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Dynamic analysis of pre-stressed elastic beams under moving mass using different beam models

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ABSTRACT

This study presents dynamic analysis of pre-stressed elastic beams under the action of moving mass loads by using Bernoulli-Euler, Rayleigh, and shear beam models. It is assumed the mass moves with a constant speed and is in continuous contact with the beam during its motion. Discrete equations of motion with time-dependent coefficients are obtained by using the assumed mode method for each beam models considered. Numerical calculations are made by Newmark method to obtain dynamic response of the beam. Effects of the pre-stressing force, rotatory inertia and transverse shear on the results for the dynamic deflection and bending moment of the beam and the interaction force between the mass and the beam are studied by depending on mass weight and speed of the moving mass.

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1. Introduction

Dynamics of beam structures under the action of moving loads and moving masses has been extensively studied for over a century in relation to the design of railroad tracks and bridges, and also machining processes. It is well known the inertia effects of a moving load cannot be ignored in the analysis when the mass weight of the moving load is large compared to the mass of the beam even if the speed of the moving mass is relatively small (Sadiku and Leipholz, 1987; Lee, 1996). Jeffcott (1929) was the first to consider inertial effects of both the moving load and the beam by the method of successive approximations. Frýba (1972) presented a comprehensive literature survey which contains analytical solutions to a large number of problems on the dynamic analysis of solids and structures under moving loads.

On the basis of the Bernoulli-Euler beam theory, Ting et al. (1974), Sadiku and Leipholz (1987), and Foda and Abduljabbar (1998) dealt with the moving mass problem of elastic beams by the use of Green's function. Stanišić (1985) derived an exact, closed form solution for a simple beam carrying a single moving mass by means of expansion of the eigenfunctions in a series. Akin and Mofid (1989) presented an analytical-numerical method to determine the dynamic behavior of beams with different boundary conditions carrying a moving mass. Assuming the solution in the form of a series in terms of eigenfunctions of the beam, they transformed the governing differential equation into a series of coupled ordinary differential equations. Esmailzadeh and Ghorashi (1995) studied the dynamic response of simply supported beams to uniform partially distributed moving masses. They found that increase in the load length makes more important the inertial effects of the moving mass. Michaltsos et al. (1996) considered the dynamic response of a simply supported beam under a moving mass with constant magnitude and speed. Using a series solution for the dynamic deflection of the beam in terms of normal modes, effects of the mass weight and speed of the moving load and other parameters were fully assessed. A new solution technique so-called discrete element method was proposed by Mofid and his co-workers for the moving mass problem of simple beams. They applied this method to Bernoulli-Euler beams with different boundary conditions under moving mass loads (Mofid and Akin, 1996; Mofid and Shadnam, 2000). Lee (1996) gave solution to the moving mass problem of an

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elastic beam. He formulated the equation of motion in matrix form by using the Lagrangian approach and the assumed mode method. According to his study, there is a possibility of the loss of contact between the mass and the beam during the course of the motion. Another study pointing out separation between the mass and the beam was carried out by Lee (1998). He investigated the onset of the separation between the mass and the beam, and took into account its effect in calculating the interaction forces and the dynamic response of the beam. A technique using combined finite element and analytical methods for determining the dynamic responses of structures to moving loads was presented by Wu et al. (2001). Bilello et al. (2004) gave an experimental investigation of a simple beam under a moving mass. They designated a small-scale model to satisfy both static and dynamic similitude with a selected prototype bridge structure. More recently, Bowe and Mullarkey (2008) used a modal and finite element model to solve the problem of moving unsprung mass traversing a beam with different boundary conditions. They highlighted the drastic effects of omitting the convective acceleration terms from the formulation of unsprung moving mass problem.

On the basis of Timoshenko beam theory, Mackertich (1992) considered dynamic response of a simply supported beam excited by a moving mass. He reported that the effect of shear deformation and rotatory inertia are significant in determining the dynamic response of a beam subjected to a high-speed moving mass. The analysis of a Timoshenko beam under the action of moving mass was studied by Esmailzadeh and Ghorashi (1997). They solved the equations of motion by using a finite difference based algorithm. They investigated effects of shear deformation, rotatory inertia and the length of distributed load on the beam response. Yavari et al. (2002) analyzed the moving mass problem of Timoshenko beams using discrete element method. Lou et al. (2006) studied response of a Timoshenko beam to a moving mass by using the finite element method in which the inertial effects of the mass are incorporated into the finite element model. In a recent study, Kiani et al. (2009) presented a comprehensive assessment of design parameters such as dynamic deflection and bending moment of elastic beams subjected to a moving mass under different boundary conditions for Bernoulli-Euler, Timoshenko and higher-order shear beam theories. They gave detailed results to clarify effects of important parameters such as beam slenderness and boundary conditions as well as the change in weight and velocity of the moving mass.

Concrete bridges are common in the world and some of them were constructed by using pre-stressed concrete which works well for long-span bridges. Due to pre-tensioning, tensile stresses appearing in the beam are reduced to desired degree or vanished completely. The physical effect of pre-stressing is to provide an additional compressive stress in structures to resist tensile stresses occurring due to external loads (Chan and Yung, 2000). Although the effects of axial loading on the vibration characteristics of beams have been well investigated, studies related to the forced vibrations of beams with axial loading under moving loads are fewer. Frýba (1972) studied the response of a Bernoulli-Euler beam subjected to an axial force and a moving load. He gave analytical solutions of two different problems: First is the moving concentrated force problem in which inertia effect of the mass is ignored, and the second is the moving continuous load problem in which inertia effect of the mass is considered. Forced vibration of a viscoelastic Bernoulli-Euler beam subjected to an eccentric compressive force and a moving harmonic load was studied by Kocatürk and Şimşek (2006a), and was extended to the cases based on Timoshenko and higher-order beam theories in their subsequent papers (Kocatürk and Şimşek, 2006b; Şimşek and Kocatürk, 2007). In a recent study by Şimşek and Kocatürk (2009), dynamic analysis of an eccentrically pre-stressed damped beam under a moving harmonic concentrated force was studied by considering geometric nonlinearity. Kahya (2009) considered inertial effect of the moving mass in analyzing dynamic behavior of a simply supported pre-stressed elastic beam. According to the studies mentioned above, pre-stressing force has remarkable effect on the beam response. However, to the author's knowledge, inertial effects of moving loads on the dynamic response of beams with axial loading by also considering rotatory inertia and shear deformation have not been studied yet.

This study presents dynamic analysis of a simply supported elastic beam subjected to an axial load and a moving mass. Using the assumed mode method, discrete equations of motion with time-dependent coefficients is derived in matrix form for Bernoulli-Euler, Rayleigh and shear beam models to investigate effects of rotatory inertia and transverse shear on the beam response. The matrix equation of motion is solved numerically by using Newmark method. Effects of pre-stressing force, rotatory inertia and transverse shear on the dynamic deflection and bending moment of the beam and the interaction force between the mass and the beam are studied by depending on weight and speed of the moving mass and the beam length.

2. Definition of the Problem

A simply supported elastic beam shown in Fig. 1 is subjected to an axial force N at its ends and a mass M moving with a constant velocity v from left to right along the beam. At the beginning of motion ($t = 0$), the beam is at rest and it is assumed the moving mass keeps contact with the beam during its motion. The pre-stressed tendon is assumed straight and unbonded with the concrete. For a pre-stressed structure with a straight tendon, the structure is under the effect of an axial force and a bending moment at its ends. Chan and Yung (2000) reported that neglecting the moments due to pre-stressing in the governing equations is a reasonable approach. Therefore, only the axial pre-stressing force is considered in the governing equations of the system.

