Challenge Journal of Structural Mechanics

Effect of severe corrosion upon natural frequencies of beam-like structures

Daniel Bobos, Zeno Iosif Praisach, Doina Frunzăverde, Gilbert Rainer Gillich *, Ionica Negru

Department of Mechanics and Material Engineering, Eftimie Murgu University of Resita, 320085 Resita, Romania

ABSTRACT

Corrosion, as the spontaneous process of material degradation produced by the environment, affects the reliability and safety of structures, both by reducing the section of the components, due to material loss and by diminution of the materials mechanical strength. The authors have found a mathematical relation between discontinuities in beams and changes of its natural frequencies and developed a method to identify these discontinuities. The present paper considers the more complex case of damage determined by corrosion, where beam thinning is accompanied by mass decrease. These impose considering natural frequency changes in both directions: decrease due damage and increase because of mass loss. FEM simulations and analytical investigations were carried out, in order to find the relation between mass change in different positions along the beam and the frequency increase. The results were correlated with the "classical" relation describing frequency decrease because of discontinuities. Finally, the authors developed a new relation, proper to be used for damage produced by severe corrosion, which was validated by laboratory experiments.

ARTICLE INFO

Article history:
Received 29 April 2015
Accepted 23 May 2015

Keywords:
Corrosion
Damage assessment
Beam
Natural frequency
Mode shape
Strain energy

1. Introduction

Corrosion is a consequence of the interaction between a material and its environment, which determines changes in the materials properties and often leads to impairment of the metals function, the environment or the technical system of which these form a part (ISO 8044-1986). The damage process is irreversible and leads to degradation of the material, accompanied by diminished performances of the technical system to which it belongs. The corrosion processes always trigger from the components surface, but sometimes penetrate deep inside the materials.

Several criteria are used to classify the different corrosion types. Regarding the corrosions mechanism, one can distinguish between: (a) chemical (dry) corrosion, as the heterogeneous corrosion process between a solid metallic phase and a gaseous phase (air, industrial gases); (b) electrochemical (wet) corrosion, which appears when metallic materials are in contact with an aggressive, liquid and conducting medium; when it occurs under the conditions of simultaneous mechanical load, result stress corrosion, fatigue corrosion or fretting corrosion and (c) biochemical corrosion, where the destruction of metal is caused by bacteria, fungi or other microbiological organisms under specific environmental conditions (DeGiorgi, 1992).

Another traditional classification (Landolfo et al., 2010) characterizes corrosion phenomena according to the appearance of the corroded area. Regarding this criterion, the basic forms of corrosion are: (i) continuous (generalized) corrosion, which can be both uniform, where the metal surface is affected at the same rate on large areas and nonuniform corrosion, characterized by different corrosion rates in different zones of the surface; (ii) localized corrosion, which is restricted to reduced areas and takes the form of pits, crevices or cavities.

From structural point of view, the loss of thickness of the cross section due to corrosion attack leads to a smaller bearing area, producing a decrease in the structural performance in terms of strength, stiffness and

* *Corresponding author.* Tel.: +40-355-405900 ; Fax: :+40-255-207501 ; E-mail address: gr.gillich@uem.ro (G. R. Gillich)
ISSN: 2149-8024 / DOI: http://dx.doi.org/10.20528/cjsmec.2015.05.010

ductility, thus shortening the designed life expectancy (Bazant, 1979; Apostolopoulos and Michalopoulos, 2007). In case of cyclic loads, the corrosion phenomenon can produce a significant reduction in the fatigue strength, mainly in zones with high stress concentration (Albrecht and Hall, 2003; Landolfo et al., 2005; Klinesmith, 2007). Especially in case of uniform corrosion the loss of material mass is significant, bringing up difficulties when dynamic methods are used for corrosion assessments. While corrosion is a major problem, it concerned the attention of researchers, but only recently they gave attention to the joined effects of stiffness and mass loss.

Our previous researches considered the influence of local, transversal damages without loss of mass upon dynamic behavior of beams and established a simple, reliable method to asses these damages. Recent investigations considering the influence of local corrosion on the dynamic behavior of beams were performed, with focus both on stiffness changes and loss of mass. This paper presents the results obtained and considerations how these can be used to improve damage detection methods.

