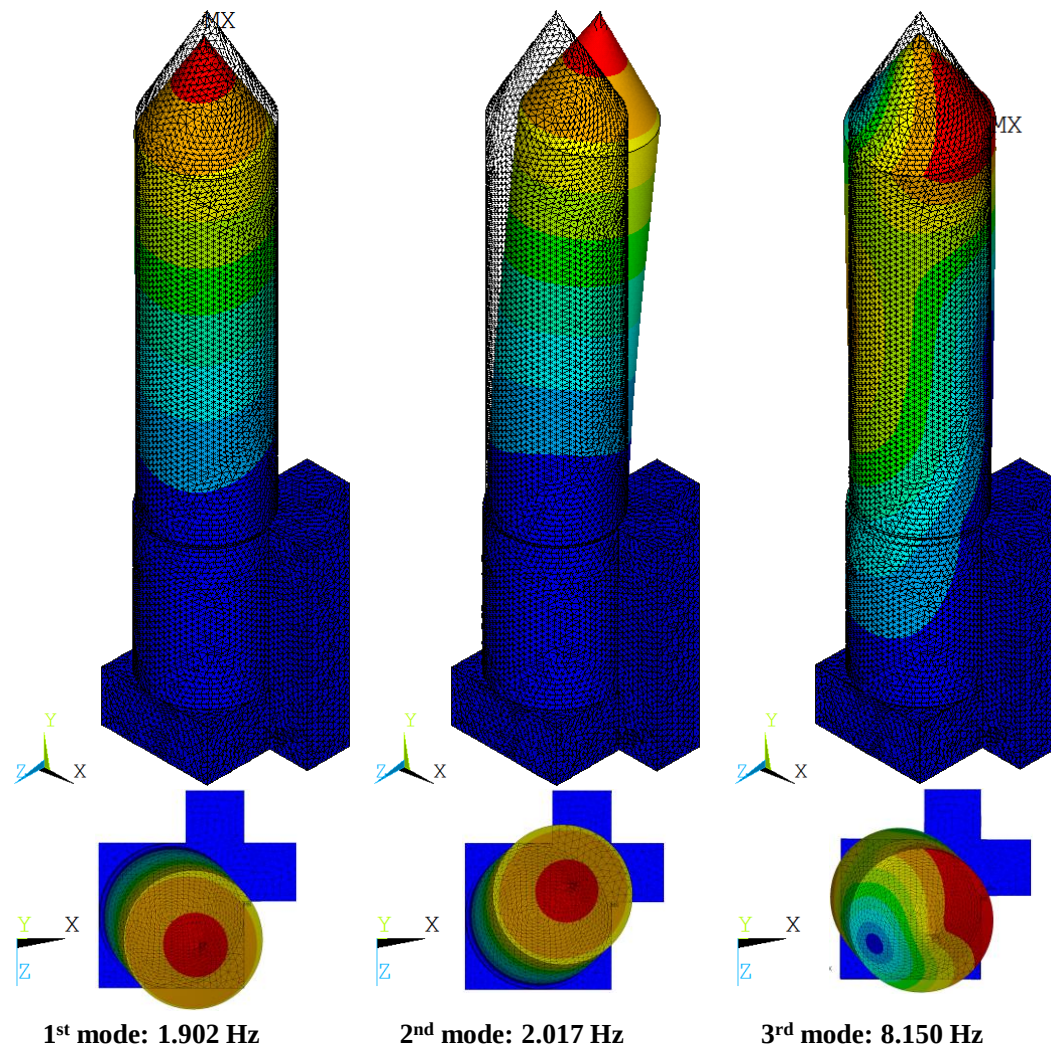


Fig. 6. Mode shapes corresponding to the first three natural vibration modes obtained for RM.

For each earthquake record, the two horizontal components (N-S and E-W) were scaled according to the provisions of the Turkish Building Earthquake Code (TBEC-2018). The scaling procedure was based on the dominant vibration period obtained from the modal analysis of the reference model. Accordingly, the average of the scaled response spectra was ensured to be not less than 90% of the target elastic design spectrum within the period range of $0.2T_p$ – $1.5T_p$. The same scaling factor was applied simultaneously to both horizontal components of each record.

Modal analyses were performed not only for the Reference Model (RM) but also for all strengthening scenarios (RET-1, RET-2, RET-3, and RET-4). Since strengthen-

ing interventions modify the dynamic properties of the structure, scaling coefficients were recalculated separately for each model. This procedure ensured that all nonlinear analyses were conducted within a code-compliant and mutually comparable analytical framework.

The main seismic parameters of the selected ground motions, including earthquake name, year, magnitude, station code, peak ground acceleration values, and the scaling factors used for each model, are presented in Table 2.

The acceleration–time histories of the selected ground motions are presented in Fig. 7, and these records were used as input for the nonlinear dynamic analyses.

Table 2. Characteristics of the selected ground motions and corresponding scaling coefficients.

ID	Event	Year	Mw	Station Code	PGA (gal)		Scale Factor				
					N-S component	E-W component	RM	RET-1	RET-2	RET-3	RET-4
GM-1	Merkez Erzincan	13.03.1992	6.8	Center	405.007	479.543	0.85	0.84	0.99	0.99	0.97
GM-2	Gölcük İzmit	17.08.1999	7.6	8101	314.340	365.930	0.98	0.98	1.04	1.04	1.02
GM-3	Düzce Bolu	12.11.1999	7.1	1401	719.330	803.830	0.55	0.54	0.57	0.57	0.58

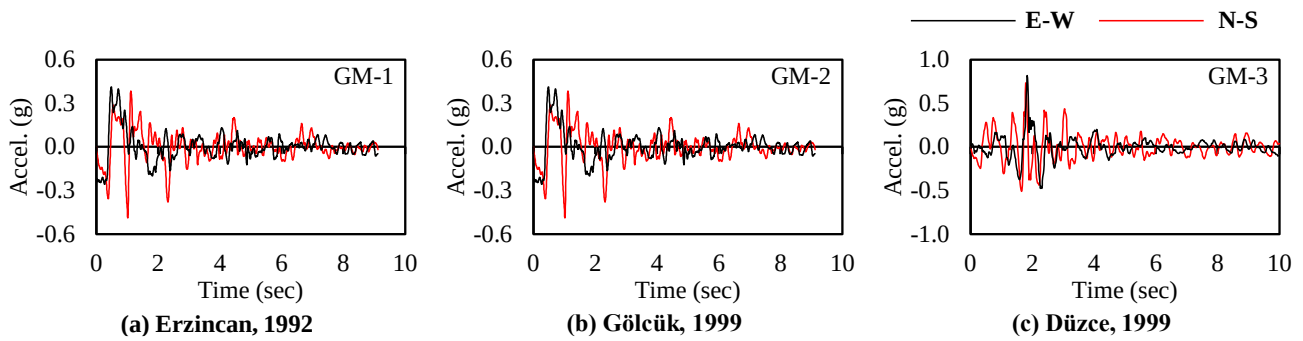


Fig. 7. Acceleration–time histories of the selected ground motions.