3. Mathematical Formulations

The dynamic behavior of the beam under the action of moving mass load is expressed by three different bam models: (a) Bernoulli-Euler beam model, (b) Rayleigh beam model, and (c) Shear beam model. For each model, discrete governing equations of motion are derived by using the assumed mode method in subsequent sections.

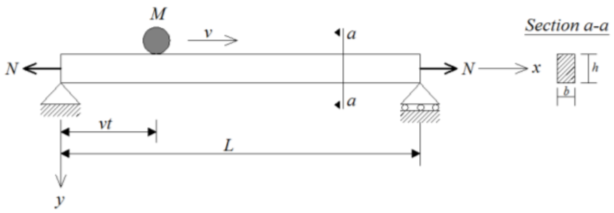

Fig. 1. Moving mass on a simply supported elastic beam with an axial load.

3.1. Bernoulli-Euler beam formulation

According to Bernoulli-Euler beam theory, governing differential equation of the beam shown in Fig. 1 can be written as follows.

$$EIy^{iv}(x,t) - Ny''(x,t) + cy'(x,t) + m\ddot{y}(x,t) = p(x,t), \quad (1)$$

where primes and dots represent derivatives with respect to spatial coordinate x and time t, respectively. $y(x,t)$ is the dynamic deflection, EI is the flexural rigidity, c is the damping coefficient and m is mass per unit length of the beam. N is the axial force (pre-stressing force) at the ends of the beam and $p(x,t)$ is transverse external force which can be defined as

$$p(x,t) = \{Mg - M[\ddot{y}(x,t) + 2v\dot{y}'(x,t) + v^2 y''(x,t)]\}\delta(x,vt), \quad (2)$$

where g is the gravitational acceleration and $\delta(\dots)$ is the Dirac delta function.

In order to solve the governing differential equation given in Eq. (1), the solution $y(x,t)$ can be assumed as

$$y(x,t) = \sum_{i=1}^{\infty} \phi_i(x) q_i(t), \quad (3)$$

where $\phi_i(x)$ and $q_i(t)$ represent modal shape function and generalized coordinate at ith mode, respectively. Substituting Eq. (3) into Eq. (1) with considering Eq. (2) gives:

$$\sum_{i=1}^{\infty} EI\,\phi_i^{iv}(x)q_i(t) - \sum_{i=1}^{\infty} N\phi_i''(x)q_i(t) + \sum_{i=1}^{\infty} c\phi_i(x)\dot{q}_i(t) + \sum_{i=1}^{\infty} m\phi_i(x)\ddot{q}_i(t) = (Mg - M\sum_{i=1}^{\infty}[\phi_i(x)\ddot{q}_i(t) + 2v\phi_i'(x)\dot{q}_i(t) + v^2\phi_i''(x)q_i(t)])\delta(x - vt). \quad (4)$$

Multiplying each term of Eq. (4) by $\phi_j(x)$, integrating it with respect to x over the beam length, and interchanging the order of integration and summation gives

$$EI\sum_{i=1}^{\infty} q_i(t)\int_0^L \phi_i^{iv}(x)\phi_j(x)dx - N\sum_{i=1}^{\infty} q_i(t)\int_0^L \phi_i''(x)\phi_j(x)dx + c\sum_{i=1}^{\infty}\dot{q}_i(t)\int_0^L \phi_i(x)\phi_j(x)dx + m\sum_{i=1}^{\infty}\ddot{q}_i(t)\int_0^L \phi_i(x)\phi_j(x)dx = Mg\,\phi_j(vt) - M\sum_{i=1}^{\infty}\{\phi_i(vt)\phi_j(vt)\ddot{q}_i(t) + 2v\phi_i'(vt)\phi_j(vt)(t)\dot{q}_i(t) + v^2\phi_i''(vt)\phi_j(vt)q_i(t)\}, \quad (5)$$

Note that the following property of the Dirac delta function is considered to obtain right-hand side of Eq. (5)

$$\int_a^b f(x)\delta(x-\xi)dx = f(\xi) \quad (a < \xi < b). \quad (6)$$

For simply supported beams, modal shape function can be written as

$$\phi_i(x) = sin\frac{i\pi x}{L}. \quad (7)$$

Substituting Eq. (7) into Eq. (4), using modal orthogonality, and carrying out integrations yield the following matrix equation of motion with time-dependent coefficients.

$$M\ddot{q} + C\dot{q} + Kq = F, \quad (8)$$

where

$$M = I + \frac{2M}{mL}diag[\phi_i(x)]\Phi,$$

$$C = diag[2\xi_i\omega_{E,i}] + \frac{4M}{mL}v\,diag[\phi_i(x)]\Phi',$$

$$K = diag[\omega_{E,i}^2] + \frac{2M}{mL}v^2 diag[\phi_i(x)]\Phi'',$$

$$F = \frac{2Mg}{mL}\{\phi_1(vt)\phi_2(vt)\dots\phi_n(vt)^T\}, \quad (9)$$

where ξ_i and $\omega_{E,i}$ is the damping ratio and the circular natural frequency for ith mode, respectively. I is unit matrix, $diag(\dots)$ is a diagonal matrix. The matrix Φ can be defined as

$$\Phi = \begin{bmatrix} \phi_1(vt) & \phi_2(vt) & \cdots & \phi_n(vt) \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \phi_1(vt) & \phi_2(vt) & \dots & \phi_n(vt) \end{bmatrix}. \quad (10)$$

For a Bernoulli-Euler beam with axial load, $\omega_{E,i}$ can be defined as

$$\omega_{E,i}^2 = \frac{EI}{m}\frac{i^4\pi^4}{L^4} + \frac{N}{m}\frac{i^2\pi^2}{L^2}, \quad i = 1, 2, \dots, n, \quad (11)$$

where n is total number of modes considered in the analysis. Bending moment for Bernoulli-Euler beam can be expressed by

$$M(x,t) = -EIy''(x,t). \quad (12)$$

3.2. Rayleigh beam formulation

Rayleigh beam model considers only rotatory inertia of the beam. The governing differential equation can thus be written as

$$EIy^{iv}(x,t) - Ny''(x,t) + c\dot{y}(x,t) - mr^2\ddot{y}''(x,t) + m\ddot{y}(x,t) = p(x,t)\,. \qquad (13)$$

Here, $r = \sqrt{\pi I/A}$ denotes the radius of gyration where I and A are the second moment of area and cross-sectional area of the beam, respectively.

Assuming the solution $y(x,t)$ as given in Eq. (3) and following the same procedure just described for Bernoulli-Euler beams yields

$$M\ddot{q} + C\dot{q} + Kq = F\,, \qquad (14)$$

where

$$M = diag[\mu_{R,i}] + \frac{2M}{mL} diag[\phi_i(x)]\Phi\,,$$

$$C = diag[2\xi_i\omega_{R,i}] + \frac{4M}{mL} v\, diag[\phi_i(x)]\Phi'\,,$$

$$K = diag[\omega_{E,i}^2] + \frac{2M}{mL} v^2 diag[\phi_i(x)]\Phi''\,,$$

$$F = diag\,\frac{2Mg}{mL}\{\phi_1(vt)\phi_2(vt)\ldots\phi_n(vt)^T\}\,, \qquad (15)$$

where

$$\mu = 1 + \frac{i^2\pi^2 r^2}{L^2}\quad,\quad \omega_{R,i}^2 = \frac{\omega_{E,i}^2}{\mu_{R,i}}\quad,\quad i = 1,2,\ldots,n\quad, \qquad (16)$$

which is the circular natural frequency for a Rayleigh beam with axial load. Bending moment of a Rayleigh beam is also expressed by Eq. (12).

