



### Research Article

## Estimation of capacity of eccentrically loaded single angle struts with decision trees

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### ABSTRACT

Single angle struts are used as compression members for many structures including roof trusses and transmission towers. The exact analysis and design of such members is challenging due to various uncertainties such as the end fixity or eccentricity of the applied loads. The design standards provide guidelines that have been found inaccurate towards the conservative side. Artificial Neural Networks (ANN) have been observed to perform better than the design standards, when trained with experimental data and this has been reported literature. However, practical implementation of ANN poses problem as the trained network as well as the knowhow regarding the application should be accessible to practitioners. In another data-driven tool, the Decision Trees (DT), the practical application is easier as decision based rules are generated, which are readily comprehended and implemented by designers. Hence, in this paper, DT was explored for the evaluation of capacity of eccentrically loaded single angle struts and was found to be robust and yielded comparable accuracy as ANN, and better than design code (AISC). This has enormous potential for easy and straightforward implementation by practicing engineers through the logic based decision rules, which would be easily programmable on computer. For this application, use of dimensionless ratios as inputs for the development of DT was found to yield better results when compared to the approach of using the original variables as inputs.

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### 1. Introduction

In construction of steel structures, use of single angle struts is quite common and often their connections lead to eccentricity of loading in both the axes. This had been attributed to factors like eccentricity of applied load, mismatch of the axis of the frame and the angle, and some degree of fixity occurring at the connections (Sakla, 2004). The complexity of behaviour of these struts arising out of the factors mentioned above, make accurate estimation of their load carrying capacity quite challenging. The possible modes of failure might include compression, flexural buckling, and torsional-flexural buckling. The exact analysis as enumerated in design codes (AISC, 2000) becomes difficult due to all these complexities and uncertainties. The design recommendations (Fisher, 2000; Page, 2005) for these members which were based on analytical models and some experimental

results had been reported in literature to be on the conservative side (Sakla, 2004; Liu and Hui, 2008).

There were various experimental investigations reported in literature wherein compressive capacity of single angle struts were determined for different eccentricities and slenderness ratios. Analytical, empirical and numerical investigations had been carried out by researchers for improving the understanding of the behaviour of single angle struts and estimation of their ultimate compressive capacity under eccentricities. Wakabayashi and Nonaka (1965) compared the experimental results from 57 tests with the buckling theories. Woolcock and Kitipornchai (1986) presented a design approach based on experimental results for single angle struts connected with a single leg. Elgaaly et al. (1991) evaluated AISC design recommendations with test results of single angle struts with low slenderness ratios.

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Bathon et al. (1993) examined the test results of ultimate load capacity of single angle struts along with the AISC design guidelines. Adluri and Madugela (1996) presented the results from tests on single angle sections for concentric compressive loads failing in flexural buckling. Rao et al. (2003) explored the design issues of such struts with numerical investigation accounting for the material and geometric nonlinearities and proposed a design approach. Sakla (2004) explored the neural network approach for estimation of the compressive capacity on the basis of published experimental data. Liu and Hui (2008) compared the experimental results with the AISC design and concluded that the AISC gave conservative results, especially for struts subjected to eccentricity about the major principal axis. In a subsequent study, Liu and Hui (2010) confirmed the earlier finding with numerical modelling based on finite element techniques.

In their study, Liu and Chantel (2011) examined the responses of single angle struts for eccentric compressive loads and concluded that the effect of eccentricity on the ultimate load decreased with increasing slenderness ratio. Bashar and Amanat (2014) investigated the ultimate axial capacity of eccentrically loaded angles with numerical technique accounting for the geometric and material nonlinearities. Barszcz (2014) developed analytical formulation for simulating the force-deflection behaviour of single angle struts and validated the model with experimental results.

For the practicing designers, simulating the behaviour of single angle struts and the end conditions in the actual frame or truss was a big challenge. Most of the design codes have guidelines that were empirical in nature, with their own limitations. The numerical approach with intricate finite element model of each strut for evaluation of the stresses and deflections was very demanding on resources to be useful for practical design. Simpler methods of estimation of the ultimate compressive load capacity of single angle struts had been proposed based on artificial neural network (ANN) by Sakla (2004). However, implementation of ANN for practical design poses challenge that is yet to be resolved.