Considering the high computational demand of nonlinear time-history analyses, the duration of the earthquake records was limited to the time interval representing the significant portion of seismic energy. For this purpose, the records were truncated according to the Arias Intensity criterion proposed by Trifunac and Brady (1975), and only the interval corresponding to 5%–95% of the cumulative Arias intensity was used in the analyses. This approach ensures that the most energy-relevant portion of the ground motion is considered while reducing computational cost.

The nonlinear dynamic response of the reference model was evaluated using displacement–time histories

obtained from the three selected ground motions (GM-1, GM-2, and GM-3). Horizontal displacements were monitored at the top of the minaret, which represents the most dynamically sensitive region due to the slender geometry and flexure-dominated structural behavior.

The displacement–time histories shown in Fig. 8 indicate that the reference model exhibits significant displacement demands under all ground motions. For GM-1, horizontal displacements at the minaret top reached approximately ± 80 – 100 mm, and the response did not return to its initial position during the later stages of the record, indicating the development of permanent deformations and stiffness degradation.

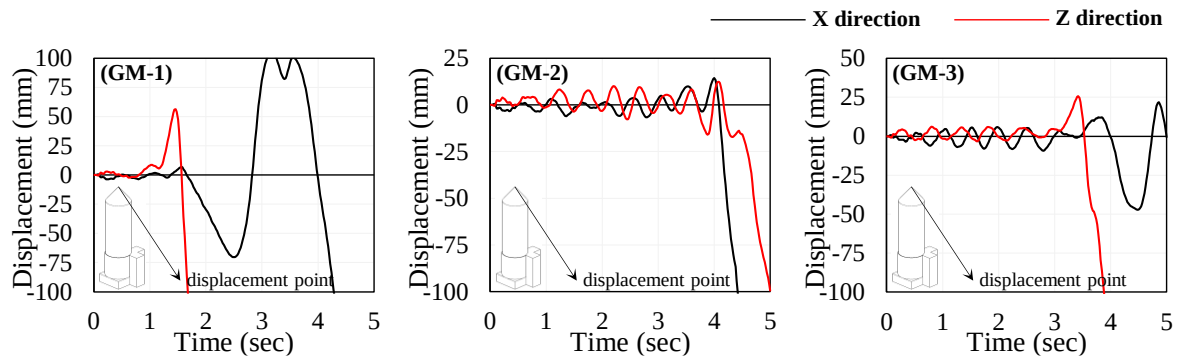


Fig. 8. Displacement–time histories obtained from dynamic analyses (for the RM model).

Under GM-2, the displacement response was slightly lower but still critical, reaching approximately ± 60 – 80 mm. The relatively long duration and low-frequency content of this ground motion interacted with the dominant vibration periods of the minaret, resulting in progressive amplification of displacement demands.

In contrast, GM-3 produced a different response pattern due to its impulsive characteristics and high peak accelerations. Although the displacement range remained within approximately ± 50 – 70 mm, sudden displacement jumps occurred over short time intervals. Such abrupt increases are particularly unfavorable for masonry structures with limited ductility.

Although specific displacement limits for minarets are not defined in the literature, comparison with performance limits proposed for historic masonry mosques and domed structures indicates that the displacement demands obtained in the reference model exceed commonly accepted thresholds (Kocaman 2024). The observed non-recoverable displacement increases suggest

that the structure cannot effectively dissipate seismic energy and that nonlinear behavior develops at an early stage.

Overall, the displacement histories demonstrate that the Yakutiye Madrasa minaret exhibits significant seismic vulnerability in its current condition. These results also provide the baseline for evaluating the effectiveness of the strengthening scenarios presented in the following sections and will be further interpreted together with the strain and damage distributions discussed subsequently.

The first principal strain (ε_1) distributions obtained from the nonlinear dynamic analyses of the reference model clearly reveal the dominant damage mechanism of the minaret. The ε_1 contours presented in Fig. 9 show that, under all ground motions (GM-1, GM-2, and GM-3), damage develops in a similar spatial pattern. This consistency indicates that damage evolution is governed primarily by the geometric and structural characteristics of the minaret rather than by the specific characteristics of the earthquake records.

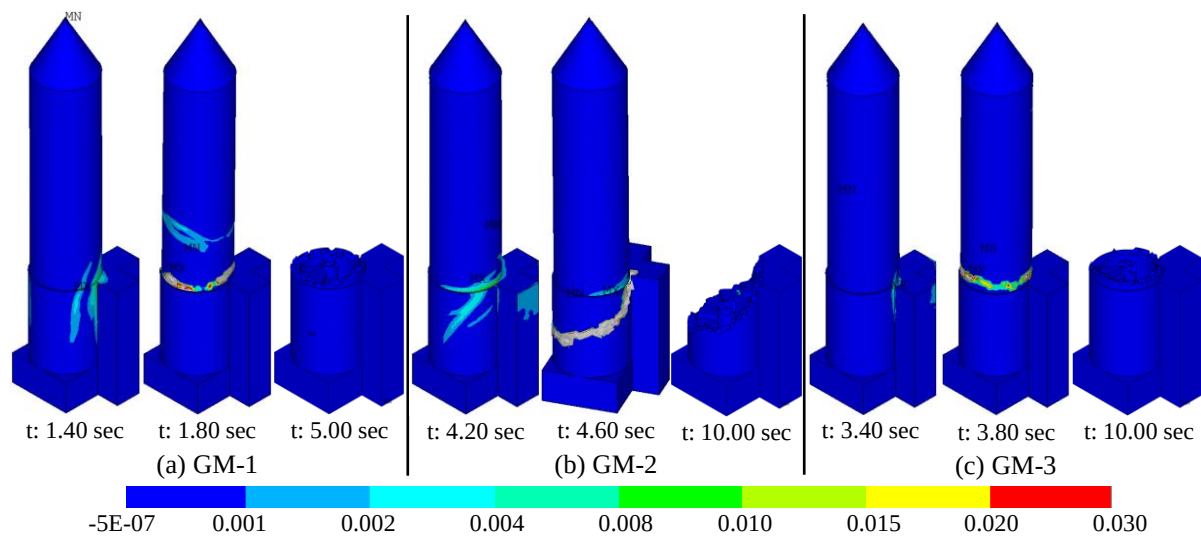


Fig. 9. Damage mechanisms obtained from the nonlinear dynamic analyses (for the RM model).

In all analyses, strain concentrations first emerge in the transition region between the minaret shaft and the base/substructure. In this zone, elevated ε_1 values form a discontinuous yet circumferential band, indicating the development of a tension-dominated strain field. Given the low tensile capacity of masonry materials, these strain levels correspond to the initiation and propagation of cracking, consistent with numerical evidence that historic stone and brick elements are particularly vulnerable in tension under seismic loading (Zardari et al. 2024).

As seismic excitation continues, the strain concentration in the transition region becomes progressively more continuous around the shaft circumference, weakening the effective load-carrying capacity of the cross-section. The circumferential distribution of elevated ε_1 values indicates that the damage mechanism involves not only localized cracking but also progressive degradation of the structural integrity of the entire cross-section. This behavior highlights the transition region—characterized by abrupt stiffness changes and slender geometry—as the most seismically vulnerable part of the minaret.

The consistent damage pattern observed under all ground motions suggests that the governing collapse mechanism of the reference model involves circumferential tensile cracking in the transition region followed by progressive weakening of the shaft–substructure interaction. These strain-based observations provide direct insight into the location and evolution of damage and form the basis for the strengthening strategies proposed in the subsequent sections.