3.3. Shear beam formulation

When only the effect of transverse shear on the dynamic behavior of the beam is considered, the following couple of governing differential equations can be written.

$$m\ddot{y}(x,t) + c\dot{y}(x,t) - kAG[y''(x,t) - \theta'(x,t)] - Ny''(x,t) = p(x,t)\,,$$

$$EI\theta''(x,t) + kAG[y'(x,t) - \theta(x,t)] = 0\,, \qquad (17)$$

where k is the shear correction factor that depends on the shape of the cross-section of the beam, G is shear modulus and $\theta(x,t)$ is the rotation of the cross-section of the beam.

Solutions for the dynamic deflection $y(x,t)$ and the rotation $\theta(x,t)$ can be assumed as

$$y(x,t) = \sum_{i=1}^{\infty}\phi_i(x)q_i(t)\ \ ,\ \theta(x,t) = \sum_{i=1}^{\infty}\psi_i(x)s_i(t)\,, \qquad (18)$$

where $q_i(t)$ and $s_i(t)$ are generalized coordinates, and $\phi_i(t)$ and $\psi_i(t)$ are modal shape functions which can be defined as

$$\phi_i(x) = sin\frac{i\pi x}{L}\quad,\quad \theta\psi_i(x) = cos\frac{i\pi x}{L}\,, \qquad (19)$$

for simply supported beams. Substituting Eq. (18) into Eq. (17) gives

$$m\sum_{i=1}^{\infty}\ddot{q}_i(t) + c\sum_{i=1}^{\infty}\phi_i(x)\dot{q}_i(t) - kAG[\sum_{i=1}^{\infty}\phi_i''(x)q_i(t) - \sum_{i=1}^{\infty}\psi_i'(x)s_i(t)] - N\sum_{i=1}^{\infty}\phi_i''(x)q_i(t)$$
$$= (Mg - M\sum_{i=1}^{\infty}[\phi_i(x)\ddot{q}_i(t) + 2v\phi_i'(x)\dot{q}_i(t) + v^2\phi_i''(x)q_i(t)])\delta(x-vt)\,,$$
$$EI\sum_{i=1}^{\infty}\psi_i''(x)s_i(t) + kAG[\sum_{i=1}^{\infty}\phi_i'(x)q_i(t) - \sum_{i=1}^{\infty}\psi_i'(x)s_i(t)\,] = 0\,. \qquad (20)$$

Multiplying the first equation of (20) by $\phi_j(x)$ and the second by $\psi_j(x)$, integrating them over the beam length, and interchanging the order of integration and summation yields

$$m\sum_{i=1}^{\infty}\ddot{q}_i(t)\int_0^L\phi_i(x)\,\phi_j(x)dx + c\sum_{i=1}^{\infty}\dot{q}_i(t)\int_0^L\phi_i(x)\,\phi_j(x)dx - kAG\left[\sum_{i=1}^{\infty}q_i(t)\int_0^L\phi_i''(x)\,\phi_j(x)dx - \right.$$
$$\left.\sum_{i=1}^{\infty}s_i(t)\int_0^L\psi_i'(x)\,\phi_j(x)dx\right] - N\sum_{i=1}^{\infty}q_i(t)\int_0^L\phi_i''(x)\,\phi_j(x)dx = Mg\,\phi_j(vt) - M\sum_{i=1}^{\infty}\{\phi_i(vt)\phi_j(vt)\ddot{q}_i(t) +$$
$$2v\phi_i(vt)\phi_j(vt)\dot{q}_i(t) + v^2\phi_i''(vt)\phi_j(vt)q_i(t)\}\,,$$
$$EI\sum_{i=1}^{\infty}s_i(t)\int_0^L\psi_i''(x)\,\psi_j(x)dx + kAG\left[\sum_{i=1}^{\infty}q_i(t)\int_0^L\phi_i'(x)\,\psi_j(x)dx - \sum_{i=1}^{\infty}s_i(t)\int_0^L\psi_i'(x)\,\psi_j(x)dx\right] = 0\,. \qquad (21)$$

Substituting Eq. (19) into Eq. (21), carrying out integrals, and using modal orthogonality, the following coupled matrix equations of motion with time-dependent coefficients can be obtained.

$$\left\{I + \frac{2M}{ml}diag[\phi_i]\Phi\right\}\ddot{q} + \left\{diag[2\xi_i\omega_{s,i}] + \frac{4M}{mL}v\,diag[\phi_i(x)]\Phi'\right\}\dot{q} + \left\{diag\left[\frac{N+kAG}{m}\frac{i^2\pi^2}{L^2}\right] + \frac{2M}{mL}v^2 diag[\phi_i(x)]\Phi''\right\}q - diag\left[\frac{kAG}{m}\frac{i\pi}{L}\right]s = F\,,$$

$$diag\left[\frac{kAG}{m}\frac{i\pi}{L}\right]q - diag\left[\frac{EI}{m}\frac{i^2\pi^2}{L^2} + \frac{kAG}{m}\right]s = 0\,. \qquad (22)$$

From the second equation of (22)

$$s = \frac{diag\left[\frac{kAGi\pi}{m\ L}\right]}{diag\left[\frac{EIi^2\pi^2}{m\ L^2}+\frac{kAG}{m}\right]} q \ . \tag{23}$$

Using Eq. (23) into the first equation of (22), after some arrangements, the following matrix equation of motion can be obtained in terms of q.

$$M\ddot{q} + C\dot{q} + Kq = F \ , \tag{24}$$

where

$$M = I + \frac{2M}{mL} dia[\phi_i(x)]\Phi \ ,$$

$$C = diag\left[2\xi_i\omega_{s,i}\right] + \frac{4M}{mL} v diag[\phi_i(x)]\Phi' \ ,$$

$$K = diag\left[\omega_{s,i}^2\right] + \frac{2M}{mL} v^2 diag[\phi_i(x)]\Phi'' \ ,$$

$$F = \frac{2Mg}{mL} \ \{\phi_1(vt)\ \phi_2(vt)\ \dots\ \phi_n(vt)^T\} \ , \tag{25}$$

where $\omega_{s,i}$ is the circular natural frequency for a shear beam with axial load and can be defined as

$$\omega_{s,i}^2 = \frac{1}{1+\frac{EI}{kAG}\frac{i^2\pi^2}{L^2}} \left\{\left(1+\frac{N}{kAG}\right)\frac{EI}{m}\frac{i^4\pi^4}{L^4} + \frac{N}{m}\frac{i^2\pi^2}{L^2}\right\} \ , i = 1, 2, \dots, n \ . \tag{26}$$

Bending moment for a shear beam can be expressed by

$$M(x,t) = -EI\theta'(x,t) \ . \tag{27}$$

4. Numerical Results

Since matrix equations of motion given by Eq. (8) for Bernoulli-Euler beam, Eq. (14) for Rayleigh beam and Eq. (24) for shear beam have time-dependent coefficients, a numerical solution algorithm is required for solution. In this study, Newmark method is employed to obtain dynamic deflection of the beam with axial load under moving mass. Once the dynamic deflection is obtained, the bending moment can easily be obtained by using Eq. (12) for Bernoulli-Euler and Rayleigh beams and Eq. (27) for shear beam.