Decision Tree (DT), concurrently known as Model Tree (MT) or Regression Tree (RT), had been utilized in some civil engineering applications including hydrology, geotechnical applications, and concrete technology in the past. Garg et al. (2008) discussed an application of Model Tree (MT or DT) for prediction of currents (in future: output) in tide-dominated areas like gulfs and creeks from past current records (inputs) and concluded that the MT was faster by orders of magnitude compared to ANN and Genetic Programming (GP) models while producing comparable forecasts. Tiraki (2008) employed multivariate statistics, ANN and regression tree (or DT) for predicting uniaxial compressive strength (output) and static modulus of elasticity (output) from other rock properties (inputs) and concluded that the DT were best for development of such predictive models. Kim and Pachepsky (2010) used regression tree (or DT) in conjunction with ANN for reconstructing the missing data (output) in daily precipitation records from adjacent precipitation data (inputs).

Using experimental data from literature, Ayaz et al. (2015) employed DT for predicting the compressive

strength (output) and ultrasonic pulse velocity (output) for HPC from ingredients (inputs) with 97% and 87% success respectively. Behnood et al. (2015) predicted the modulus of elasticity (output) of recycled aggregate concrete from the ingredients (inputs) using DT with 80% accuracy. Gharaei-Manesh et al. (2016) employed ANN and DT for estimating the snow depth (output) from terrain parameters (inputs) in the Sakhvid Basin in Iran and concluded that DT had superiority over ANN for that application. Dauji (2016) utilized the DT for prediction of compressive strength of concrete (output) from its ingredients (inputs) and concluded that performance of DT was superior to that of ANN reported in literature. Furthermore, it was noted that for certain applications, DT had been reported to give faster and superior performance as compared to other data driven tools like ANN.

In this article it is proposed to explore application of another data-driven tool, the DT for this particular application and compare its accuracy with that of the ANN. ANN had been reported to be quite accurate in estimation of compressive capacity of eccentrically loaded single angle struts. If DT model has similar accuracy as ANN, it would hold comparative advantage over ANN in implementation by practising designers, as it gives a set of rules for estimation of the target variable. In this paper, the data-driven tool Decision Tree (DT) was employed for predicting the capacity of eccentrically loaded single angles under compressive loads.

## 2. Data and Methodology

This study was based on the published experimental data obtained from literature (Wakabayashi and Nonaka, 1965; Ishida, 1968; Mueller and Erzurumlu, 1983; Bathon et al., 1993; Adluri and Madugula, 1996; Liu and Hui, 2008). The accumulated data was filtered so as to arrive at a collection of ultimate compressive strength of single angle struts under various eccentricities, with the end conditions being hinged at both ends. The resulting database contained 153 sets of data which were then used for development and evaluation of the DT. The data was taken from the experimental studies reported by: (a) Wakabayashi and Nonaka (1965); (b) Ishida (1968) (c) Mueller and Erzurumlu (1983); (d) Bathon et al. (1993); (e) Adluri and Madugula (1996); (f) Liu and Hui (2008). The test setup and other details of the experiments can be found in aforementioned literature.

The regression tree (RT) alternately known as DT modelling had been described as an exploratory technique based on uncovering structure in data (Clarke, 1991). The popularity of DT is increasing for their simplicity and interpretability, their low computational cost and for the possibility of being graphically represented (Rodriguez-Galiano et al., 2015). In the model space (Fig. 1a), the decision tree process the information from the root node (decision box) to other nodes (decision boxes) or leaves (representing the models or expressions) based on the decision output: 'yes' or 'no' (Fig. 1b). In this way, the model space is progressively subdivided into smaller spaces (Fig. 1a) such that one decision rule prevails in each sub-space. The division of the model space into sub-spaces or domain

splitting is conducted by some algorithm, such as, minimum entropy in sub-domain, or collecting as many samples as possible in the class, or any other. In the popular M5 algorithm of DT, this task is performed by minimizing the standard deviation of the class value reaching a node (Breiman et al., 1984; Quinlan, 1993; Jekabsons G, 2016) and this approach has been adopted in this study.