Collapse was not defined solely by the termination of the nonlinear solution or by the loss of numerical convergence. Instead, the occurrence and time of collapse were determined through a combined and mutually corroborating evaluation of the global displacement response, literature-based drift limits, first-principal-strain distributions, crack-line continuity, and the development of physically plausible regional failure mechanisms.

The global drift ratio was calculated as the relative lateral displacement between the top and reference base levels divided by the corresponding structural height.

Because specific and universally accepted drift limits are not available for historic masonry minarets, the calculated drift demands were evaluated with reference to the ranges reported for comparable historic masonry structures. This structure-specific interpretation is consistent with evidence that damage-state thresholds can significantly affect masonry vulnerability estimates and should not be transferred directly between different structures (Yesilyurt 2025). Kocaman and Kazaz (2023), based on nonlinear analyses of eleven historic masonry mosques, reported global drift-ratio ranges of 0.01–0.07%, 0.06–0.25%, and 0.08–0.45% for the Damage Limitation, Significant Damage, and Near Collapse states, respectively. These ranges were used as comparative performance benchmarks rather than as a single deterministic collapse threshold, since drift capacity is strongly affected by structural geometry, material stiffness, boundary conditions, and the governing damage mechanism.

Accordingly, the displacement-time histories and force-displacement response were first examined for rapid displacement growth, accumulation of non-recoverable deformation, loss of stable cyclic behavior, and pronounced stiffness or strength degradation. These global-response indicators were then evaluated together with the first-principal-strain distributions. Isolated peak strain values occurring in a limited number of finite elements were not interpreted as collapse. Instead, attention was given to the magnitude, spatial extent, propagation, and continuity of tensile-strain localization bands and to whether these bands formed a mechanically meaningful crack pattern.

For the reference model, the governing collapse mechanism was identified by the formation and propagation of an approximately continuous circumferential tensile-damage band in the shaft–substructure transition region. This regional damage pattern indicated progressive loss of sectional integrity and weakening of load transfer between the minaret shaft and the supporting substructure. Therefore, collapse was assigned only when the literature-based global drift demand, the deterioration observed in the global response, and the strain-based regional damage mechanism provided consistent and simultaneous evidence of structural instability.

The reported collapse time, (t_c), corresponds to the earliest analysis time step at which this combined collapse condition became established. It does not necessarily correspond to the final converged time step. In some analyses, the numerical solution continued after the structural response had already satisfied the adopted collapse criteria. Conversely, numerical non-convergence was not accepted as evidence of collapse unless it was supported by the displacement, drift, strain-localization, crack-propagation, and regional damage findings described above.

6. Strengthening Scenarios

The nonlinear dynamic analyses of the reference model (RM) clearly indicated that the Yakutiye Madrasa minaret exhibits significant seismic vulnerability in its current condition. The combined evaluation of displacement demands, first principal strain distributions, and circumferential cracking patterns revealed that seismic demands concentrate primarily in the shaft-substructure transition region. These findings demonstrate that strengthening intervention is necessary to improve the seismic performance of the structure.

However, strengthening strategies for historic masonry structures must not rely solely on structural capacity enhancement. Interventions must comply with international conservation principles defined by ICOMOS, including minimum intervention, reversibility, respect for original materials, and preservation of historical authenticity. For the Yakutiye Madrasa minaret, this requirement is particularly important due to the highly refined Seljuk stone ornamentation on the exterior surface. Consequently, all strengthening strategies considered in this study were designed to be implemented entirely within the internal volume of the minaret without altering the original exterior stone texture.

The analyses of the reference model further indicated that the seismic response of the minaret is governed by a global damage mechanism initiated by strain concentrations in the shaft-substructure transition region. Therefore, strengthening strategies limited to localized repair would be insufficient; instead, interventions capable of redistributing stresses and reorganizing load transfer mechanisms were required.

Within this framework, four strengthening scenarios (RET-1, RET-2, RET-3, and RET-4) were developed. Rather than representing incremental variations of a single solution, these scenarios were formulated as alternative engineering strategies addressing the same structural problem with different levels of stiffness continuity and structural influence. The layouts of the strengthening systems are illustrated schematically in Fig. 10.

The same geometric and material assumptions were maintained for the steel components in all strengthening scenarios. All internal steel plates used in RET-1, RET-2, RET-3, and the plate component of RET-4 were assigned a uniform thickness of 30 mm. The plates and the 30 mm diameter longitudinal bars used in RET-4 were modeled using SOLID185 elements and the MAT-5 steel properties reported in Table 1.

The strengthening systems investigated herein represent comparative numerical configurations rather than fully detailed construction designs. Mechanical anchors were therefore not represented as discrete finite elements. Instead, adequate anchorage was idealized through the fully bonded steel–masonry interface condition described in Section 3. Accordingly, the analyses do not simulate local anchor failure, plate debonding, pull-out, or relative slip. The type, diameter, spacing, embedment length, corrosion protection, and installation sequence of the mechanical anchors should be determined through a separate site-specific application design if any of the investigated systems is considered for implementation.

6.1. RET-1 strengthening scenario: Discontinuous steel plate system along the internal spiral staircase

In RET-1, the discontinuous modules consist of 30 mm thick steel plates arranged along the internal spiral staircase from the first to the last step. Each module is approximately 750 mm high and follows the internal circumference of the shaft at the corresponding stair level. The modules remain structurally discontinuous from one another, while each module is fully bonded to the adjacent masonry surface through the adopted interface constraint.

RET-1 represents the first strengthening strategy developed to mitigate tensile strain concentrations while maintaining minimal intervention. In this scenario, a discontinuous internal banding system composed of steel plate modules was introduced along the spiral staircase within the interior of the minaret.

The steel plates are arranged along the stair levels, extending from the first to the last step, with each module approximately 750 mm in height. The modules are positioned against the inner surface of the shaft and follow the staircase geometry. This configuration allows the strengthening elements to remain concealed while providing additional confinement through the interaction between the internal stair core and the shaft.

The engineering concept of RET-1 is to partially restrain circumferential opening of the masonry walls by introducing localized internal banding. These steel modules aim to slow crack propagation, preserve sectional integrity, and reduce displacement demands by distributing tensile stresses along the shaft.

6.2. RET-2 strengthening scenario: Concentrated internal steel plate at the transition region

In RET-2, a continuous cylindrical steel plate with a uniform thickness of 30 mm is placed along the internal circumference of the shaft between approximately +5.00 m and +12.00 m elevation. The entire plate–masonry contact surface is modeled as fully bonded. Continuity is maintained within the strengthened region, whereas no steel lining is provided above or below these elevations.

RET-2 was developed based on the results obtained from RET-1. Although RET-1 redistributed stresses along the shaft, the nonlinear analyses indicated that circumferential cracking persisted in the shaft-substructure transition region.