The interaction (contact) force between the moving mass and the beam can be calculated by

$$F_c = Mg - M[\ddot{y}(x,t) + 2v\dot{y}'(x,t) + v^2y''(x,t)]_{x=vt} \ , \tag{28}$$

which is defined to be positive if the force acting on the beam is pointing downward, i.e., in the positive y direction. Changing sign of the contact force from positive to negative would indicate that the mass separated from the beam (Lee, 1996).

Numerical calculations are made for n=10 modes, and the parameters of the beam are assumed to be E=35 GPa, v=0.2, m=1500 kg/m, b=0.4 m, h=0.4 m, and k=5/6 (Şimşek and Kocatürk, 2007). The damping ratio (ξ) is considered as 2.5% for all vibration modes.

Comparison of fundamental frequency of the beam for various slenderness ratios L/h according to different beam models is given in Table 1. Axial force is normalized by $N_{cr} = \pi^2L^2/EI$ which is the critical buckling force for simply supported elastic beams. As seen in Eqs. (11), (16) and (26) which express natural frequencies for the considered beam models; the axial force has a direct effect on the natural frequency of the beam. As the axial force increases in tension, the fundamental frequency increases, too. On the other hand, the fundamental frequency decreases with increasing the axial force in compression. The latter case is called as the compression softening which can be explained as reduction in beam stiffness due to the pre-stressing (Chan and Yung, 2000; Şimşek and Kocatürk, 2007). In Table 2, critical velocities of the moving mass, which is defined as $v_{cr} = \omega_1 L/\pi$, are given for considered beam models and various L/h values. Variation of the critical velocity with the normalized axial force is similar to that of the fundamental frequency. From Tables 1 and 2, and Figs. 2 and 3, it can be said that shear beam model always gives smaller fundamental frequency and critical velocity in comparison to the other beam models when the slenderness ratio L/h of the beam is small

Table 1. Comparison of fundamental frequencies of the beam according to different beam models.

N/N_{cr}	$\omega_1/2\pi$ (Hertz)								
	L/h=5			L/h=10			L/h=20		
	BE	*R*	*S*	*BE*	*R*	*S*	*BE*	*R*	*S*
-1.0	-	-	-	-	-	-	-	-	-
-0.8	24.78	24.38	23.12	6.20	6.17	6.09	1.55	1.55	1.54
-0.6	35.05	34.49	33.10	8.76	8.73	8.63	2.19	2.19	2.18
-0.4	42.93	42.24	40.70	10.73	10.69	10.59	2.68	2.68	2.67
-0.2	49.57	48.77	47.09	12.39	12.34	12.23	3.10	3.09	3.09
0.0	55.42	54.53	52.71	13.85	13.80	13.68	3.46	3.46	3.45
0.2	60.68	59.74	57.79	15.18	15.11	14.98	3.79	3.80	3.78
0.4	65.56	64.51	62.45	16.34	16.32	16.19	4.10	4.09	4.09
0.6	71.10	69.00	66.79	17.52	17.45	17.31	4.38	4.38	4.37
0.8	74.34	73.15	70.87	18.59	18.51	18.35	4.65	4.64	4.63
1.0	78.36	77.11	74.72	19.59	19.51	19.35	4.90	4.89	4.88

Table 2. Comparison of critical velocities of the moving mass according to different beam models.

N/N_{cr}	v_{cr} (m/s)								
	L/h=5			L/h=10			L/h=20		
	BE	*R*	*S*	*BE*	*R*	*S*	*BE*	*R*	*S*
-1.0	-	-	-	-	-	-	-	-	-
-0.8	247.83	243.84	231.20	124.00	123.40	121.80	62.00	62.00	61.60
-0.6	350.52	344.87	331.01	175.20	174.60	172.60	87.6.0	87.60	87.20
-0.4	429.27	422.40	406.99	214.60	213.80	211.80	107.20	107.20	106.80
-0.2	495.66	487.79	470.94	247.80	246.80	244.60	124.00	123.60	123.60
0.0	554.18	545.26	527.14	277.00	276.00	273.60	138.40	138.40	138.00
0.2	606.18	597.41	578.01	303.60	302.20	299.60	151.60	152.00	151.20
0.4	655.60	645.07	624.56	326.80	326.40	323.80	164.00	163.60	163.60
0.6	701.04	689.95	667.79	350.40	349.00	346.20	175.20	175.20	174.80
0.8	743.40	731.50	708.70	371.80	370.20	367.00	186.00	185.60	185.20
1.0	783.60	771.10	747.20	391.80	390.20	387.00	196.00	195.60	195.20

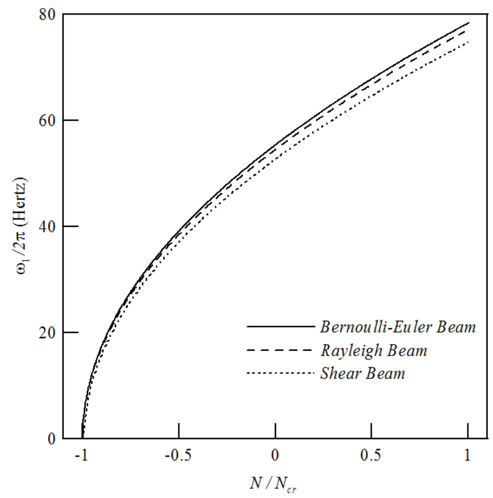

Fig. 2. Fundamental frequency of the beam vs. the normalized axial force for different beam models (L/h=5).

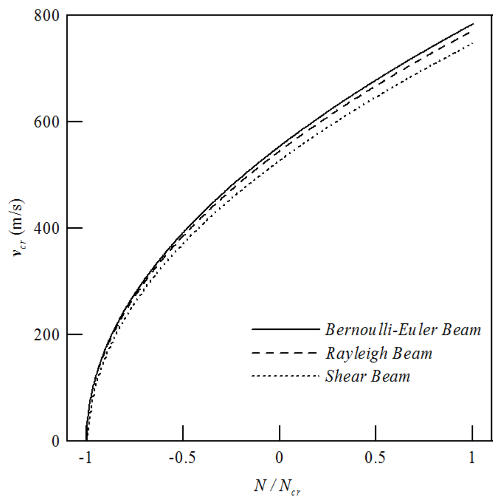

Fig. 3. Critical velocity of the moving mass vs. the normalized axial force for different beam models (L/h=5).

Figs. 4(a-b) show variation of maximum dynamic deflection and bending moment of the beam at its midspan with dimensionless velocity of the moving mass for Bernoulli-Euler beam, L/h=20 and M/mL=0.15, respectively. When the axial compressive force increases, both the deflection and the bending moment increase because of the compression softening effect. On the contrary, the deflection and the bending moment decrease with increasing the axial force in tension.