2. Analytical Investigations

For this paper we have chosen to present the cantilever beam, which due to its asymmetry is more complex, but uniquely defines the damage/corrosion location. For demonstration we used a steel cantilever beam with rectangular cross section (Fig. 1) having the following geometry: length L = 600 mm; width B = 50 mm and height H = 5 mm. Consequently, for the undamaged state, the beam has the cross-section A = 250-10^6 m^2 and the moment of inertia I = 520.833-10^{-12} m^4. The mechanical parameters of the specimens' material are: mass density ρ = 7850 kg/m^3; Young's modulus E = 2.0-10^{11} N/m^2 and Poisson's ratio m = 0.3. The earth gravity is considered g = 9.806 m/s^2 and the mass of the beam is m = 1.1775 kg. On the cantilever beam acts only its own mass.

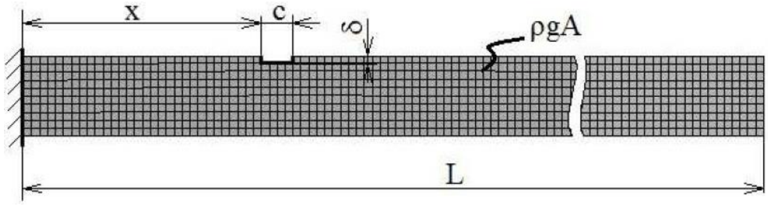

Fig. 1. Cantilever beam with damage.

The natural frequencies Eq. (1) for the cantilever beam is presented below:

$$f_n = \frac{a_n^2}{2\pi}\sqrt{\frac{EI}{mL^3}}, \qquad (1)$$

where, the wave numbers a_n of Euler-Bernoulli model are the solution of Eq. (2) for n vibration modes:

$$cos\, a + cos\, h + 1 = 0\,. \qquad (2)$$

At distance x from clamped end (Fig. 1) it is considered a damage/corrosion area of 2 mm wide during the whole width of the beam with a small depth of 8% from beam height, respectively 0.4 mm depth.

Taking in consideration that for the cantilever beam the deflection v at the free end is:

$$v = \frac{pgAL^4}{8EI} = \frac{mL^3}{8EI}, \qquad (3)$$

for the damaged beam the deflection v_D can be written:

$$v_D = \frac{m(x,\delta)L^3}{8EI(x,\delta)}, \qquad (4)$$

and consequently, the natural frequencies equation for the damaged beam f_{Dn} becomes:

$$f_{Dn} = \frac{a_n^2}{2\pi}\sqrt{\frac{EI(x,\delta)}{m(x,\delta)L^3}}. \qquad (5)$$

According to relation (5), the natural frequencies for the damaged/corroded beam depends of moment of inertia $I(x,\delta)$ and mass $m(x,\delta)$. In the corroded area on the beam, when mass loss is significant compared to reducing the moment of inertia we can consider that $I(x,\delta) \sim I$. The natural frequencies for the corroded beam depends only the terms $m(x,\delta)$.

In this case, the relative frequency shift (Gillich and Praisach, 2012; Gillich et al., 2012a) can be written:

$$\Delta f = \frac{f - f_D}{f} = 1 - \frac{f_D}{f}, \qquad (6)$$

but the relative frequency shift for the corroded beam Δf_C is,

$$\Delta f_C = 1 - \frac{f_D}{f} = 1 - \frac{\frac{a_n^2}{2\pi}\sqrt{\frac{EI}{m(x,\delta)L^3}}}{\frac{a_n^2}{2\pi}\sqrt{\frac{EI}{mL^3}}} = 1 - \frac{\sqrt{m}}{\sqrt{m(x,\delta)}} = \frac{\sqrt{m(x,\delta)} - \sqrt{m}}{\sqrt{m(x,\delta)}}, \qquad (7)$$

and taking in consideration the relation of natural frequency for the damaged beam (Gillich and Praisach, 2012), the natural frequency for the corroded beam becomes:

$$f_{Cn} = f_n\left(1 - \frac{\sqrt{m(x,\delta)} - \sqrt{m}}{\sqrt{m(x,\delta)}}(\bar{\phi}(x))^2\right)$$

$$= f_n\left(1 + \frac{\sqrt{m} - \sqrt{m(x,\delta)}}{\sqrt{m(x,\delta)}}(\bar{\phi}(x))^2\right), \qquad (8)$$

where $\bar{\phi}(x)$ is the normalized mode shape curvature of the cantilever beam.