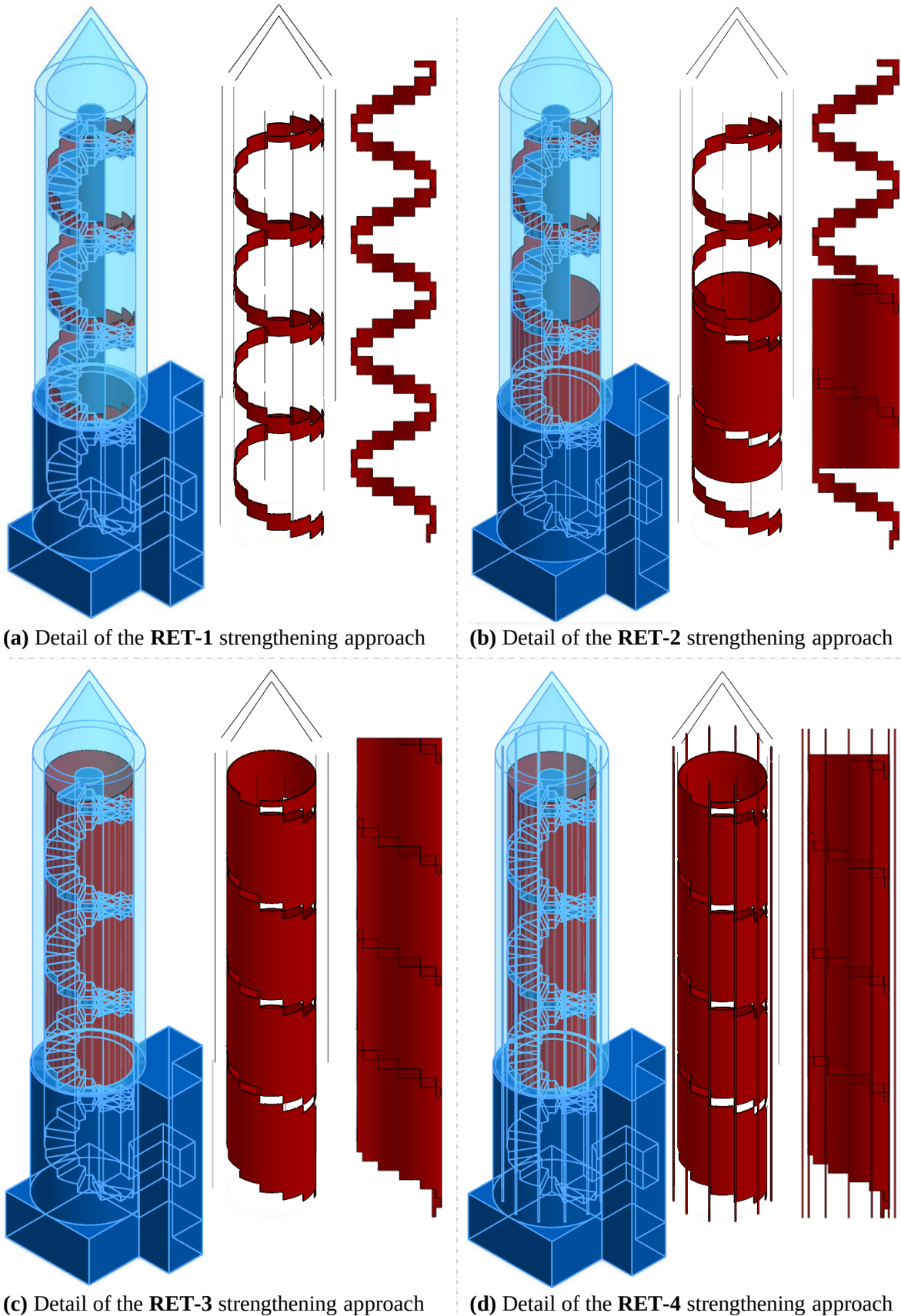


Fig. 10. Geometric and numerical definitions of the proposed internal strengthening configurations:
 (a) RET-1, discontinuous 30 mm thick steel plate modules with an approximate module height of 750 mm;
 (b) RET-2, continuous 30 mm thick internal steel plate between +5.00 m and +12.00 m elevation;
 (c) RET-3, continuous 30 mm thick internal steel lining along the entire shaft;
 (d) RET-4, RET-3 lining supplemented with twelve 30 mm diameter longitudinal steel bars anchored to the foundation. Steel-masonry and connected steel-steel interfaces were assumed to be fully bonded.

To address this limitation, RET-2 introduces a continuous cylindrical steel plate located specifically in the transition region. The plate extends approximately from +5.0 m to +12.0 m elevation and is placed along the inner surface of the shaft.

The objective of this strengthening system is to increase circumferential stiffness at the sectional level and limit crack propagation within the transition region. By providing continuous confinement, the steel plate promotes more integrated structural behavior and delays stiffness degradation under seismic loading. The strengthening element remains confined within the internal volume and therefore does not affect the exterior stone surface, maintaining compatibility with conservation principles.

6.3. RET-3 strengthening scenario: Continuous internal steel plate along the entire shaft

In RET-3, a 30 mm thick continuous steel lining is extended along the entire internal shaft surface, from the substructure level to the base of the spire. All adjoining steel regions are modeled with full displacement compatibility, and the lining is fully bonded to the masonry along its complete contact surface.

The RET-3 scenario was developed after examining the nonlinear response of RET-2. Although the transition region strengthening reduced damage concentrations locally, new strain concentrations were observed at the boundaries of the strengthened zone due to abrupt stiffness changes.

To eliminate these secondary weakness planes, RET-3 extends the internal steel plate along the entire height of the shaft. The continuous steel lining covers the inner surface from the base to the base of the spire. This configuration aims to homogenize stiffness distribution along the shaft and prevent localized strain concentrations. By acting as an internal steel shell, the system enables more uniform distribution of bending, shear, and torsional demands. Despite being a more extensive intervention than previous scenarios, RET-3 remains compatible with conservation requirements because all elements are located inside the minaret and do not interfere with the exterior architectural features.

6.4. RET-4 strengthening scenario: Hybrid system with longitudinal steel reinforcements

RET-4 retains the 30 mm thick continuous internal lining defined for RET-3 and supplements it with twelve longitudinal steel bars, each 30 mm in diameter. The bars are uniformly distributed around the shaft to preserve sectional symmetry, extend continuously along the full height of the minaret, and are anchored to the foundation in the numerical model. The bar-masonry and bar-plate connections are assumed to be fully bonded. The foundation anchorage is represented by displacement compatibility rather than by explicitly modeled anchor plates, bolts, or embedment zones.

RET-4 represents the most comprehensive strengthening strategy considered in this study. Although RET-3 significantly improved stiffness distribution, nonlinear analyses indicated that increasing stiffness alone was in-

sufficient to fully prevent damage mechanisms governed by the low tensile capacity of masonry.

Therefore, RET-4 introduces a hybrid strengthening system combining the continuous internal steel lining of RET-3 with additional longitudinal reinforcement elements that directly increase tensile capacity.

In this configuration, twelve steel bars with a diameter of 30 mm are installed along the full height of the shaft and anchored to the foundation. The reinforcement layout preserves sectional symmetry and avoids amplification of torsional effects while remaining compatible with the internal staircase geometry.

The resulting system allows the minaret to behave as a composite structure in which the internal steel shell regulates stiffness distribution and the longitudinal reinforcements enhance tensile resistance and limit crack development. RET-4 therefore provides simultaneous control of stiffness distribution and tensile-dominated masonry failure mechanisms.