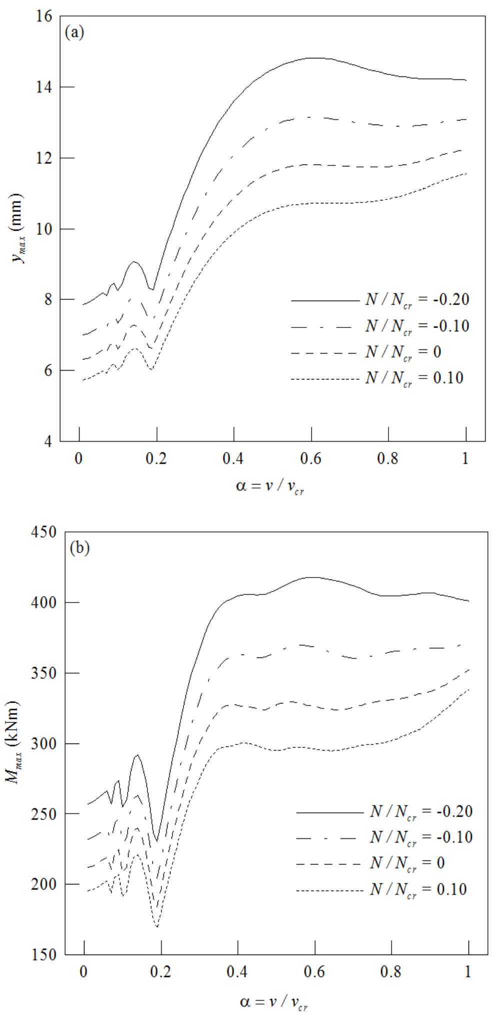

Fig. 4. Maximum dynamic response of the beam at its midspan vs. dimensionless velocity of the mass for different pre-stressing forces; (a) Deflection; (b) Bending moment (L/h=20, M/mL=0.15).

In Figs. 5 and 6, variation of maximum dynamic deflection and bending moment response of the beam with the dimensionless velocity for different beam models is shown for N/N_{cr}=-0.10, M/mL=0.15 and L/h=5, 10, 15 and 20. According to Fig. 5, rotatory inertia has almost no effect on deflection response of the beam at its midspan. However, shear deformation affects the midspan deflection considerably for shorter beams. Midspan deflections increase with increasing the slenderness ratio L/h. Deflection curves come closer to each other with increasing L/h since effect of shear deformation disappears for slender beams. According to Fig. 6, shear deformation has little effect on the bending moment response compared to the deflection response.

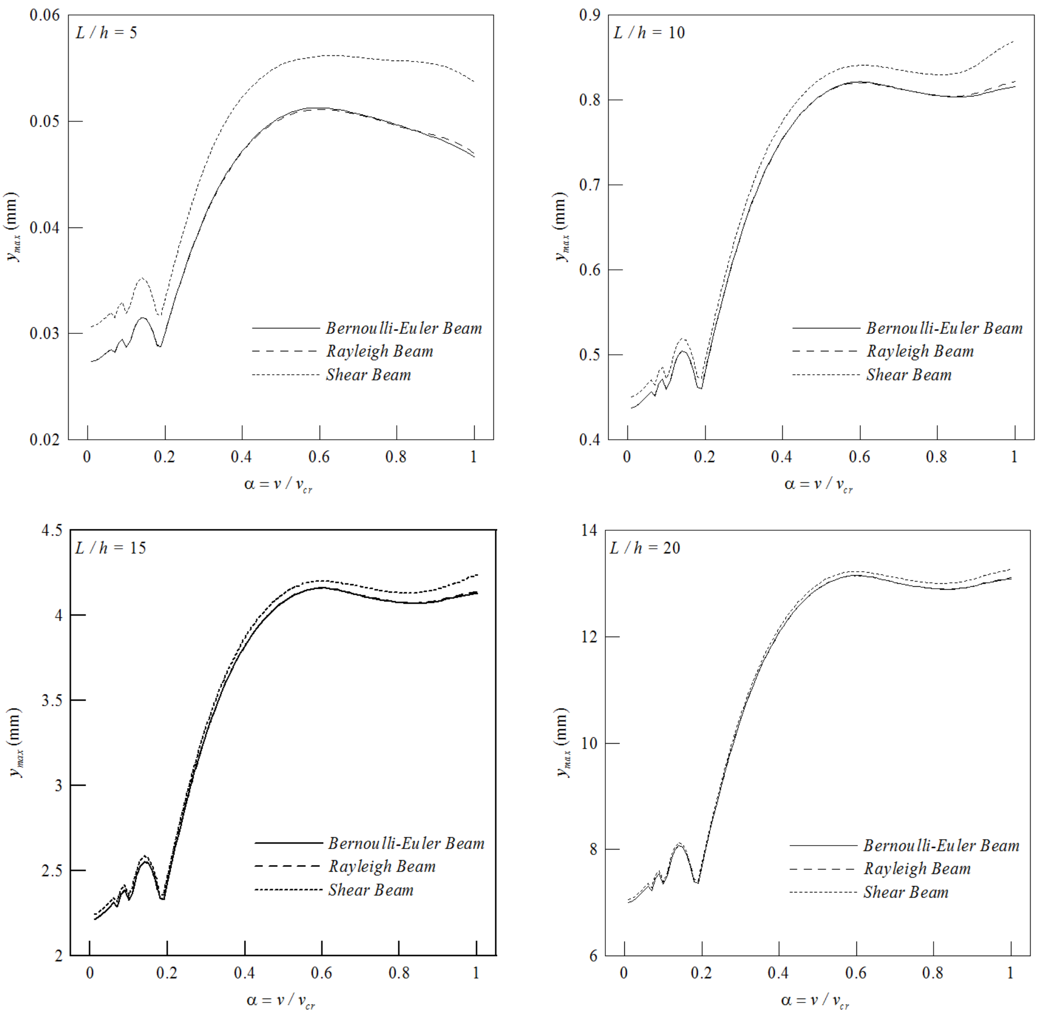

Fig. 5. Maximum deflection of the beam at its midspan vs. dimensionless velocity of the mass for different beam lengths (N/N_{cr}=-0.10, M/mL=0.15).

Variation of maximum deflection and bending moment of the beam at its midspan with the mass parameter M/mL is given in Fig. 7 for Bernoulli-Euler beam, L/h=20 and α=0.20. Both the deflection and the bending moment increase with increasing M/mL. As pre-stressing force increases, deflections and bending moments of the beam increase, too.

Figs. 8 and 9 give variation of maximum deflection and bending moment at midspan of the beam with M/mL for N/N_{cr}=-0.10, α=0.20, and L/h=5 and 10 considering different beam models. According to these figures, consideration of shear deformation in calculations increases both the deflection and the bending moment while rotatory inertia has almost no effect. Shear deformation has greater effect on the deflection response compared to the bending moment response, especially for beams with smaller L/h values.

When the mass moves along the beam length, the deflection and the bending moment at midspan and the contact force between the mass and the beam are given in Fig. 10 for various pre-stressing forces. Here, L/h=20, M/mL=0.15, α=0.20 and Bernoulli-Euler beam model is considered. As stated above, an increase in compressive axial forces (pre-stressing forces) causes greater deflections and bending moments. However, the contact force between the mass and the beam is not considerably affected by increasing pre-stressing forces.