Accordingly, Fig. 1a shows the domain splitting for two variables ( $x_1$  &  $x_2$ ) and the decision tree structure is depicted in Fig. 1b, where the diamonds represent the decision boxes and the rectangles represent the decision rules. The ability of a root node to maximise the reduction in standard deviation is taken as the attribute for its selection. Many possible subdivisions of inputs are explored during the model development and the one that

results in the maximum reduction in standard deviation is selected to build linear models within each sub-domain. There are many methods reported in literature, which may be employed for avoiding too many domain splits or large discontinuities between neighbouring models (Witten and Frank, 2000; Rokach and Maimon, 2015; Jekabsons, 2016). For a dataset, the model tree enables subdivision of the data hyperspace and definition of linear models in each sub-domain by the adopted objective function (maximum reduction of standard deviation in M5 algorithm) and the linear decision rules together would describe the relationship between the input and output variables. Further details for the development of decision trees may be obtained from standard text books (Witten and Frank, 2000; Rokach and Maimon, 2015).

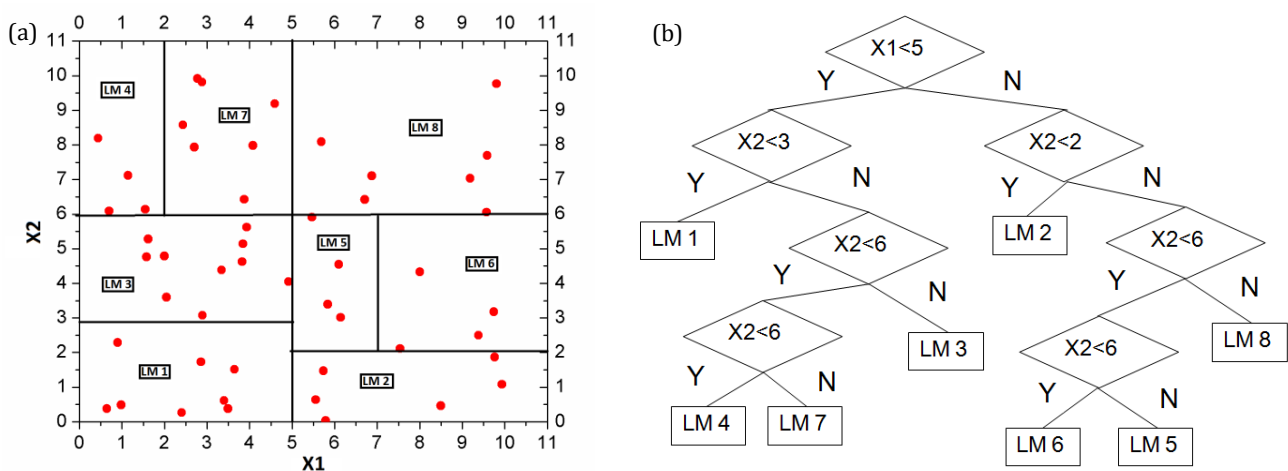


Fig. 1. Decision tree: (a) Domain sub-division; (b) Tree structure.

The DT modelling results in distinct step-wise linear rules to represent the model domain, and hence can be easily implemented by simple programming for new data points within the model domain even without the knowledge of DT development. The modelling by ANN involves establishing the relationship between the input/s (in input layer) and the output/s (in output layer) through the hidden layer/s of neurons, which ensure the desired degree of non-linearity in the relationship. In general, each neuron of a layer is connected with all the neurons in the next layer and the strength of the connection is denoted by weights, which, along with the bias terms of each neuron are ascertained during the process of training process by back propagation of errors. Different algorithms may be applied for the back-propagation of errors such as steepest descent, conjugate gradient, resilient propagation, Levenberg-Marquardt, etc. and for further details regarding development of ANN models, textbooks may be referred (Wasserman, 1993; Bose and Liang, 1993). In case of ANN, the practitioner needs to be acquainted with the concept of modelling in ANN as well as access to the trained ANN & modelling software for implementation of the ANN model for new data points in the model space. When compared with the DT, this is a particular drawback for the ANN models for application by designers. This motivated the present study in which DT is applied for estimation of the capacity of eccentrically

loaded single angle struts, for which successful ANN application has already been reported in literature (Sakla, 2004).

For the development of the DT in this study, two approaches were adopted. In the first, the length of the angle leg ( $b$ ), the thickness of the angle leg ( $t$ ), the slenderness ratio ( $l/r$ ), the yield strength of steel ( $f_y$ ), the two eccentricities ( $e_1$ ,  $e_2$ ) were the input variables (total six) and the ultimate load ( $P$ ) was the target variable. In the second approach, the dimensionless ratios were used as inputs, namely, ratio of the length to the thickness of the angle leg ( $b/t$ ), slenderness ratio ( $l/r$ ), the yield strength of steel ( $f_y$ ), the two relative eccentricities ( $e_1/b$ ,  $e_2/b$ ) were the input variables (total five) and the ratio of the ultimate load to the yield capacity of the angle ( $P/Af_y$ ) was the target variable. The DT were developed with random assignment of 80% of the data for modelling (123 nos.) and remaining 20% of the data for evaluation of the developed models (30 nos.). Multiple such runs were taken to demonstrate the robustness of the approach. The results of three such runs in either approach are presented and discussed in the subsequent sections.