7. Comparative Analysis Results

In this section, the seismic performance of the reference model (RM) and the four strengthening scenarios (RET-1–RET-4) is comparatively evaluated. The analyses include both modal characteristics and nonlinear time-history responses obtained under the three selected ground motions (GM-1, GM-2, and GM-3). First, the natural vibration frequencies and mode shapes of the models are compared to examine the influence of strengthening interventions on global stiffness. Subsequently, displacement-time histories and first principal strain distributions are evaluated to investigate the evolution of collapse time, maximum displacement demands, and damage mechanisms. This comparison provides a quantitative basis for assessing the effectiveness of the proposed strengthening strategies.

7.1. Modal analysis results

The modal analysis results of the reference model (RM) and the strengthening scenarios (RET-1–RET-4) are comparatively presented in Table 3, while the deformation patterns corresponding to the first mode are illustrated in Fig. 11.

Examination of Table 3 indicates that the strengthening interventions lead to gradual and significant increases in natural frequencies. Compared to the RM model, the increase in the first natural frequency was calculated as 3.6% for RET-1, 45.9% for RET-2, 74.3% for RET-3, and 82.5% for RET-4. A similar increasing trend is observed in the second and third modes, with the increase in the second mode frequency reaching approximately 96% in the RET-4 model. These results demonstrate that, particularly following RET-2, the interventions result in a substantial increase in overall system stiffness.

The modal analyses were performed by extracting 100 vibration modes. The cumulative effective modal mass participation ratios reached approximately 78% in the X direction and 82% in the Z direction, indicating that the structural mass contribution is distributed over a rel-

atively broad range of modes. The first three modes, representing the fundamental lateral-bending and torsional characteristics of the minaret, provide cumulative horizontal mass participation ratios of approximately 30.17% and 30.92% in the X and Z directions, respectively. Therefore, although these modes are important for describing the principal global deformation characteristics, higher-order modes also contribute substantially to the overall dynamic behavior. It should be em-

phasized that the nonlinear time-history analyses were performed using direct time integration rather than modal superposition; consequently, the nonlinear response was not restricted to the modes extracted in the modal analysis. The third mode was identified as a torsion-dominated mode based on its deformation pattern. A numerical ROTY participation ratio was not reported because a consistently normalized rotational effective-mass ratio was not available for all models.

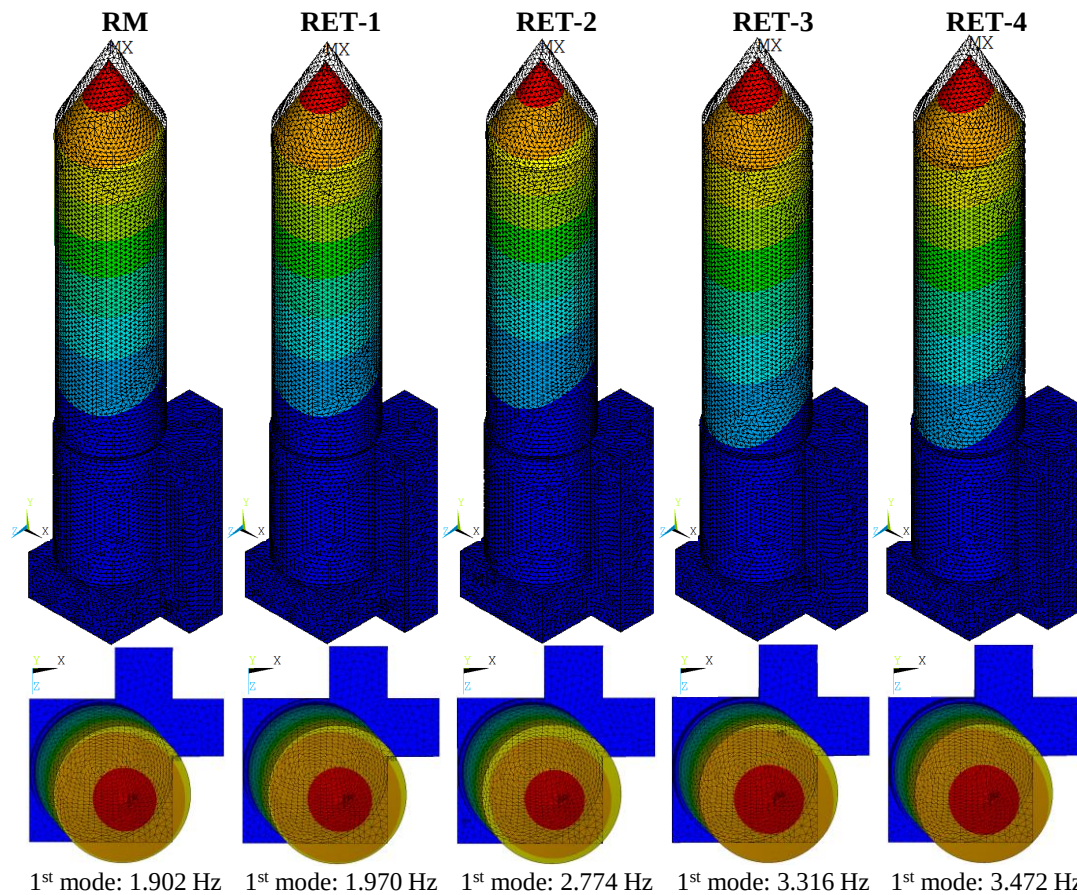


Fig. 11. Comparison of the first mode shapes of the reference model (RM) and the strengthening scenarios.

Table 3. Comparison of natural frequencies and modal mass participation ratios (MPR) of the RM and RET models.

Model	f_1 (Hz)	MPR (%)	Δf_1 (%)	f_2 (Hz)	MPR (%)	Δf_2 (%)	f_3 (Hz)	Δf_3 (%)
RM	1.902	15.84 (Z)	–	2.017	15.00 (X)	–	8.150	–
RET-1	1.970	15.46(Z)	+3.6	2.091	14.61 (X)	+3.7	8.571	+5.2
RET-2	2.774	15.26 (Z)	+45.9	3.052	13.52 (X)	+51.3	10.082	+23.7
RET-3	3.316	17.01 (X)	+74.3	3.772	15.46 (Z)	+87.0	13.759	+68.8
RET-4	3.471	17.35 (Z)	+82.5	3.957	15.736 (X)	+96.2	14.255	+74.9

In contrast, examination of the first mode shapes presented in Fig. 11 indicates that the dominant behavior remains globally flexural in all models, and the modal deformation patterns are nearly identical. No significant change in torsional dominance or modal shape distribution is observed following the strengthening interventions. The maximum modal displacements continue to concentrate in the upper regions of the minaret shaft across all models.

This observation demonstrates that, although the strengthening scenarios increase the global stiffness of the system, they do not alter the fundamental dynamic behavior of the structure. Accordingly, the collapse mechanisms observed in the nonlinear dynamic analyses are associated not with changes in modal characteristics but rather with localized tensile cracking and compressive crushing developing within the masonry units. This finding clearly indicates that an increase in natural

frequencies alone is not sufficient to ensure improvement in seismic performance.

7.2. Nonlinear time-history analyses

The five different models (RM, RET-1, RET-2, RET-3, and RET-4) were subjected to nonlinear time-history analyses under three different ground motion records (GM-1, GM-2, and GM-3). The analysis results were comparatively evaluated to reveal the influence of the strengthening scenarios on the global dynamic response and the evolution of local damage within the masonry units. In this context, the global response was first exam-

ined through displacement-time histories, followed by an assessment of strain distributions and the progression of damage mechanisms.