In Fig. 11, a comparison of the deflection and the bending moment at midspan of the beam and the contact force distribution under the moving mass for different beam lengths is given for N/N_{cr}=-0.10, M/mL=0.15, α=0.20 considering different beam models. Results for the deflection and the bending moment in this figure are in good agreement those of Figs. 5, 6, 8 and 9. As can be seen in these figures, midspan deflections significantly increase by considering shear deformation for short beams. However, shear deformation has little effect on the bending moment and the contact force compared to the deflection response**.**

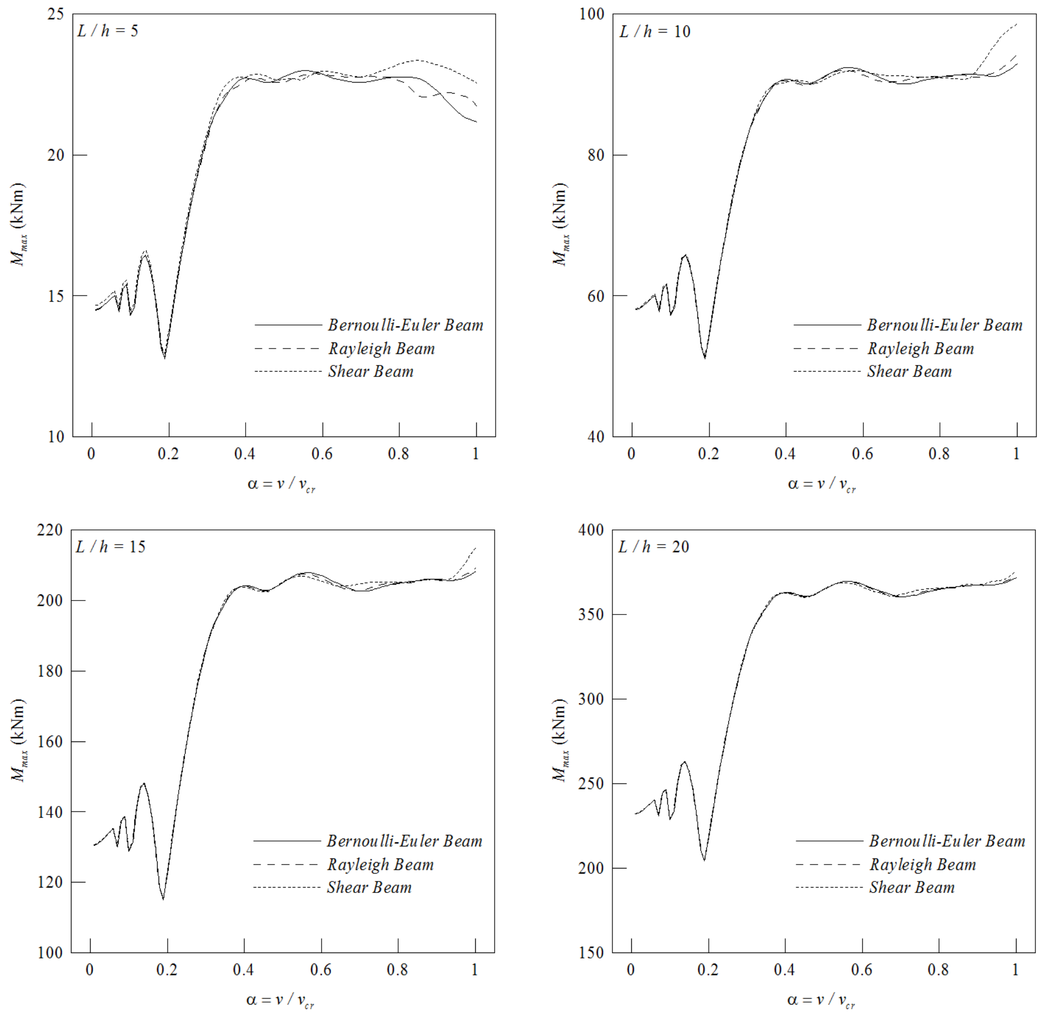

Fig. 6. Maximum bending moment at midspan of the beam vs. dimensionless velocity of the mass for different beam lengths (N/N_{cr}=-0.10, M/mL=0.15)

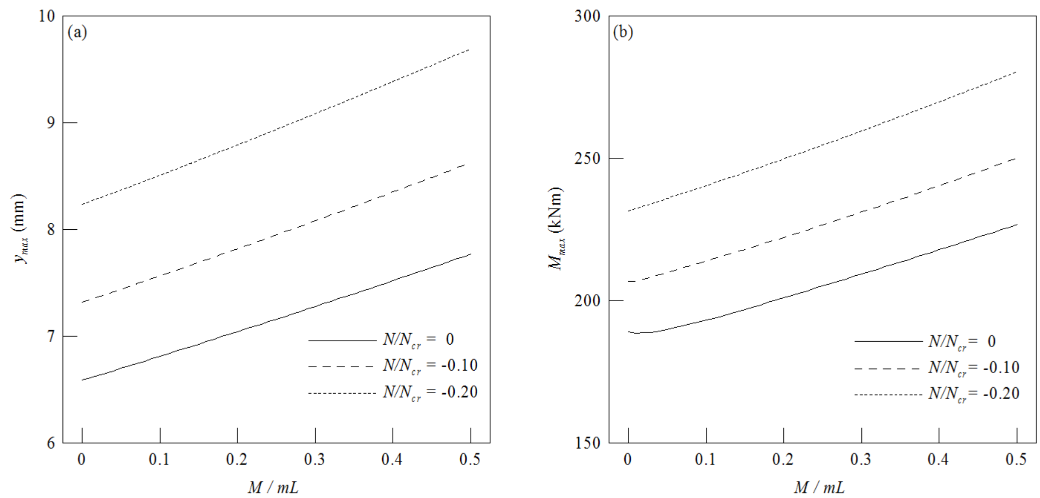

Fig. 7. Maximum dynamic response of the beam at its midspan vs.   for various pre-stressing forces; (a) Deflection; (b) Bending moment (L/h=20, α=0.20)

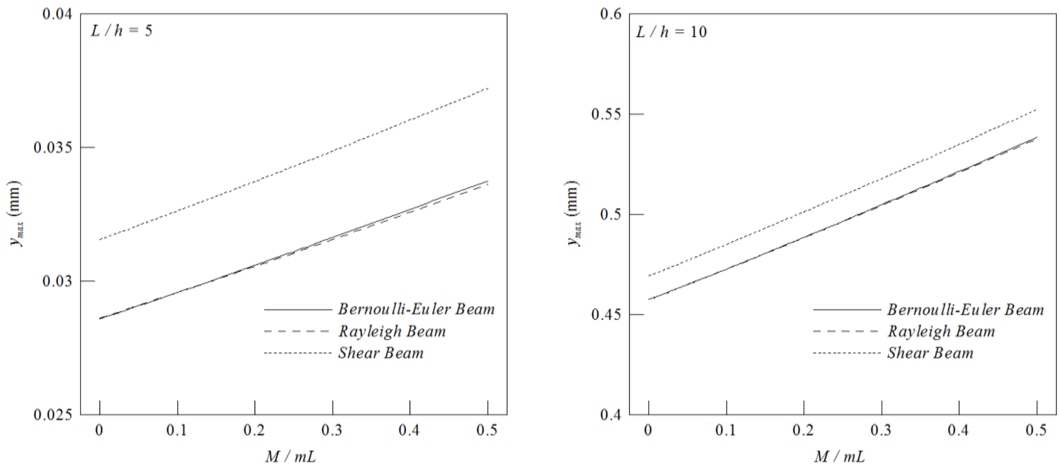

Fig. 8. Maximum deflection of the beam at its midspan vs. M/mL for different beam lengths (N/N_{cr}=-0.10, α=0.20)

