Challenge Journal of Structural Mechanics

Research Article

Parametric investigation for discrete optimal design of a cantilever retaining wall

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ABSTRACT

In this paper, discrete design optimization of a cantilever retaining wall has been submitted associated with a detailed parametric study of the wall. In optimal design, the minimum wall weight is treated as the objective function. Through design algorithm, the optimal design variables (base width, toe width, thickness of base slab and angle of front face) yielded minimum structural weight of the wall and satisfied stability conditions have been determined for different soil parameter values. At the end, a detail parametric study searching the effect of change of soil parameters on the retaining wall design has been conducted with 120 optimized wall designs for different values; eight values of the angle of internal friction, three values of the unit volume weight and five values of wall heights. The obtained results from optimization analyses indicate that change of the angle of internal friction more effective than change of the unit volume weight on the optimal wall weight. Economic wall design with optimization analysis is achieved in a shorter time than the traditional method.

ARTICLE INFO

Article history:
Received 16 July 2019
Revised 27 August 2019
Accepted 6 September 2019

Keywords:
Cantilever retaining wall
Structural optimization
Metaheuristics optimization method
Discrete design optimization

1. Introduction

In geotechnical engineering, the retaining walls are employed to resist lateral soil load in case of constructing works like an excavation, slopes, railway or highway as lateral support. In conventional design of a retaining wall, stability conditions like sliding and overturning are checked by using selected wall dimensions, firstly. If selected wall dimensions do not ensure stability conditions, this trial and error process is continued, till satisfying stability conditions. Even though safe wall dimensions have been obtained in plenty of time, it is not certain that obtained wall design is the most economic among all possible solutions. On the other hand, conditions of worksite like ground water level, soil height to be supported laterally or intended use of structure and soil properties such as bearing capacity or behavior of settlement under loads of soil should be considered in case of design. Existing of all mentioned situations in wall design with reasonable cost make this design a challenging engineering problem with many unknowns. Optimization methods have been commonly employed to obtain optimal solution of these kind of complex engineering problems by Rhomberg and Street (1981) and Keskar and Adidam (1989).

In real world problems, the existence of some cases like the sophisticated characteristics of problems with many unknowns, an infinite solution space, or the numerous iterations have given metaheuristic optimization methods prominence. The metaheuristic optimization algorithms, which are quite popular in recent years, have been used effectively in solving such problems over the last two decades. Popularity of metaheuristics that mimics the natural phenomenon is based on being simple, compatible and effective. While, preliminary information is required to solve the problem normally, such advantages eliminate this necessity even in the case of a broad array of optimization problems. Metaheuristics have been commonly utilized for solving engineering problems with multiple variables in case of deterministic

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ISSN: 2149-8024 / DOI: https://doi.org/10.20528/cjsmec.2019.03.004

and conventional methods are insufficient to obtain solutions. By using these algorithms, such as the genetic algorithms (GA) by Chau and Albermani (2003), the simulated annealing algorithm (SA) by Ceranic et al. (2003), the particle swarm optimization (PSO) by Khajehzadeh et al. (2010), the big bang-big crunch algorithm (BB-BC) by Camp and Akın (2012), the firefly algorithm (FA) by Sheikholeslami et al. (2014), the charged system search algorithm (CSS) by Talatahari and Sheikholeslami (2014), the gravitational search algorithm (GSA) by Khajehzadeh and Eslami (2012) and the teaching learning-based optimization (TLBO) by Temür and Bekdaş (2016), have all been investigated in the optimal design of a retaining wall. For this purpose, the particle swarm optimization (PSO), firefly algorithm (FA) and cuckoo search (CS) algorithm, known as swarm intelligence, have been employed to compare the results of studies by Gandomi et al. (2015). Parameters as the total wall weight, the angle of internal friction and the unit volume weight play an important role in retaining wall design which must satisfy stability conditions and must be economical. Effect of those parameters on the optimal design of a wall has been investigated as a parametric study and results have been presented by Yepes et al. (2008) and Molina-Moreno et al. (2017).

In this study, harmony search algorithm (HSA), which is a relatively contemporary metaheuristic optimization method, has been employed with the aim of investigating the safe and economic wall design. The algorithm firstly proposed by Geem et al. (2001) is based on harmony of sounds coming from each musical instrument in an orchestra. The HSA is more favorable than other metaheuristic algorithms and makes it possible to get results in less time without trapping local optimals. In addition, it can be used both continues and discrete variables and is easy to use in optimization process. It is proved that the HSA is a steady and fertile technique, which is impressively performed to derive solutions for a broad array of real-sized optimum design problems (Lee et al., 2005). In the literature, prosperous studies have been conducted like in structural optimization by Saka and Çarbaş (2009) and Bekdaş and Niğdeli (2016), in hydraulics by Ayvaz and Elçi (2013), in vehicle routing by Geem et al. (2005) and in geotechnical engineering by Cheng et al. (2011). Besides, improved, modified and hybrid version of HS algorithm which increase robustness and convergence of algorithm has been proposed with better optimum results of benchmark problem. New versions of HSA have been employed for optimum design of foundation presented by Khajehzadeh et al. (2011) and assignation of critical surface of slope presented by Cheng (2009) and Fattahi (2015). HSA is utilized successfully for optimization of reinforced cantilever retaining walls in realized studies by Akın and Saka (2010) and Uray et al. (2015).