7.2.1. Displacement-time histories

Fig. 12 presents the displacement-time histories of the RM and RET-1–RET-4 models in the X and Z directions under the three different ground motion records. The results indicate that the strengthening interventions influence the dynamic response in different ways depending on the characteristics of the ground motion record.

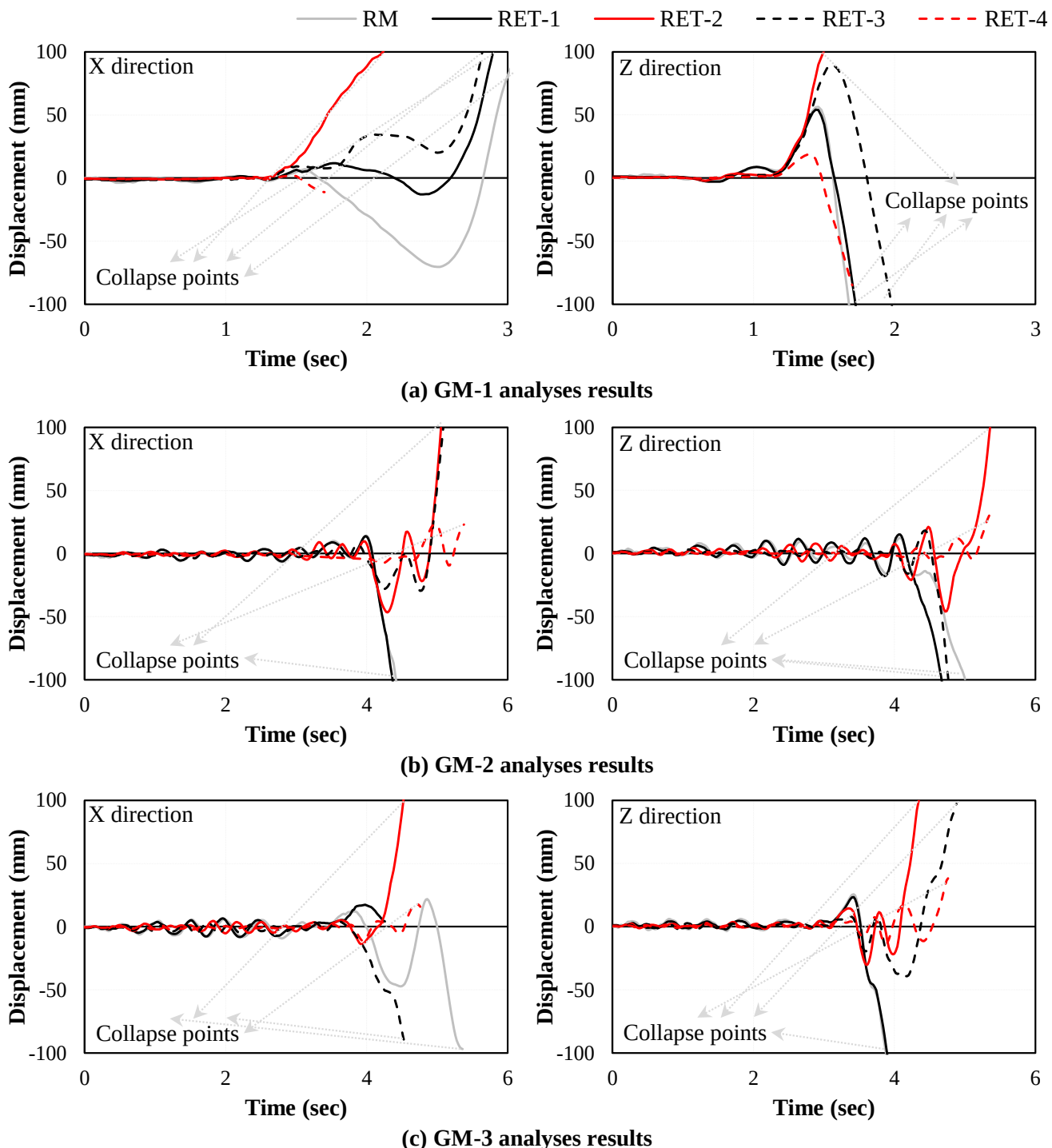


Fig. 12. Comparison of displacement-time histories of the reference model (RM) and the strengthening scenarios.

Under GM-1, the RET-1 model reduced the maximum displacement amplitude in the X direction compared to the RM; however, this improvement was insufficient to prevent global collapse. In the Z direction, the response of RET-1 remained largely similar to that of the RM. This indicates that the discontinuous internal steel plate system defined along the spiral staircase (RET-1) partially redistributed local strain demands but did not eliminate the dominant damage mechanism concentrated in the transition region.

For the RET-2 model under GM-1, a more critical outcome was observed. Although the continuous internal steel plate in the transition region increased global stiffness, earlier divergence was detected in the displacement histories. This suggests that stiffness enhancement may, in certain cases, adversely affect stress redistribution and trigger premature instability. In other words, a targeted yet partially continuous steel plate application may introduce secondary weakness planes at elevations where stiffness discontinuities occur.

The RET-3 model, owing to the continuous internal steel plate system along the shaft, distributed displacement demands more uniformly; however, it was unable to fully eliminate the collapse mechanism. For RET-4, although complete data in the X direction were not available, a certain reduction in post-elastic displacement amplitudes was observed in the Z direction. Nevertheless, global instability developed as the loading cycles progressed, resulting in collapse. This outcome indicates that, despite the increase in tensile capacity provided by the longitudinal reinforcements, the system-level collapse behavior could not be entirely prevented.

Under GM-2, the RET-1 model exhibited a dynamic response in both X and Z directions that was very similar to the RM. This finding indicates that the discontinuous internal banding approach does not significantly modify global behavior under certain ground motion characteristics. RET-2 and RET-3 produced comparable displacement histories under GM-2, demonstrating that increasing the extent of strengthening elements does not necessarily result in proportional improvement in nonlinear dynamic performance. In particular, stiffness enhancement concentrated in the transition region did not fundamentally alter system behavior for some ground motions.

Although complete collapse-time data for RET-4 under GM-2 could not be fully tracked, it is evident that lower displacement amplitudes were achieved compared to the other models. Nevertheless, examination of strain distributions indicates that the structure ultimately collapsed. This result suggests that RET-4 is relatively more effective in limiting displacement demands; however, it does not entirely eliminate global instability.

Under GM-3, none of the RET models exhibited earlier collapse than the RM in the X direction. This indicates that the tendency for early instability observed in GM-1 is record-dependent. In the Z direction, a different pattern emerged. The RET-1 model displayed a collapse behavior similar to that of the RM, with no significant delay. In contrast, RET-2 and RET-3 delayed the time of collapse to a certain extent. Particularly, strengthening systems providing greater continuity along the shaft (RET-3) managed displacement growth in a more controlled

manner. The RET-4 scenario also delayed collapse; however, ultimate instability could not be prevented.