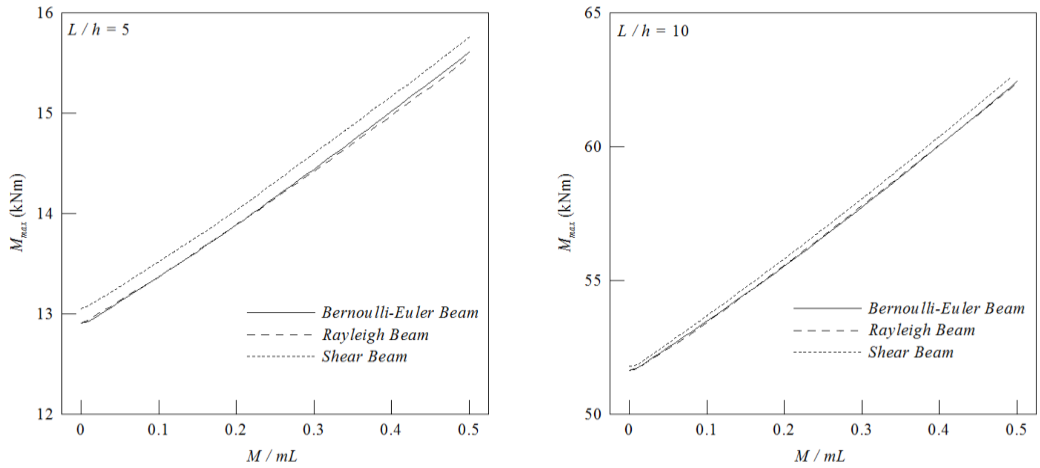

Fig. 9. Maximum bending moment at midspan of the beam vs. M/mL for different beam lengths (N/N_{cr}=-0.10, α=0.20)

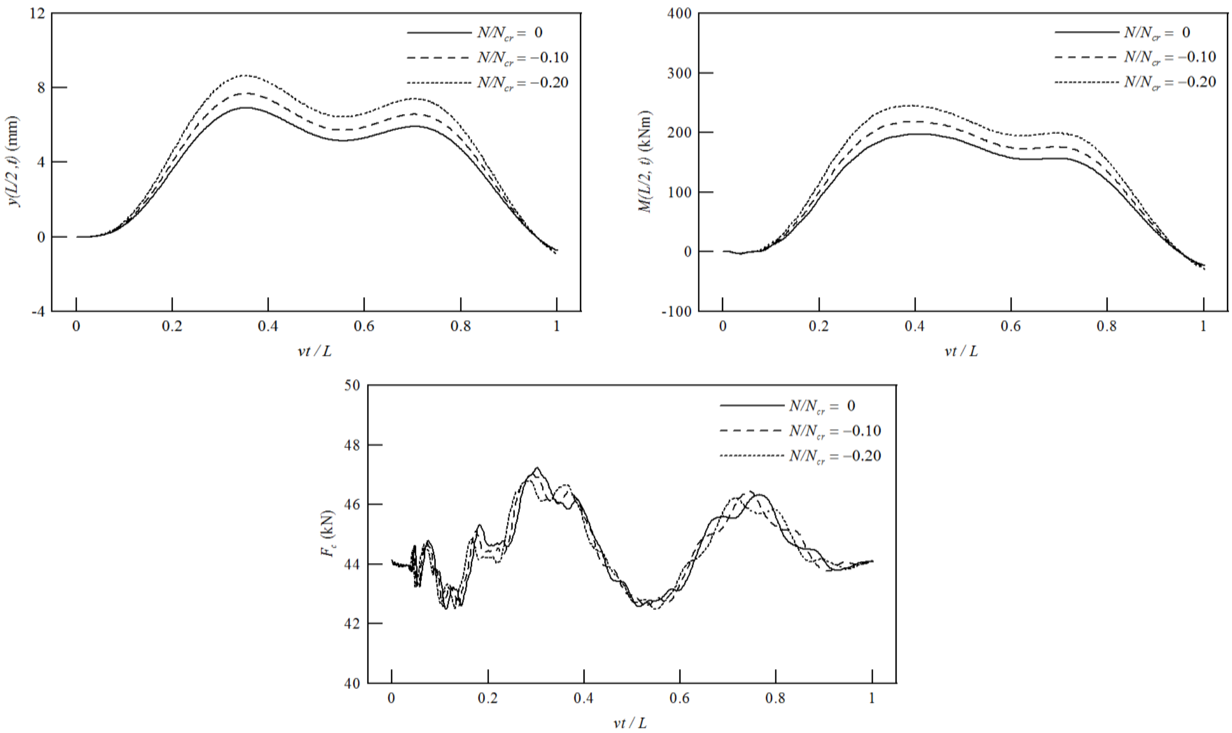

Fig. 10. Deflection, bending moment and contact force distributions under the moving mass for various pre-stressing forces (L/h=20, M/mL=0.15, α=0.20)