In this paper, the performance of the DT developed was evaluated with measures like Root Mean Square Relative Error (RMSRE), Mean Absolute Relative Error (MARE), and correlation coefficient ( $R$ ). RMSRE is a relative error index which is sensitive to the extreme values.

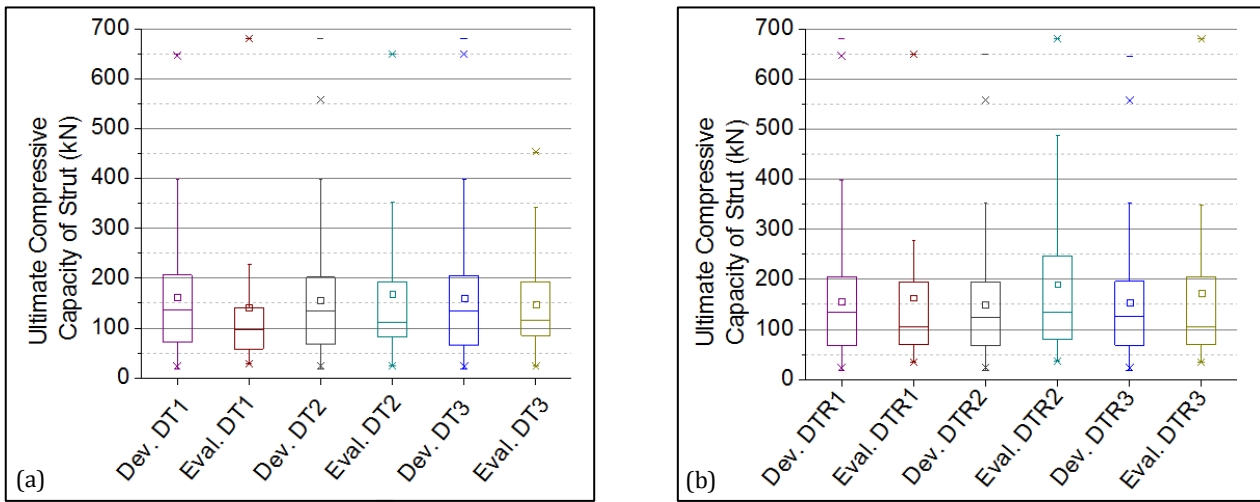
MARE gives an estimate of the relative accuracy of prediction in the absolute scale. The correlation coefficient indicates the degree of linear association between the estimation and the observation and while being sensitive to outliers it is insensitive to proportional or additive differences. The scatter plots of the measured and predicted compressive capacity of struts helped visual evaluation of the accuracy of the models. The ratio of the capacity estimated by the DT and the yield capacity of the angle were evaluated as a measure of the relative accuracy of the DT.

### 3. Results and Discussion

In the DT-s developed with the actual variables are indicated by 'DTX' and the DT-s developed with ratios as inputs and target are indicated by 'DTRX' in the following sections where 'X' indicates the serial number of the run presented.

The box plots shown in Fig. 2 present the comparison of the various percentiles (mean: small box, median: horizontal in box, 25, 75: extents of box, 1, 99: cross mark, and maximum & minimum: horizontal outside box) for the various random assignment of the data in different runs. It can be noticed that the median is always less than the mean indicating that the tail is longer towards the higher values. For DT cases, it is observed that all the evaluation sets (Eval.) have less spread and standard deviation as compared to the development set (Dev.). However, the range of the first evaluation set is more towards the lower values as compared to the development set. This indicates that the model performance of the first DT case might be affected.

