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Parametric analysis of thick plates subjected to earthquake excitations

Yaprak Itır Özdemir [a], Yusuf Ayvaz [b,*], Tayfun Dede [a]

[a] Department of Civil Engineering, Karadeniz Technical University, 61080 Trabzon, Turkey
[b] Department of Civil Engineering, Yıldız Technical University, 34220 İstanbul, Turkey

ABSTRACT

Plates are structural elements commonly used in the building industry. A plate is considered to be a thin plate if the ratio of the plate thickness to the smaller span length is less than 1/20; it is considered to be a thick plate if this ratio is larger than 1/20. The purpose of this paper is to study shear locking-free analysis of thick plates using Mindlin's theory and to determine the effects of the thickness/span ratio, the aspect ratio and the boundary conditions on the linear responses of thick plates subjected to earthquake excitations. Finite element formulation of the equations of the thick plate theory is derived by using second order displacement shape functions. A computer program using finite element method is coded in C++ to analyze the plates clamped or simply supported along all four edges. In the analysis, 17-noded finite element is used. Graphs and tables are presented that should help engineers in the design of thick plates subjected to earthquake excitations.

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1. Introduction

In the past 30 years, using of the plate bending elements based on the Mindlin (1951), and Reissner (1947) theory (the first-order shear deformation theory) have interested many researchers. Numerous methods have been proposed by earlier researchers for solving plate bending problem. In Mindlin-Reissner plate theory formulation's the deflection w and rotations βx, βy are generally considered to be independent function therefore only C0-continuity is required. The most encountered phenomenon is shear locking which acts as the plate becomes incrementally thinner.

In order to avoid this problem, the method of reduced and selective reduced integration (Zienkiewicz et al., 1971; Hughes et al., 1977; Ozkul and Ture, 2004) are chosen instead of the full integration, the substitute shear strain method proposed by Hinton et al, (1986), free formulation method proposed by Bergan et al. (1984). The same problem can also be prevented by using higher order displacement shape function (Özdemir et al., 2007). This paper that improved element is used for the vibration analysis of the plate.

The aim of this paper is to study forced vibration analysis of thick plates using Mindlin's theory and to determine the effects of the thickness/span ratio, the aspect ratio and the boundary conditions on the linear responses of thick plates subjected to earthquake excitations.

A computer program using finite element method is coded in C++ to analyze the plates clamped or simply supported along all four edges. In the program, the finite element method is used for spatial integration and the Newmark-β method is used for time integration. In the analysis, 17-noded finite elements are used to construct the stiffness and mass matrices.

2. Finite Element Modeling

The governing equation for a flexural plate subjected to an earthquake excitation without damping can be given as

$$[M]\{\ddot{w}\} + [K]\{w\} = [F] = -[M]\{\ddot{u}_g\}, \quad (1)$$

* *Corresponding author.* Tel.: +90-212-3835213 ; Fax: +90-212-3835213 ; E-mail address: yayvaz@yildiz.edu.tr (Y. Ayvaz)
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where [K] and [M] are the stiffness matrix and the mass matrix of the plate, respectively, w and $\ddot{w}$ are the lateral displacement and the second derivative of the lateral displacement of the plate with respect to time, respectively; and $\ddot{u}_g$ is the earthquake acceleration.

In order to do forced vibration analysis of a plate, the stiffness, [K], mass matrices, [M], and equivalent nodal loads vector, [F], of the plate should be constructed. The evaluation of these matrices is given in the following sections.

2.1. Evaluation of the stiffness matrix

In this study, 17-noded quadrilateral serendipity element (MT17) (Fig. 1) is used. The stiffness matrix for this element can be obtained by the following equation (Cook et al., 1989; Özdemir et al., 2007),

$$K = \int_A [B]^T[D][B]dA = \int_{-1}^{1}\int_{-1}^{1} B^T DB\,|J|drds\,, \qquad (2)$$

which must be evaluated numerically (Weaver and Jahston, 1984).

As seen from Eq. (2), in order to obtain the stiffness matrix, the strain–displacement matrix, [B], and the flexural rigidity matrix, [D], of the element need to be constructed and can be seen Özdemir and Ayvaz (2007).