In the damaged beam the reducing of moment of inertia and loss mass is significant. Taking in consideration only the reducing of moment of inertia $m(x,\delta)\sim m$ and relations (3) and (4) the relative frequency shift Δf_D is:

$$\Delta f_D = 1 - \frac{f_D}{f} = 1 - \frac{\frac{a_n^2}{2\pi}\sqrt{\frac{EI}{m(x,\delta)L^3}}}{\frac{a_n^2}{2\pi}\sqrt{\frac{EI}{mL^3}}} = 1 - \frac{\sqrt{\frac{1}{8v_D}}}{\sqrt{\frac{1}{8v_D}}}$$

$$= 1 - \frac{\sqrt{v}}{\sqrt{v_D}} = \frac{\sqrt{v_D}-\sqrt{v}}{\sqrt{v_D}}, \tag{9}$$

and the natural frequency for the damaged beam (Gillich et al., 2012b) without taking in consideration the mass loss:

$$f_{Dn} = f_n\left(1 - \frac{\sqrt{v_D}-\sqrt{v}}{\sqrt{v_D}}\right)\left(\frac{\partial^2\bar{\phi}(x)}{\partial x^2}\right)^2. \tag{10}$$

Taking in consideration the mass loss (8) and reducing the moment of inertia (10), the natural frequency for the damaged beam can be written:

$$f_{Dn} = f_n\begin{pmatrix}\left(1 - \frac{\sqrt{v_D}-\sqrt{v}}{\sqrt{v_D}}\right).\left(\frac{\partial^2\bar{\phi}(x)}{\partial x^2}\right)^2 \\ + \frac{\sqrt{m}-\sqrt{m(x,\delta)}}{\sqrt{m(x,\delta)}}(\bar{\phi}(x))^2\end{pmatrix}. \tag{11}$$

The damage location index (12) can be obtained from fracture mechanics by using the following formula:

$$\frac{\sqrt{v_D}-\sqrt{v}}{\sqrt{v_D}} = 5.346h\pi.P\left(\frac{\delta}{h}\right), \tag{12}$$

where (Ostachowicz and Krawczuk, 1991)

$$P\left(\frac{\delta}{h}\right) = 1.862\left(\frac{\delta}{h}\right)^2 - 3.95\left(\frac{\delta}{h}\right)^3 + 16.375\left(\frac{\delta}{h}\right)^4 - 37.226\left(\frac{\delta}{h}\right)^5 + 76.81\left(\frac{\delta}{h}\right)^6 - 126\left(\frac{\delta}{h}\right)^7 + 172\left(\frac{\delta}{h}\right)^8 - 172\left(\frac{\delta}{h}\right)^9 - 66.56\left(\frac{\delta}{h}\right)^{10}. \tag{13}$$

3. Numerical Investigations

Investigations using the finite element method were performed on a cantilever beam, first case for the undamaged beam, second case for a cantilever beam with damage and mass loss (Fig. 1) and third case a geometrical undamaged beam where in the presumed damaged area of wide c = 2 mm the original material was replaced with a new one (Fig. 2), having the mass density reduced in order to assure the same total beam mass like that of the damaged beam.

The 3D beam with the geometrical characteristics and material parameters presented in chapter 2, was meshed by 0.5 mm elements in all cases. In order to compare analytical investigations with numerical results, were considered 197 locations of the damage on the beam length, both for damaged beam with mass loss (case two) presented in Fig. 1 and theoretical undamaged beam (case three) presented in Fig. 2. For all the cases were determined the maximum deflection and the first ten natural frequencies of the weak-axes bending vibration modes, used to highlight the frequency changes in a graphical way.