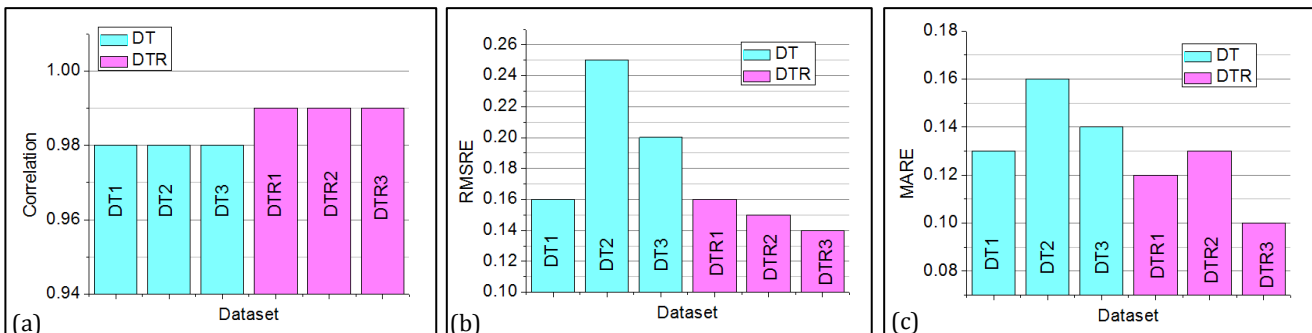
In case of the DTR cases, the spread and the standard deviation are more than, almost equal and less than the respective development sets for the second, third and first cases. This indicates that the model performance of the second DTR case might be affected.



**Fig. 2.** Data characteristics for the development set (Dev.) and the evaluation (Eval.) set for two approaches: (a) Actual variables; (b) Ratios.

The performance metrics, namely, correlation, RMSRE and MARE, for the different runs are presented in Figs. 3(a-c), respectively. For the DT models, the correlation is around 0.98 while for the DTR models, it rises to 0.99. In general, the RMSRE and the MARE are higher in DT models (between 0.16 – 0.25 and 0.13 – 0.16 respectively) as compared to the DTR models (between

0.14 – 0.16 and 0.10 – 0.13 respectively). This clearly indicates that the use of dimensionless ratios in development of the models have been very beneficial in improving the accuracy of the estimation in the evaluation sets. Among the DT models, the first one is the better and the third DTR model is better one. Overall, the third DTR model appears to be the best amongst all.



**Fig. 3.** Performance of the evaluation runs for the two approaches: (a) Correlation; (b) RMRSE; (c) MARE.

The ratio of the estimated ultimate strength and the experimental value has been evaluated for each estimation in the individual runs. The statistics of this relative accuracy is presented as box plots (mean: small box, median: horizontal in box, 25, 75: extents of box, 1, 99: cross mark, and maximum & minimum: horizontal outside box) in Fig. 4. Here it is evident that all the DTR cases are better balanced on either side of unity as compared to the DT cases. The second DTR case has

the highest spread and standard deviation among all cases. This could be due to the higher spread and standard deviation of the evaluation set than the development set.

Visual appreciation of the estimation accuracy is possible in the scatter plots shown in Fig. 5 for the DT cases and Fig. 6 for the DTR cases. As indicated earlier, the first DT model appears better and the third DTR model appears better than the rest.

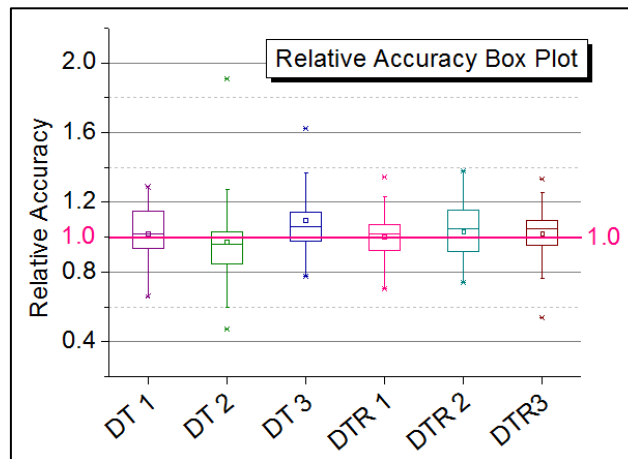


Fig. 4. Data characteristics for the relative accuracy of the different runs in two approaches.