The weight of the cantilever retaining wall has been optimized by using the HSA in this paper. In the discrete optimal design procedure, the wall dimensions selected from pre-dimension sets must satisfy the stability conditions and must be cost efficient. For this reason, numerous iterations and computational time are required to obtain the optimal dimensions. In the optimization problem, the dimensions of the retaining wall are taken as discrete design variables along with the effect of the soil properties of the backfill. To obtain the safe design which satisfy stability conditions, safety factors of sliding and overturning have been taken into consideration as design constraints with geometric restrictions. Finally, the optimal dimensions of the wall that are given minimum wall weight and satisfy the constraints are obtained. Furthermore, a detail parametric study has been performed for different design parameters to investigate effect of parameter change on optimal wall design. In the design optimization process of a cantilever retaining wall, only the weight of concrete has been taken into consideration.

2. Materials and Method

2.1. Formulation of the optimization problem

The cantilever retaining wall model is acquired with reference to the provisions of Building Code Requirements for Structural Concrete (ACI 318-08, 2008) and LRFD Bridge Design Specifications (AASHTO, 2010) so that it satisfies the structural stability. The general mathematical formulation of optimal retaining wall design is given below by Eq. (1).

Minimum value of objective function: $f_{min}(x) = W_{wall}$

Constraints to be employed:

$$g_i(x) = g_1(x),\ g_2(x),\ g_3(x),\ g_4(x) \le 0 \quad (x_l \le x_i \le x_u) \tag{1}$$

where x_l and x_u present lower and upper borders of design variables, whose number is equal to i.

In the optimal design problem, the base width (X_1), the toe width (X_2), thickness of base slab (X_3) and the angle of front face (X_4) are treated as discrete design variables tabulated in Fig. 1, and also the acting loads are shown in the same figure. Due to the design variables correspond to dimensions of the cantilever retaining wall, these variables and their intervals has been selected as discrete to achieve integer wall dimensions.

The lower and upper borders of the variables of optimal design, designated in Table 1, are determined by taking into consideration of the design provisions. In optimum design of the wall, design variables have been determined interrelated each other to obtain reasonable dimensions except the angle of front face (X_4). For instance, the base width (X_1) depend on the wall height (H), which changes range between 0.30H and 1.0H, similar to other design variables.

The bottom thickness of the stem, b_b, is given by Eq. (2).

$$b_b = (H - X_3)\, X_4 + b \tag{2}$$

As the optimal design weight is more significant than the optimal design cost (Temur and Bekdaş, 2016), the goal function has retrieved as the total weight of the cantilever retaining wall. The weight W_1, W_2 and W_3, of Eq. (3) are explained with regard to design variables as shown in Eqs. (4) to (6). The soil weight of backfill (W_4), the active soil pressure (P_a), the passive soil pressure (P_p), the active soil pressure coefficient (K_a) and the passive soil pressure coefficient (K_p) are given in Eqs. (7) to (11) respectively.

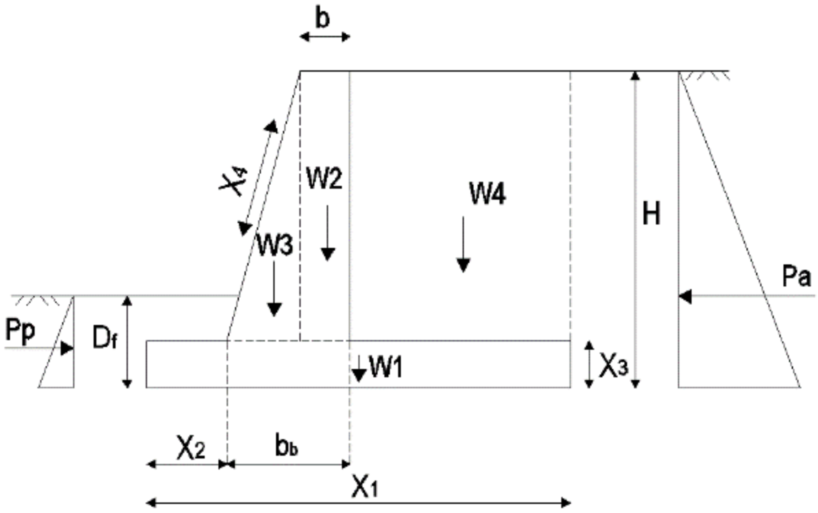

Fig. 1. Cantilever retaining wall and design variables.