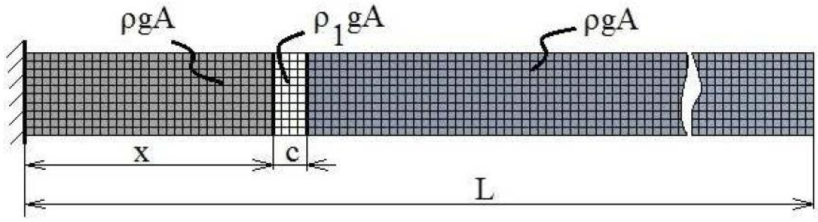

Fig. 2. Undamaged cantilever beam.

Fig. 3 presents the natural frequencies for the damaged beam with mass loss for the first, second, third and fourth vibration mode. The dashed line represents the natural frequencies obtained by numerical analysis and the continuous line represents the natural frequencies obtained analytic with relation (10).

Fig. 4 presents the natural frequencies for the geometrical undamaged beam but with loss of mass, for the first, second, third and fourth vibration mode. The dashed line represents the natural frequencies obtained by numerical analysis and the continuous line represents the natural frequencies obtained analytic with relation (8).

Fig. 5 presents the natural frequencies for the damaged beam without mass loss, for the first, second, third and fourth vibration mode. The dashed line represents the natural frequencies obtained by numerical analysis for damaged beam and undamaged beam with reduced mass and the continuous line represents the natural frequencies obtained analytic with relation (10).

For Figs. 3 to 5 the dash-dotted line represents the natural frequency of the undamaged beam.

The dashed lines presented in Fig. 5 are obtained as difference between numerical results of natural frequencies for the damaged beam with mass loss (Fig. 3) and numerical results of natural frequencies for the undamaged beam with reduced mass (Fig. 4). It can be observes the good concordance of the corrected results obtained by numerical analysis and that obtained by means of fracture mechanics.

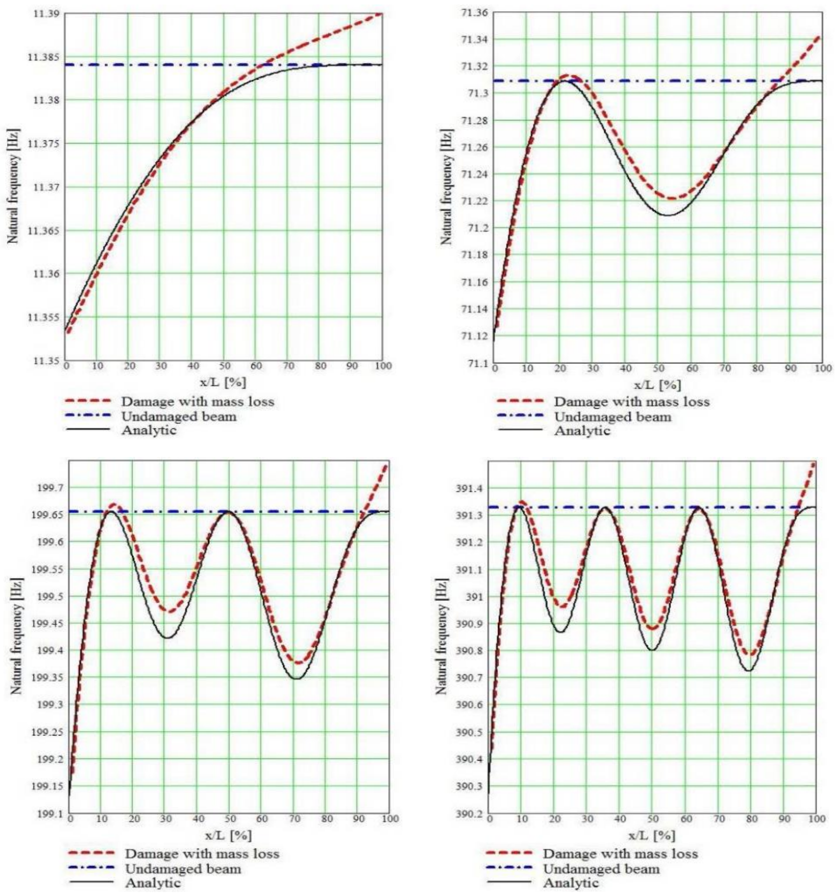

Fig. 3. Natural frequencies for the damaged beam with mass loss (first, second, third and fourth vibration modes).