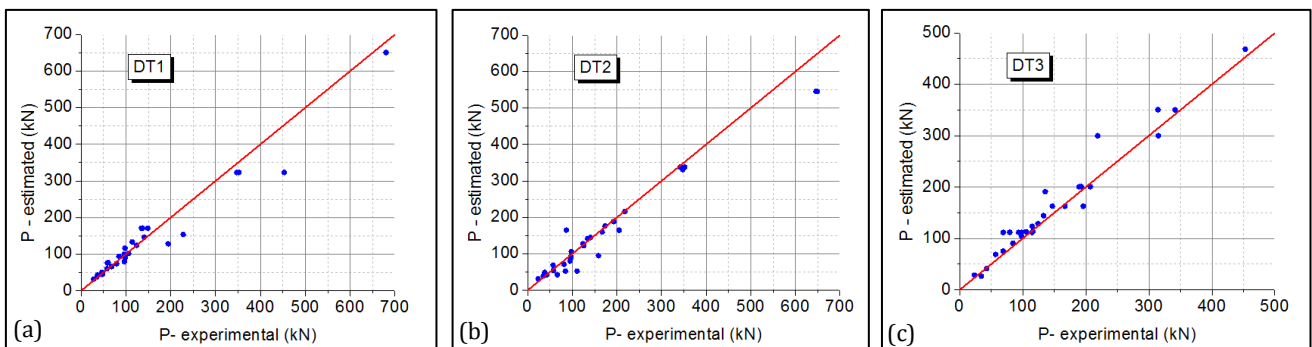


Fig. 5. Scatter plot for the evaluation cases for DT models: (a) Run 1; (b) Run 2; (c) Run 3.

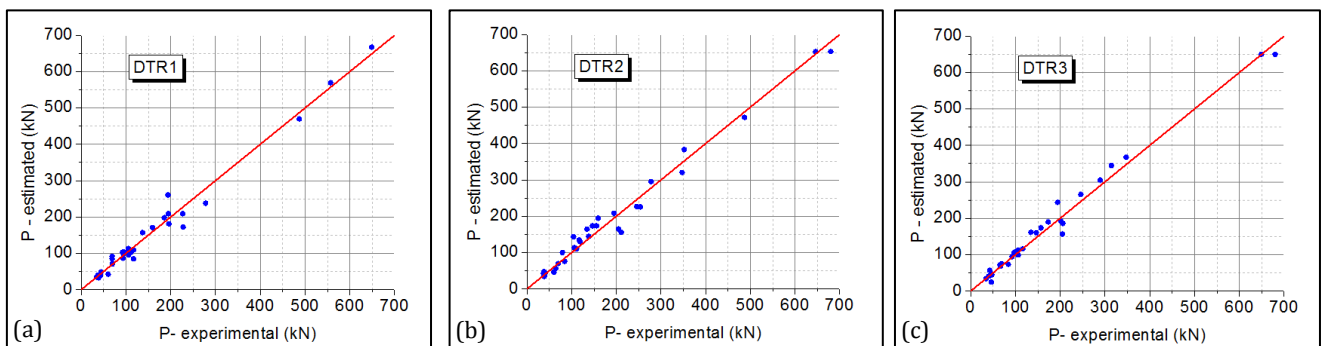
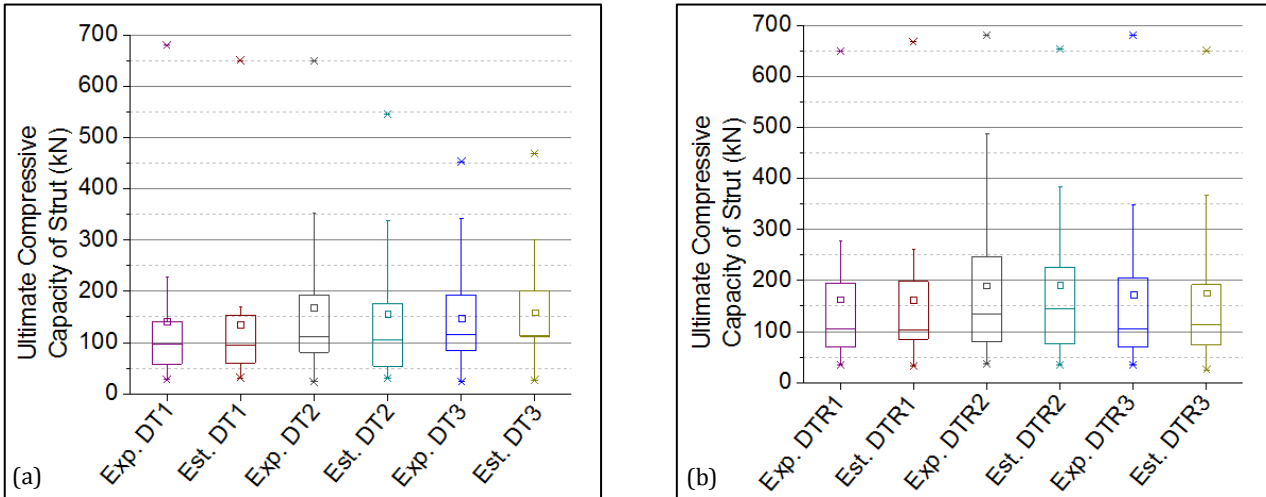