These results demonstrate that the effectiveness of strengthening strategies is sensitive to the frequency content and duration characteristics of the ground motion. Discontinuous or regionally concentrated stiffness increases may lead to early instability under certain records, whereas more comprehensive strengthening approaches can delay collapse. However, as long as the tensile-dominated damage mechanism developing within masonry units is not fully controlled, ultimate collapse cannot be entirely prevented.

When the displacement–time histories are evaluated collectively, it is evident that global instability persists in all strengthening scenarios despite the significant increases observed in natural frequencies. This finding indicates that stiffness enhancement alone is insufficient in historic masonry minarets and that collapse behavior is primarily governed by nonlinear material response and strain localization.

7.2.2. Strain distributions and damage mechanism

The maximum principal strain distributions obtained under the three ground motions are summarized in Fig. 13. The results indicate that the instability behavior observed in the displacement–time histories is associated with tensile-dominated strain localization occurring in specific regions of the structure.

For the reference model (RM), the dominant damage mechanism under all ground motions appears as a circumferential strain band concentrated in the shaft–substructure transition region. Tensile strain accumulation develops on one side of the cross-section due to bending effects and eventually leads to loss of sectional integrity. The consistency of this damage location across different ground motions indicates that the mechanism is primarily controlled by structural geometry and stiffness distribution.

In the RET-1 scenario, the discontinuous steel plates installed along the spiral staircase partially redistributed the strain field, but the circumferential damage band remained. Although peak strain values were reduced, the governing damage mechanism was not eliminated, indicating limited effectiveness of the discontinuous internal banding system.

In the RET-2 model, the continuous steel plate located in the transition region reduced strain intensity locally; however, new strain concentrations developed at the boundary of the strengthened zone due to stiffness discontinuities. This behavior explains the early instability observed under GM-1.

The RET-3 configuration, consisting of a continuous internal steel lining along the shaft, resulted in a more uniform strain distribution and reduced local peaks. Nevertheless, the circumferential tensile band mechanism persisted, indicating that improved stiffness continuity alone cannot fully prevent damage governed by the low tensile capacity of masonry.

In the RET-4 hybrid system, the addition of longitudinal steel reinforcements further reduced circumferential strain concentrations under some ground motions. However, new strain patterns associated with longitudinal

cracking developed in certain cases and collapse still occurred.

Overall, while all strengthening scenarios modified the strain distribution and reduced strain magnitudes to varying degrees, none completely eliminated the tensile-

dominated damage mechanism. The results indicate that the ultimate seismic performance of the minaret is largely controlled by the limited tensile strength and damage-softening characteristics of the masonry material.

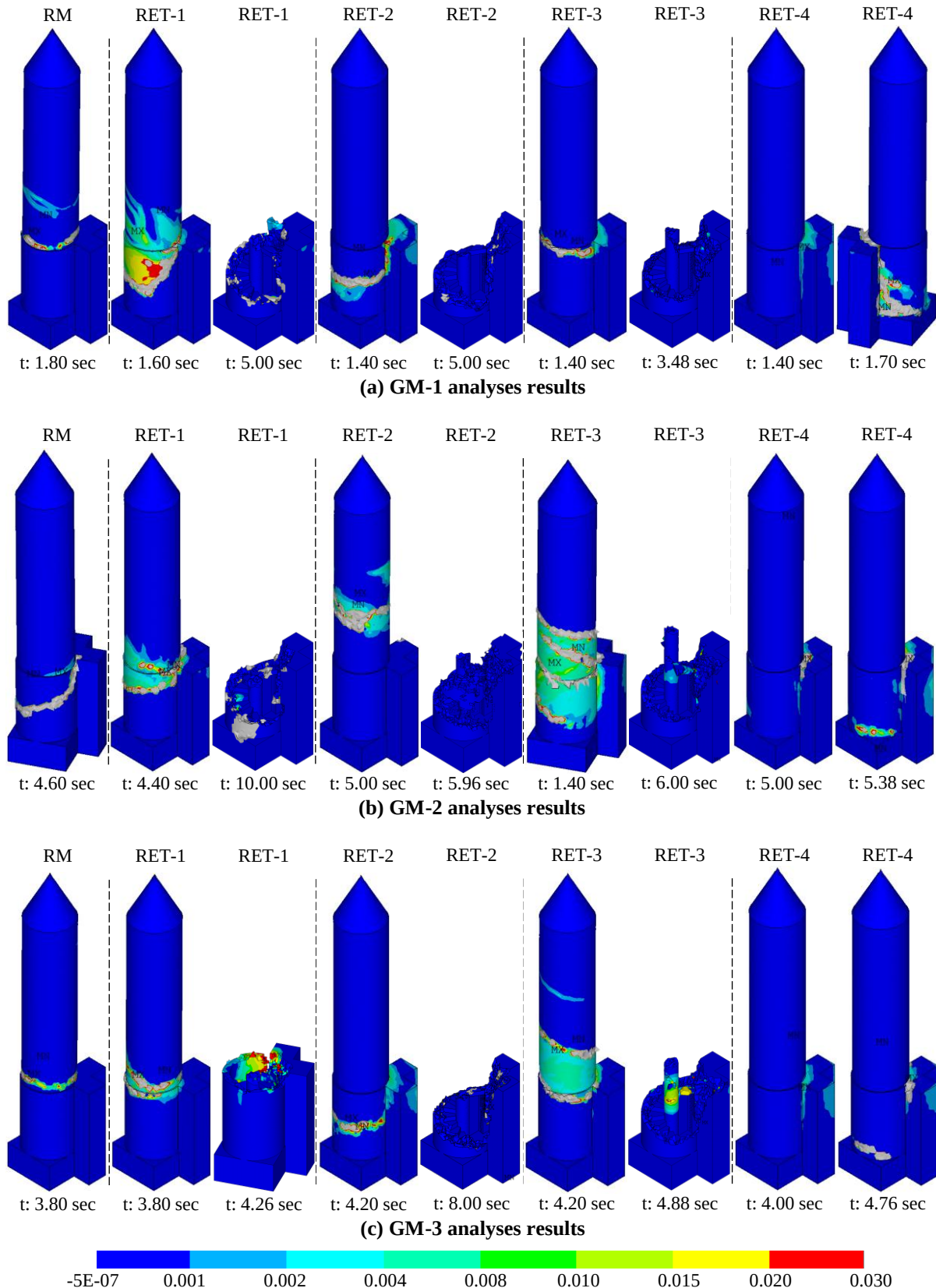


Fig. 13. Comparison of damage distributions of the reference model (RM) and the strengthening scenarios.

7.2.3. Integrated quantitative comparison and availability of response measures

For a consistent comparison, the response of each model was evaluated at two distinct times: the collapse time, (t_c), defined according to the combined criterion introduced in Section 5.2, and the final solved time, (t_{end}). These quantities should not be interpreted interchangeably because several analyses continued after the adopted structural collapse condition had already been reached.

Under GM-1, the (t_c/t_{end}) values were 1.68/5.00 s for RM, 1.72/5.00 s for RET-1, 1.50/5.00 s for RET-2, 1.98/3.48 s for RET-3, and 1.70/1.70 s for RET-4, with the latter determined from the Z-direction displacement response. Relative to RM, RET-3 delayed the collapse condition by approximately 17.9%, whereas RET-2 reduced the collapse time by approximately 10.7%. The changes obtained for RET-1 and RET-4 were limited to approximately 2.4% and 1.2%, respectively.