Table 1. Design variables and limit bounds.

Design variables	Lower bound	Upper bound	Interval
X_1: Base width	$0.30H$	$1.0H$	$0.02H$
X_2: Toe width	$0.15X_1$	$0.55X_1$	$0.02X_1$
X_3: Thickness of base slab	$0.06H$	$0.16H$	$0.005H$
X_4: Angle of front face	%2	%7	%0.5

$$W_{wall} = W_1 + W_2 + W_3 \tag{3}$$

$$W_1 = X_1 X_3 \gamma_c \tag{4}$$

$$W_2 = b\,(H - X_3)\,\gamma_c \tag{5}$$

$$W_3 = (b_b - b)(H - X_3)\,0.5\,\gamma_c \tag{6}$$

$$W_4 = (X_1 - X_2 - b_b)(H - X_3)\,\gamma_s \tag{7}$$

$$P_a = 0.5H^2 \gamma_s K_a \tag{8}$$

$$P_p = 0.5{D_f}^2 \gamma_s K_p \tag{9}$$

$$K_a = tan^2(45 - \emptyset/2) \tag{10}$$

$$K_p = tan^2(45 + \emptyset/2) \tag{11}$$

The design constraints in the formulation of design optimization problem are so-called safety factors of sliding and overturning and the geometric constraints of the wall. The factor of safety against sliding and overturning is taken as 1.3 and the constraints of normalized mathematical expressions are given by the Eqs. (12) and (13), respectively.

$$g_1(x) = 1.3 - \frac{F_{resistant}}{F_{sliding}} \leq 0 \tag{12}$$

$$g_2(x) = 1.3 - \frac{M_{resistant}}{M_{overturning}} \leq 0 \tag{13}$$

Here, $F_{resistant}$, in Eq. (14), is a resistant force against to sliding of the wall and $F_{sliding}$, in Eq. (15), is a shift force, which causes sliding the wall. Similarly, $M_{resistant}$, in Eq. (16), is a resistant moment against to overturning of the wall and $M_{overturning}$, in Eq. (17), is an overturning moment which causes overturning the wall. To satisfy safety factor of sliding and overturning stability of the wall, $g_1(x)$ and $g_2(x)$ must be equal or greater than 1.3. In addition, the normalized mathematical expressions of the geometric constraints of the wall, $g_3(x)$ and $g_4(x)$, are given by the Eqs. (18) and (19).

$$F_{resistant} = (W_1 + W_2 + W_3 + W_4)\,tan\,\delta + P_P \tag{14}$$

$$F_{sliding} = P_a \tag{15}$$

$$M_{resistant} = W_1\left(\frac{X_1}{2}\right) + W_2\left(b_b - \frac{b}{2} + X_2\right) + W_3\left(\frac{2}{3}(b - b_b) + X_2\right) + W_4\left(\frac{X_1 + X_2 + b_b}{2}\right) + P_p\frac{D_f}{3} \tag{16}$$

$$M_{overturning} = P_a\left(\frac{H}{3}\right) \tag{17}$$

$$g_3(x) = \frac{b}{b_b} - 1 \leq 0 \tag{18}$$

$$g_4(x) = \frac{X_2 + b_b}{X_1} - 1 \leq 0 \tag{19}$$

In Table 2, design parameters used in the optimization analyses are given. In design of the cantilever retaining wall, the wall height has an important role for calculation of the acting loads to the wall depends on directly wall height. Other important parameters are the soil properties of backfill, the unit volume weight and the angle of internal friction. Coefficient of friction (δ) between wall and soil is taken as equal to the angle of internal friction during optimization process.

Table 2. Design parameters.

Parameter	Value
H: Wall height (m)	4-5-6-7-8
γ_s: Unit volume weight (kN/m^3)	16-18-20
Ø: Angle of internal friction (°)	20-22-24-26-30-35-40-45

2.2. Harmony search algorithm

Heuristic methods are algorithms that inspired from solutions produced by nature for difficult problems. HSA (Geem, 2001) which is a relatively novel metaheuristic optimization algorithm is related to find the best tonality in music process similar to producing the best solutions for complex optimization problems. In the optimal design of a cantilever retaining wall, the design variable values are decided by utilizing a design pool comprising the discrete variables as depicted in Table 1. The HSA has been employed in order to solve the design problem with inequality constraints containing those discrete design variable values and to reach the optimal objective function productively.