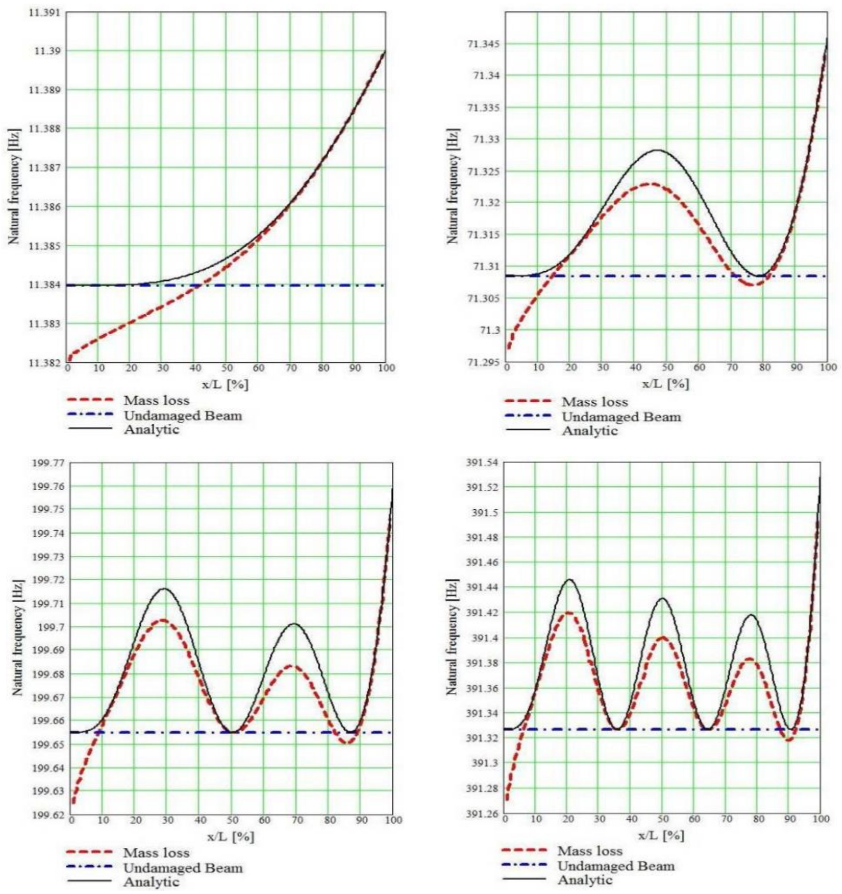

Fig. 4. Natural frequencies for the undamaged beam with reduced mass (first, second, third and fourth vibration modes).