Under GM-2, the corresponding (t_c/t_{end}) values were 4.98/10.00 s, 4.66/10.00 s, 5.36/5.96 s, 4.76/6.00 s, and 5.34/5.38 s for RM and RET-1–RET-4, respectively. Compared with RM, RET-2 and RET-4 delayed the collapse condition by approximately 7.6% and 7.2%, whereas RET-1 and RET-3 reduced the collapse time by approximately 6.4% and 4.4%, respectively.

Under GM-3, the (t_c/t_{end}) values were 3.88/10.00 s for RM, 3.88/4.26 s for RET-1, 4.34/8.00 s for RET-2, 4.86/4.88 s for RET-3, and 4.76/4.76 s for RET-4. Accordingly, RET-2, RET-3, and RET-4 delayed the collapse condition by approximately 11.9%, 25.3%, and 22.7%, respectively, whereas RET-1 produced no change in the identified collapse time. These comparisons quantitatively demonstrate that the influence of strengthening on collapse evolution was strongly ground-motion dependent and was not proportional to the increase in natural frequency.

Maximum top displacement was evaluated only over the interval ($t-t_c$), because displacement growth occurring after the formation of a global collapse mechanism does not represent stable structural capacity. The absolute pre-collapse peak top displacement reached approximately 100 mm, corresponding to a response envelope extending to approximately (± 100) mm. Residual displacement was not defined as a comparable response quantity because none of the model–ground-motion combinations reached a stable post-earthquake equilibrium state after satisfying the adopted collapse criterion. Therefore, the displacement at (t_{end}) should not be interpreted as a conventional residual displacement.

The first-principal-strain distributions were plotted using the same contour interval, ranging from $-5E-07$ to 0.030, for all models and ground motions. However, a model-wide scalar maximum tensile strain was not used as an independent performance measure. In a smeared-crack formulation, isolated integration-point peaks are highly sensitive to mesh discretization and may not represent a physically meaningful structural damage state. The strain assessment was therefore based on the magnitude, spatial extent, propagation, and continuity of the localized tensile-strain bands rather than on an isolated numerical maximum.

A damaged-element ratio was not calculated because SOLID65 identifies cracking and crushing at individual integration points and in different principal directions. Consequently, classification of an entire element as either damaged or undamaged would require an additional arbitrary threshold that was not included in the original analysis framework. In the strengthened configurations, the calculated equivalent stress demand in the steel components exceeded the adopted yield strength, indicating that the yielding threshold was reached. Exact peak steel-stress values were not introduced as an additional ranking parameter because stresses beyond the onset of yielding are strongly governed by the adopted constitutive idealization and do not independently characterize the effectiveness of the strengthening system.

Base-shear histories were not retained among the original time-history output requests and would require reconstruction from the time-dependent foundation reactions. Dissipated energy was also not reported as a scalar response quantity because the adopted Willam–Warnke smeared-crack model was not formulated using calibrated fracture-energy regularization. Consequently, a numerical energy-dissipation comparison would be mesh dependent and could imply a level of physical accuracy not supported by the adopted constitutive model.

Collectively, the available quantitative results show that the strengthening systems substantially modified stiffness, displacement development, and the timing of damage progression, but their effectiveness was record dependent. The decisive performance indicator was not the stiffness increase alone, but whether the intervention prevented or delayed the development of a continuous tensile-damage mechanism at the shaft–substructure transition.

8. Conclusions

The four strengthening scenarios (RET-1–RET-4) developed in this study significantly increased the global stiffness of the minaret and resulted in noticeable increases in natural frequencies. However, the nonlinear time-history analyses demonstrated that stiffness enhancement alone does not necessarily improve seismic performance. Although the onset of instability varied among the strengthening scenarios, none of the interventions completely eliminated the ultimate collapse mechanism. This indicates that global stiffness primarily modifies the dynamic response but does not control the governing damage evolution.

Strain distribution analyses revealed that collapse behavior is mainly governed by tensile-dominated strain localization concentrated in the shaft–substructure transition region. Under all ground motions, the dominant damage mechanism developed as a circumferential strain band in this zone. While the strengthening scenarios altered the magnitude and spatial distribution of strain concentrations, they did not eliminate the mechanism itself. These findings highlight the decisive role of sectional tensile capacity and damage-softening behavior in the seismic response of masonry minarets.

The early instability observed in the RET-2 scenario also emphasizes the importance of stiffness continuity in

strengthening design. Although the localized steel plate reduced strain intensity in the transition region, it introduced a stiffness discontinuity that triggered secondary damage planes. This outcome demonstrates that localized stiffness enhancement may cause unintended stress redistribution within the structural system.

The analyses further indicate that structural response is highly sensitive to masonry material parameters, including compressive strength (f_c), tensile strength (f_t), damage-softening characteristics, and energy dissipation capacity. The survival of the structure through historical earthquakes suggests that the adopted parameters may represent conservative assumptions. Future studies should therefore incorporate material models calibrated through field observations and experimental data, together with systematic parameter sensitivity analyses.

Overall, the findings indicate that strengthening strategies for historic masonry structures should not focus solely on increasing global stiffness. Instead, effective interventions should enhance tensile resistance, ensure stiffness continuity, and mitigate local strain concentrations to improve seismic performance.

This study is subject to several limitations. The steel-masonry interfaces were idealized as fully bonded, the material parameters were defined primarily based on literature data rather than structure-specific experimental characterization, and soil-structure interaction was not considered. Moreover, the strengthening alternatives examined in this study were limited to internal intervention strategies. A formal mesh-sensitivity or convergence analysis was not performed; therefore, the numerical results may retain some dependence on the adopted nominal element size of 15–20 cm. This dependence is expected to be particularly relevant to localized response quantities, including first principal strain concentrations and the predicted spatial distribution of cracking, which are generally more sensitive to mesh discretization than global parameters such as natural frequencies and overall displacements. Accordingly, the strain contours should be interpreted as comparative indicators of potential damage localization rather than as mesh-objective predictions of crack width or highly lo-

calized failure. Future studies should incorporate in situ dynamic measurements, structure-specific material characterization, and experimental testing of representative masonry components and steel-masonry connection details to calibrate and validate the numerical model. Systematic comparisons of coarse, intermediate, and refined meshes should also be conducted to evaluate the convergence of natural frequencies, global displacements, collapse times, and strain-localization patterns. Further research should additionally investigate material and interface sensitivity, soil-structure interaction, and alternative strengthening configurations within a performance-based conservation framework.

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Conflict of Interest

The authors declare no potential conflicts of interest with respect to the research, authorship, and/or publication of this manuscript.

Data Availability

The datasets generated and/or analyzed during the current study are not publicly available but are available from the corresponding author upon reasonable request.

AI Assistance

During the preparation of this manuscript, ChatGPT-5 (OpenAI) was used to support various aspects of the writing process, including improving clarity, grammar, spelling, and academic tone. All content generated with the assistance of this tool was carefully reviewed and edited by the authors, who take full responsibility for the final content of the manuscript.

Author Contributions

All authors made substantial contributions to the conception and design of the study, acquisition of data, analysis and interpretation of data; drafted or critically revised the manuscript for important intellectual content; and approved the final version to be published.

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