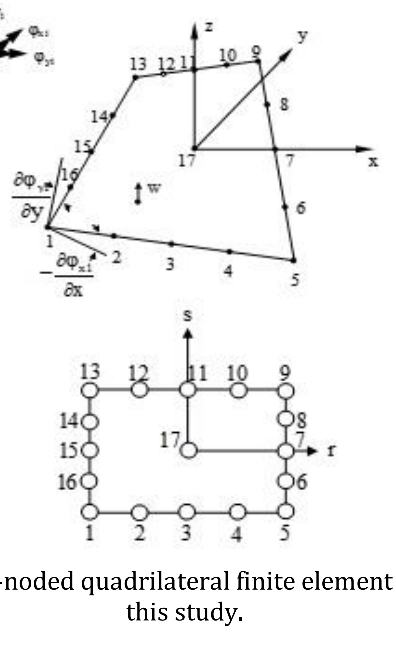

Fig. 1. 17-noded quadrilateral finite element used in this study.

2.2. Evaluation of the mass matrix

The formula for the consistent mass matrix of the plate may be written as

$$M = \int_\Omega H_i^T \mu H_i\, d\Omega\,. \qquad (3)$$

In this equation, μ is the mass density matrix of the plate (Özdemir et al., 2007) and H_i can be written as follows,

$$H_i = [dh_i/dx \quad dh_i/dy \quad h_i] \quad i = 1,\dots,17\,. \qquad (4)$$

It should be noted that the rotation inertia terms are not taken into account. By assembling the element mass matrices obtained, the system mass matrix is obtained.

2.3. Evaluation of equivalent nodal loads vector

Equivalent nodal loads, [F], can be obtained by the following equation.

$$[F] = \int H_i^T \bar{q} d\Omega\,. \qquad (5)$$

In this equation, H_i can be obtained by Eq. (4), and $\bar{q}$ denotes;

$$\bar{q} = -[M]\{\ddot{u}_g\}\,. \qquad (6)$$

It should be noted that, the Newmark-β method is used for the time integration of Eq. (1) by using the average acceleration method.

3. Numerical Examples

3.1. Data for numerical examples

In the light of the results given in references (Özdemir and Ayvaz, 2007), the aspect ratios, b/a, of the plate are taken to be 1, 1.5, 2.0, and 3.0. The thickness/span ratios, t/a are taken as 0.05, 0.1, 0.2, and 0.3 for each aspect ratio. The shorter span length of the plate is kept constant to be 3 m. The mass density, Poisson's ratio, and the modulus of elasticity of the plate are taken to be 2.5 kN/m^2, 0.2, and 2.8x10^7 kN/m^2 for both analysis. In order to obtain the response of each plate in the analysis, the first 8 s of the East-West component of the March 13, 1992 Erzincan earthquake in Turkey is used since the peak value of the record occurred in this range.

For the sake of accuracy in the results, rather than starting with a set of a finite element mesh size and time increment, the mesh size and time increment required to obtain the desired accuracy were determined before presenting any results. This analysis was performed separately for the mesh size and time increment. It was concluded that the results have acceptable error when equally spaced 4x4 mesh sizes are used for a 3 m x 3 m plate, if the 0.01 s time increment is used. Length of the elements in the x and y directions are kept constant for different aspect ratios as in the case of square plate.

3.2. Results

The absolute maximum values of displacements and bending moments of the plates modeled using MT17 element for different aspect ratios are presented in this study. The absolute maximum displacements of the plates for different aspect ratios, and thickness/span ratios are given in Fig. 2.

The absolute maximum bending moments M_x and M_y at the center of the plates simply supported and clamped plates along all four edges for different aspect ratios, and thickness/span ratios are given in Figs. 3 and 4, respectively.

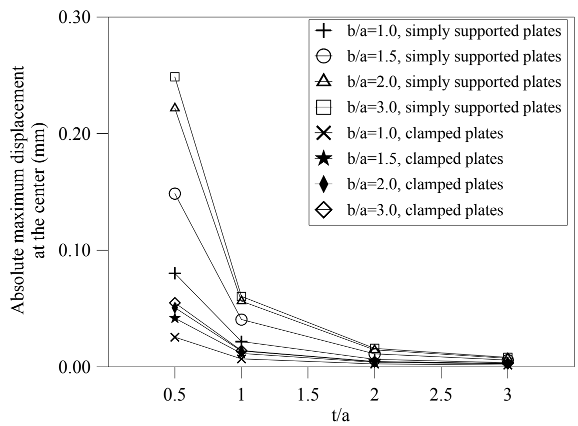

Fig. 2. Absolute maximum displacement of the simply supported and clamped plates for different aspect ratios and thickness/span ratios.