Fig. 6. Scatter plot for the evaluation cases for DTR models: (a) Run 1; (b) Run 2; (c) Run 3.

The data statistics for the experimental and the estimated values of the ultimate compressive capacity in the evaluation sets are shown in the Fig. 7 for both the approaches. Here again it is noted that the best match

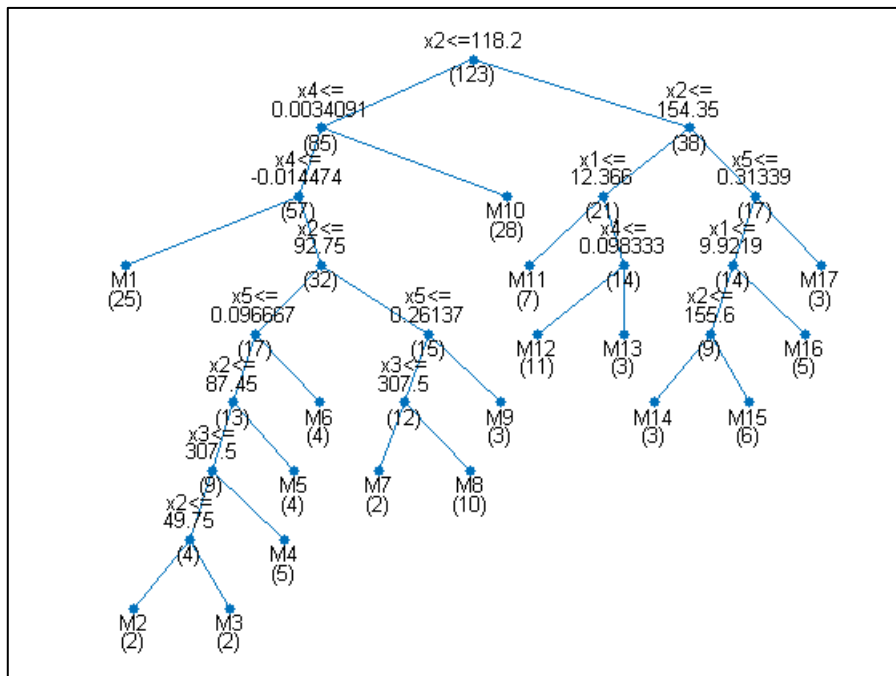
between the experimental and the estimated happened for the first DT case and the third DTR case. Overall, the statistics of the third DTR matches most closely with those of the experimental evaluation set.



**Fig. 7.** Data statistics for the experimental and estimated ultimate compressive capacity in evaluation sets (kN): (a) DT models; (b) DTR models.

From the aforementioned discussion, it is concluded that the third DTR model is the best model among all. The structure of the decision tree for the third DTR case is presented in Fig. 8. The rules for the hierarchical splitting are shown at all the intermediate nodes of the decision tree, the numbers in brackets indicate the number of cases passing through the respective nodes. The number of cases passing to the terminal nodes vary between 2 and 11. The explicit models for 'M1' to 'M17' obtained

from the DTR3 are presented in Appendix. Read together, the Fig. 8 and the appendix yields a clear-cut evaluation scheme for the ultimate compressive capacity of single angle struts. It may be noted in Table 1 from the performance comparison of DT developed in this study and the ANN (from literature: Sakla, 2004) that the accuracy achieved by the DTR3 is comparable to that reported for ANN and better than the AISC formulations reported in literature (Sakla, 2004).