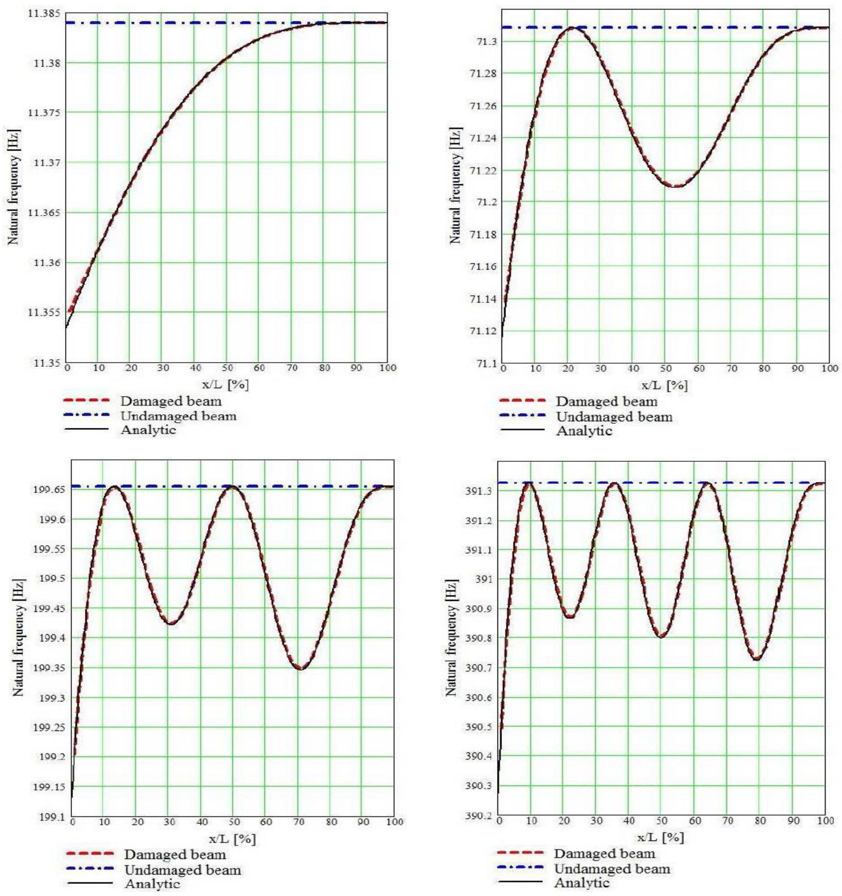

Fig. 5. Natural frequencies for the damaged beam (first, second, third and fourth vibration modes).

4. Laboratory Experiments

We conducted experiments on the cantilever beam to find the first ten natural frequencies of the undamaged and damaged beam. During the tests, the beams where fixed in a milling machine and the excitation of the structure was realized by hitting with a hammer. The measurement system was composed by a laptop, a NI cDAQ-9172 compact chassis with NI 9234 four-channel dynamic signal acquisition modules and a Kistler 8772 accelerometer mounted near the free end of the beam. Using virtual instruments created in LabVIEW we acquired the acceleration time history and identified the first ten weak-axis bending vibration modes. For the damaged case, the effect of corrosion was simulated by saw cuts, of around 2 mm wide and 0.5 mm depth. The damage is located at x/L = 0.595.

The natural frequencies for the first ten vibration mode, for the undamaged and damaged beam obtained analytically (10), numerically by finite element method and measured are presented in Table 1.

To compare the obtained results presented in Table 1 is useful to represents the normalized frequency shift versus vibration mode (Fig. 5). The normalized frequency shift (Table 2) is the relative frequency shift divided by maximum of the relative frequency shift (highlighted with bold characters in Table 1) of the ten vibration modes.

Table 1. Natural frequencies for the undamaged *(f_U)* and damaged (f_D) beam with damage located at x/L = 0.595.

Vibration mode	Natural frequencies					
	Analytic Euler-Bernoulli		Numerical		Measured	
n	f_{U_A} [Hz]	f_{D_A} [Hz]	f_{U_FEM} [Hz]	f_{D_FEM} [Hz]	f_{U_M} [Hz]	f_{D_M} [Hz]
1	11.3247	11.3237	11.3840	11.3834	11.3564	11.3552
2	70.9709	70.8803	71.3084	71.2263	71.0215	70.9302
3	198.7205	198.5839	199.6549	199.5339	199.0212	198.8811
4	389.4129	389.2844	391.3266	391.2133	389.5088	389.3814
5	643.7275	642.9087	647.1184	646.4253	648.027	647.1995
6	961.6174	961.611	967.0326	967.0125	966.0025	965.9832
7	1343.0854	1341.5112	1350.994	1349.6439	1345.3657	1343.7714
8	1788.1315	1787.2082	1798.7638	1797.9662	1792.2048	1791.1582
9	2296.7556	2295.7555	2309.9327	2309.0230	2306.3294	2305.4299
10	2868.9577	2865.4446	2883.9323	2880.9621	2878.4419	2874.9624

Table 2. Relative frequency shift (Δf) and normalized frequency shift ($N\Delta f$) for damaged beam with damage at x/L = 0.595.