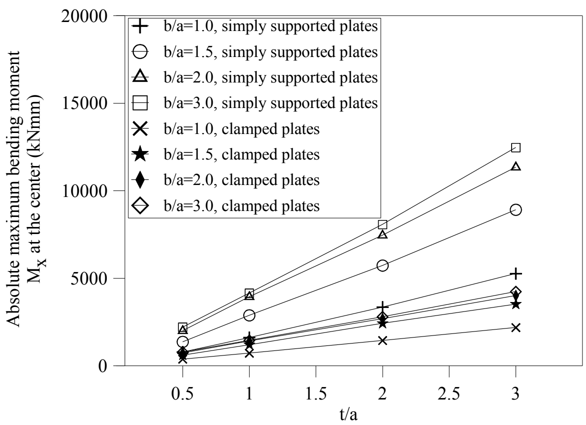

Fig. 3. Absolute maximum bending moment M_x at the center of the simply supported and clamped plates for different aspect ratios and thickness/span ratios.

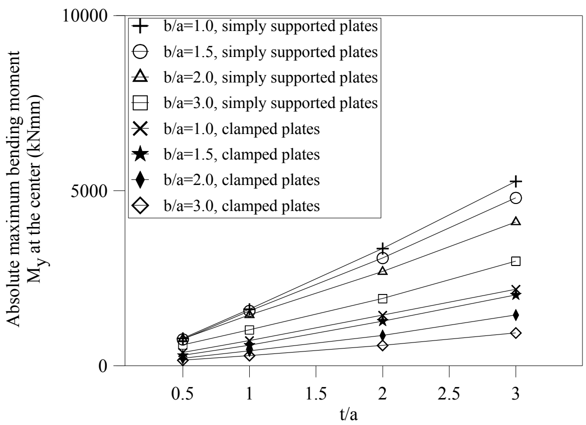

Fig. 4. Absolute maximum bending moment M_y at the center of the simply supported and clamped plates for different aspect ratios and thickness/span ratios.

As seen from Fig. 3, the absolute maximum bending moments M_x at the center of simply supported and clamped plates increase with increasing aspect ratio and thickness/span ratio. The increase in the maximum bending moment M_x decreases with increasing aspect ratio, and increases with increasing thickness/span ratio. In general, the effects of the changes in the aspect ratios on the absolute maximum bending moment, M_x, are larger than the changes in the thickness/span ratios.

As seen from Fig. 4, the absolute maximum bending moments M_y at the center of simply supported and clamped plates decreases with increasing aspect ratio and thickness/span ratio. The decrease in the maximum bending moment M_y increases with increasing aspect ratio, and decreases with decreasing thickness/span ratio. In general, the effects of the changes in the thickness/span ratios on the absolute maximum bending moment, M_y, are larger than the changes in the aspect ratios.

In this study, the absolute maximum bending moments M_x at the center of the edge in the y direction and the maximum bending moment M_y at the center of the edge in the x direction are not presented for the thick plates clamped along all four edges. It should be noted that the variations of these moments are similar to the absolute maximum bending moments M_x at the center of the thick clamped plates.

The effectiveness of the aspect and thickness/span ratios on the maximum responses considered in this study depends on the values of them. But, in general, the thickness/span ratio is more effective on the maximum responses than the aspect ratio.

4. Conclusions

The purpose of this paper was to study shear locking-free analysis of thick plates using Mindlin's theory by using 17-noded finite elements and to determine the effects of the thickness/span ratio, the aspect ratio and the boundary conditions on the maximum displacements and bending moments of thick plates subjected to earthquake excitations. It is concluded that the coded program can effectively be used in the earthquake analysis of the thick plates by using 17-noded finite element. The following conclusions can also be drawn from the results obtained in this study.

The absolute maximum displacements of the thick plates increase with increasing aspect ratio for a constant t/a ratio. The same displacements decrease with increasing t/a ratio for a constant b/a ratio. The effects of the changes in the thickness/span ratios on the absolute maximum displacement are generally larger than the changes in the aspect ratios.

The absolute maximum bending moment, M_x, at the center of the thick simply supported and clamped plates increases with increasing aspect ratio and thickness/span ratio. The effects of the changes in the aspect ratios on the absolute maximum bending moment, M_x, of the thick simply supported and clamped plates are generally larger than the changes in the thickness/span ratios.

The absolute maximum bending moment, M_y, at the center of the thick simply supported and clamped plates decreases with increasing aspect ratio and increases with increasing thickness/span ratio. The effects of the changes in the thickness/span ratios on the absolute maximum bending moment, M_y, of the thick simply supported and clamped plates are generally larger than the changes in the aspect ratios. The effectiveness of the aspect and thickness/span ratios on the maximum responses considered in this study depends on the values of them.

In general, degrees of decreases and increases depend on the changes in the aspect and thickness/span ratios, and the changes in the thickness/span ratio are more effective on the maximum responses considered in this study than the changes in the aspect ratio.

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