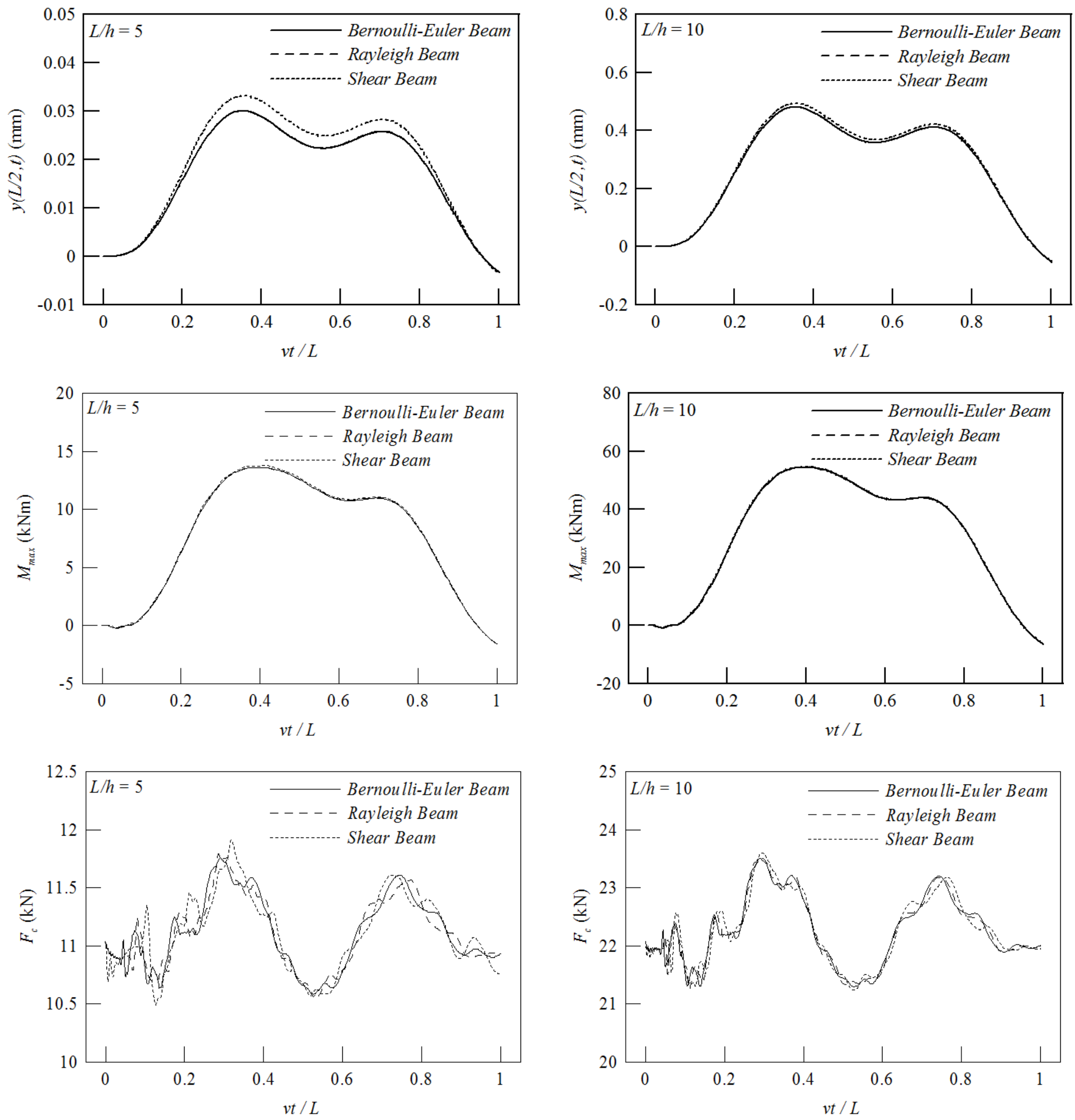

Fig. 11. Deflection, bending moment and contact force distributions under the moving mass for different beam lengths (N/N_{cr}=-0.10, M/mL=0.15, α=0.20)

5. Conclusions

This study presents analytical solution of moving mass problem for pre-stressed elastic beams using different beam models. The assumed mode method is used to derive equations of motion with time-dependent coefficients in matrix form. Numerical solution is, then, performed by using Newmark method to obtain dynamic deflections of the beam. Once deflections are obtained, the bending moment and the contact force between the mass and the beam are easily calculated. Effects of the axial force, rotatory inertia and shear deformation on dynamic behavior of the beam are studied by depending on several parameters such as weight and speed of the moving mass and the beam length.

Results show that axial force is important in calculation of beam deflections and bending moments. Results also show that shear deformation has considerable effect on the response of beams having small slenderness ratio, i.e., short beams, while rotatory inertia has almost no effect. The contact force is not affected significantly by axial force, rotatory inertia and shear deformation of the beam. In addition, results obtained from this study are also in good agreement with those of previous works.

References

Akin JE, Mofid M (1989). Numerical solution for the response of beams with moving mass. *ASCE Journal of Structural Engineering*, 115, 120-131.

Bilello C, Bergman LA (2004). Vibration of damaged beams under a moving mass: theory and experimental validation. *Journal of Sound and Vibration*, 274, 567-582.

Bowe CJ, Mullarkey TP (2008). Unsprung wheel-beam interactions using modal and finite element models. *Advances in Engineering Software*, 39, 911-922.

Chan THT, Yung TH (2000). A theoretical study of force identification using pre-stressed concrete bridges. *Engineering Structures*, 23, 1529-1537.

Esmailzadeh E, Ghorashi M (1995). Vibration analysis of beams traversed by uniform partially distributed moving masses. *Journal of Sound and Vibration*, 184, 9-17.

Esmailzadeh E, Ghorashi M (1997). Vibration analysis of a timoshenko beam subjected to a travelling mass. *Journal of Sound and Vibration*, 199, 615-628

Foda MA, Abduljabbar Z (1998). A dynamic green function formulation for the response of a beam structure to a moving mass. *Journal of Sound and Vibration*, 210, 295-306.

Frýba L (1972). Vibration of Solids and Structures Under Moving Loads. Nordhoff international Publishing, Groningen, The Netherlands.

Jeffcott HH (1929). On the vibration of beams under the action of moving loads. *Philosophical Magazine*, Series 7, 8, 66-67.

Kahya V (2009). Dynamic analysis of pre-stressed elastic beams under moving mass loads. *TUMTMK 16th National Mechanics Conference*, June 22-26, Kayseri, Turkey.

Kiani K, Nikkhoo A, Mehri B (2009). Prediction capabilites of classical and shear deformable beam models cited by a moving mass. *Journal of Sound and Vibration*, 320, 632-648.

Kocatürk T, Şimşek M (2006). Dynamic analysis of eccentrically prestressed viscoelastic timoshenko beams under a moving harmonic load. *Computers and Structures*, 84, 2113-2127.

Kocatürk T, Şimşek M (2006). Vibration of viscoelastic beams subjected to an eccentric compressive force and a concentrated moving harmonic force. *Journal of Sound and Vibration*, 291, 302-322.

Lee HP (1996). Dynamic response of a beam with a moving mass. *Journal of Sound and Vibration*, 191, 289-294.

Lee U (1998). Separation between the flexible structure and the moving mass sliding on it. *Journal of Sound and Vibration*, 209, 867-877.

Lou P, Dai GL, Zeng QY (2006). Finite-element analysis for a Timoshenko beam subjected to a moving mass. Proceedings of Institution of Mechanical Engineers-Part C: *Journal of Mechanical Engineering Science*, 220, 669-677.

Mackertich S (1992). Response of a beam to a moving mass. *Journal of Acoustical Society of America*, 92, 1766-1769.

Michaltsos G, Sophianopoulos D, Kounadis AN (1996). The effect of a moving mass and other parameters on the dynamic response of a simply supported beam. *Journal of Sound and Vibration*, 191, 357-362.

Mofid M, Akin JE (1996). Discrete element response of beams with travelling mass. *Advances in Engineering Software*, 25, 321-331.

Mofid M, Shadnam M (2000). On the response of beams with internal hinges, under moving mass. *Advances in Engineering Software*, 31, 323-328.

Sadiku S, Leipholz HHE (1987). On the dynamics of elastic systems with moving concentrated masses. *Ingenieur-Archiv*, 57, 223-242.

Stanišić MM (1985). On a new theory of the dynamic behavior of the structures carrying moving masses. *Ingenieur-Archiv*, 55, 176-185.

Şimşek M, Kocatürk T (2007). Dynamic analysis of eccentrically prestressed damped beam under moving harmonic force using higher order shear deformation theory. *ASCE Journal of Structural Engineering*, 133, 1733-1741.

Şimşek M, Kocatürk T (2009). Nonlinear dynamic analysis of an eccentrically pre-stressed damped beam under a concentrated moving harmonic load. *Journal of Sound and Vibration*, 320, 235-253.

Ting EC, Genin J, Ginsberg JH (1974). A general algorithm for moving mass problem. *Journal of Sound and Vibration*, 33, 49-58.

Wu JJ, Whittaker AR, Cartmell MP (2001). Dynamic responses of structures to moving bodies combined finite element and analytical methods. *International Journal of Mechanical Sciences*, 43, 2555-2579.

Yavari A, Nouri M, Mofid M (2002). Discrete element analysis of dynamic response of Timoshenko beams under moving mass. *Advances in Engineering Software*, 33, 143-153.