Vibration mode n	Relative frequency shift & Normalized frequency shift					
	Analytic Euler-Bernoulli		Numerical		Measured	
	Δf_A [%]	$N\Delta f_A$	Δf_{FEM} [%]	$N\Delta f_{FEM}$	Δf_M [%]	$N\Delta f_M$
1	0.0088	0.068912	0.0053	0.042535	0.0102	0.079377
2	0.1277	1.000000	0.1152	1.000000	0.1285	1.000000
3	0.0688	0.538763	0.0606	0.526042	0.0704	0.547860
4	0.0330	0.258418	0.0290	0.251736	0.0327	0.254475
5	0.1272	0.996085	0.1071	0.929688	0.1277	0.993774
6	0.0007	0.005482	0.0021	0.018229	0.0020	0.015564
7	0.1172	0.917776	0.1000	0.868056	0.1185	0.922179
8	0.0516	0.404072	0.0443	0.384549	0.0584	0.454475
9	0.0435	0.340642	0.0394	0.342014	0.0390	0.303502
10	0.1224	0.958496	0.1030	0.894097	0.1209	0.940700

It can be observed a very good correlation between analytic method, numerical analysis and measured results for a damaged beam. The normalized frequency shift in respect to the vibration mode presented in Fig. 6 uniquely characterizes the frequency changes for an asymmetrical beam. Thus, the damage location indexes can be used to precisely identify the location of a crack, without caring about its depth.

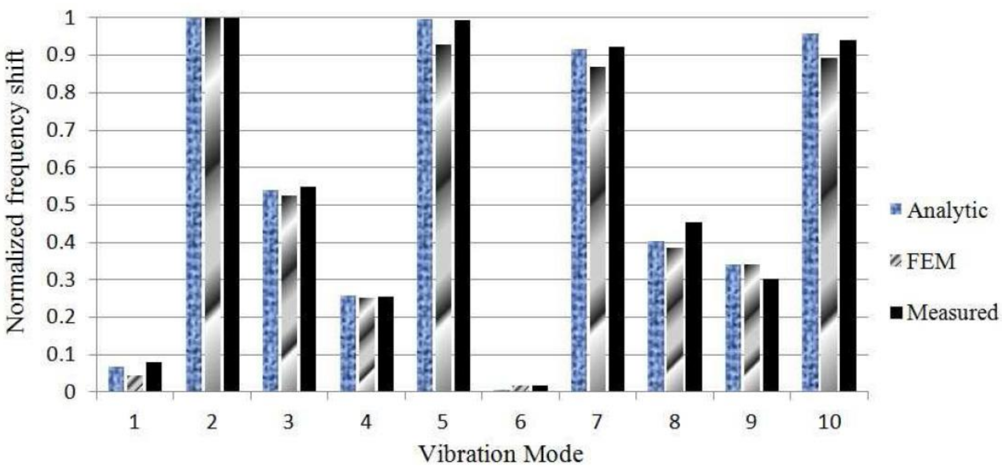

Fig. 6. Normalized frequency shift versus vibration mode.

5. Conclusions

Corrosion, as degeneration of materials, affects the reliability and safety of structures by producing loss of material together with thinning of components and consequently reduction of mechanical strength and stiffness. These two aspects are rarely considered together, since the phenomenon is complex and solutions in this case require intensive time and resources consumption.

This paper consider the case of damage due corrosion, where beam thinning is accompanied by mass loss. The relation prior contrived between deflection and frequency changes is used, completed with one found between mass loss in a given location and the resulting frequency increase. This fact reveals that for damages with mass loss, the natural frequency of the undamaged structure as a reference value is inadequate, being necessary to adjust it with a term dependent on the loss of mass and the value of the mode shape at the damage location.

Acknowledgements

The work has been co-funded by the Sectoral Operational Programme Human Resources Development 2007-2013 of the Romanian Ministry of Labour, Family and Social Protection through the Financial Agreement POSDRU/89/1.5/S/62557.

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