**Fig. 8.** Structure for the best Decision Tree (DTR3).

**Table 1.** Comparison of performance of DT (this study) with ANN (Sakla, 2004) and AISC (Sakla, 2004).

Performance Measure	ANN (Sakla, 2004)	AISC (Sakla, 2004)	DT (This study)
Mean Absolute Percentage Error	5.8	12.8	10
Average Ratio of Predicted and Actual Capacity	1.026	0.885	1.02

#### 4. Conclusions

Single angle struts are used in many structures like the roof trusses and transmission towers. The exact analysis and design of single angle struts become difficult owing to uncertainties in evaluation of the eccentricity of the applied load and the end conditions. In this study, experimental data for ultimate compressive capacity of single angle struts for different eccentricity of loads had been collected from various published articles and using them, attempt had been made to develop an accurate model with DT.

From this study, the following conclusions are drawn:

- The ANN models developed by Sakla (2004) had been demonstrated to be much superior to available AISC formulations. The accuracy of the DT model developed in this study for estimation of the ultimate compressive capacity of single angle struts are comparable to that of ANN models (Sakla, 2004) developed for the same purpose.
- The use of dimensionless ratios in place of actual variables improve the performance of the DT (correlation 0.98, RMSRE 0.16 - 0.25, MARE 0.13 - 0.16 for actual variables; correlation 0.99, RMSRE 0.14 - 0.16, MARE 0.10 - 0.13 for ratio variables).
- For the best DT model developed, correlation of 0.99, RMSRE of 0.14 and MARE of 0.10 could be achieved which indicates very good performance.
- Like other data driven tools, the DT is poor in extrapolation as was seen in the case of DTR2.
- The model DTR3 as presented in Fig. 8 and the appendix may be used by practicing engineers for estimation of the ultimate capacity of single angle struts without any sophisticated analysis like intricate numerical modelling or ANN approach, and thus could be very handy and useful.

Similar performance of the three sets of models in either approach indicates the robustness of the DT approach.

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#### Appendix

The input variables:

- x1: Ratio of the length to the thickness of the angle leg ( $b/t$ ),  
 x2 : Slenderness ratio ( $l/r$ ),  
 x3 : Yield strength of steel ( $f_y$ ),  
 x4, x5 : two relative eccentricities ( $e_1/b$ ,  $e_2/b$ ).

The target variable:

Y : ratio of the ultimate load to the yield capacity of the angle ( $P/Af_y$ ).

Number of rules: 17

Number of original input variables used: 5 (x1, x2, x3, x4, x5)

#### The Rules:

```

if x2 <= 118.2
if x4 <= 0.0034091
if x4 <= -0.014474
y = 0.66957 -0.0031293*x2 +0.38258*x4 (25)
else
if x2 <= 92.75
if x5 <= 0.096667
if x2 <= 87.45
if x3 <= 307.5
if x2 <= 49.75
y = 0.91844 (2)
else
y = 0.89938 (2)
else
y = 0.81902 (5)
else
y = 0.70902 (4)
else
y = 0.60708 -0.0015359*x2 (4)
else
if x5 <= 0.26137
if x3 <= 307.5
y = 0.64014 (2)
else
y = 0.47126 (10)
else
y = 0.15279 (3)
else
y = 0.92135 -0.031689*x1 -0.0016546*x2 -0.4013*x4 (28)
else
if x2 <= 154.35
if x1 <= 12.366
y = 0.59979 -0.0021783*x2 -0.078047*x4 (7)
else
if x4 <= 0.098333
y = 0.31092 +0.64838*x4 (11)
else
y = 0.19385 (3)
else
if x5 <= 0.31339
if x1 <= 9.9219
if x2 <= 155.6
y = 0.16353 (3)
else
y = 0.18237 (6)
else
y = 0.37906 -0.00068514*x3 (5)
else
y = 0.10157 (3)

```

#### The Models (refer Fig. 8):

M1 = 0.66957 -0.0031293\*x2 +0.38258\*x4  
 M2 = 0.91844  
 M3 = 0.89938  
 M4 = 0.81902  
 M5 = 0.70902  
 M6 = 0.60708 -0.0015359\*x2  
 M7 = 0.64014  
 M8 = 0.47126  
 M9 = 0.15279  
 M10 = 0.92135 -0.031689\*x1 -0.0016546\*x2 -0.4013\*x4

$M11 = 0.59979 - 0.0021783 * x_2 - 0.078047 * x_4$   
 $M12 = 0.31092 + 0.64838 * x_4$   
 $M13 = 0.19385$   
 $M14 = 0.16353$   
 $M15 = 0.18237$   
 $M16 = 0.37906 - 0.00068514 * x_3$   
 $M17 = 0.10157$

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