Steps of harmony search algorithm whose flowchart is demonstrated in Fig. 2 are as follows:

Step 1: Determination of algorithm parameters (*HMS*, *HMCR* and *PAR*), maximum iteration number and pool of design parameters;

Step 2: Initialization of harmony memory matrix (*HM*);

Step 3: Improvisation of new harmony based on three rules (NCHV);

Step 4: Updating of harmony memory matrix;

Step 5: Checking of termination criterion.

Algorithm parameters as harmony memory size (*HMS*), the harmony memory considering rate (*HMCR*), the pitch adjusting rate (*PAR*) and the maximum number of iterations are selected and design pool is formed by using design variables of optimization problem. Harmony memory matrix (*HM*) which is given by Eq. (20) is formed randomly by using design variables from design pool. In this equation, while the row number of *HM* corresponds to *HMS*, number of design variables (N) are equal to the columns number of matrix. For values of x_{ij} given by Eq. (20), values of i and j are respectively from 1 to *HMS* and from 1 to N. For current value of jth (j=1,…,N) are selected randomly design pool in ith possible solution and this process is repeated for each rows (i=1,…..,*HMS*). And then values of objective function are calculated for each row of *HM* which is a potential solution and sorted ascending or descending order according to aim of minimization or maximization objective function.

$$HM = \begin{bmatrix} x_{1,1} & x_{2,1} & \cdots & \cdots & x_{N-1,1} & x_{N,1} \\ x_{1,2} & x_{2,2} & \cdots & \cdots & x_{N-1,2} & x_{N,2} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{1,HMS-1} & x_{2,HMS-1} & \cdots & \cdots & x_{N-1,HMS-1} & x_{N,HMS-1} \\ x_{1,HMS} & x_{2,HMS-1} & \cdots & \cdots & x_{N-1,HMS} & x_{x_{N,HMS}} \end{bmatrix} \tag{20}$$

In improvisation of a new harmony memory matrix, it is checked whether or not there is better solution. For this process, possibilities of *HMCR*, *PAR* and (1-*HMCR*) are taken into consideration. If randomly selected number between 0 and 1 is smaller than *HMCR*, the index number of current design variable is changed with possibility of *HMCR* selected value from *HM*. If it is not, the index number is selected randomly with the possibility of (1-*HMCR*) from design pool. Revision of pitch adjusting rate (*PAR*) is just applied for changed values with *HMCR* possibility. This improvisation is repeated for all design variables (j=1,…,N) and then new solution harmony is obtained.

After improvisation of a new harmony, it is checked whether it should be good solution or not according to value of calculated objective function. If the new value of objective function is better than the worst value of objective function, new solution is saved in *HM* and the worst solution is deleted from *HM*. This process is continued until current iteration number reaches to maximum iteration number.

3. Analysis and Results

In optimization analyses, optimization algorithm is coded by using MATLAB software and 120 retaining wall designs in cohesionless soil have been carried out for different values of the wall height (H=4, 5, 6, 7, 8 m), the unit volume weight (γ_s=16, 18, 20 kN/m^3), the angle of internal friction (Ø=20, 22, 24, 26, 30, 35, 40, 45°). Firstly, value of H, γ_s, Ø and *HMS*, *HMCR*, *PAR* and maximum iteration number have been identified for each design. The discrete design variables (X_1, X_2, X_3 and X_4) given in Fig. 1 have been employed and design pool is formed by considering their upper and lower borders given Table 1. In the solution of the optimization problem, a new solution is obtained by using values of the discrete design variables selected from the design pool randomly. According to a new solution, which satisfies the constraints given by the Eqs. (12), (13), (18) and (19), the minimum goal function given by Eq. (3) value and the optimal wall dimensions have been calculated. The top thickness of the stem is taken as b=0.25 m and the depth of the foundation is taken as D_f=1.5 m for all optimization analyses.

In this study, the parameters of the HSA are chosen as *HMS*=20, *HMCR*=0.95 and *PAR*=0.15. The values of those parameters are allocated at the beginning and they stay unchanged during the optimization process. The most suitable range of parameter values are included in the studies of Lee et al. (2005). According to separate optimization problems from various fields, it is asserted that the values of the parameters are related with search space dimension. That's why the impact of the selected parameters is examined in each design example in current study. Thus, the proposed HSA is executed several times for each design problem by taking into account the various set of parameters. Afterward, carrying out the sufficient amount of run for sensitivity of the predetermined parameters, abovementioned HSA parameters are decided to utilize for having the least wall weight. To ensure the optimal values, which are obtained with the algorithm, the numerous iterations have been performed and the optimal values have been found with 5,000 iterations. In process of the optimal design, it has been observed that the optimal result remains the same after 5,000 iterations.

In the sequent sections, the optimization analysis results are given in two parts as design examples and a parametric study.

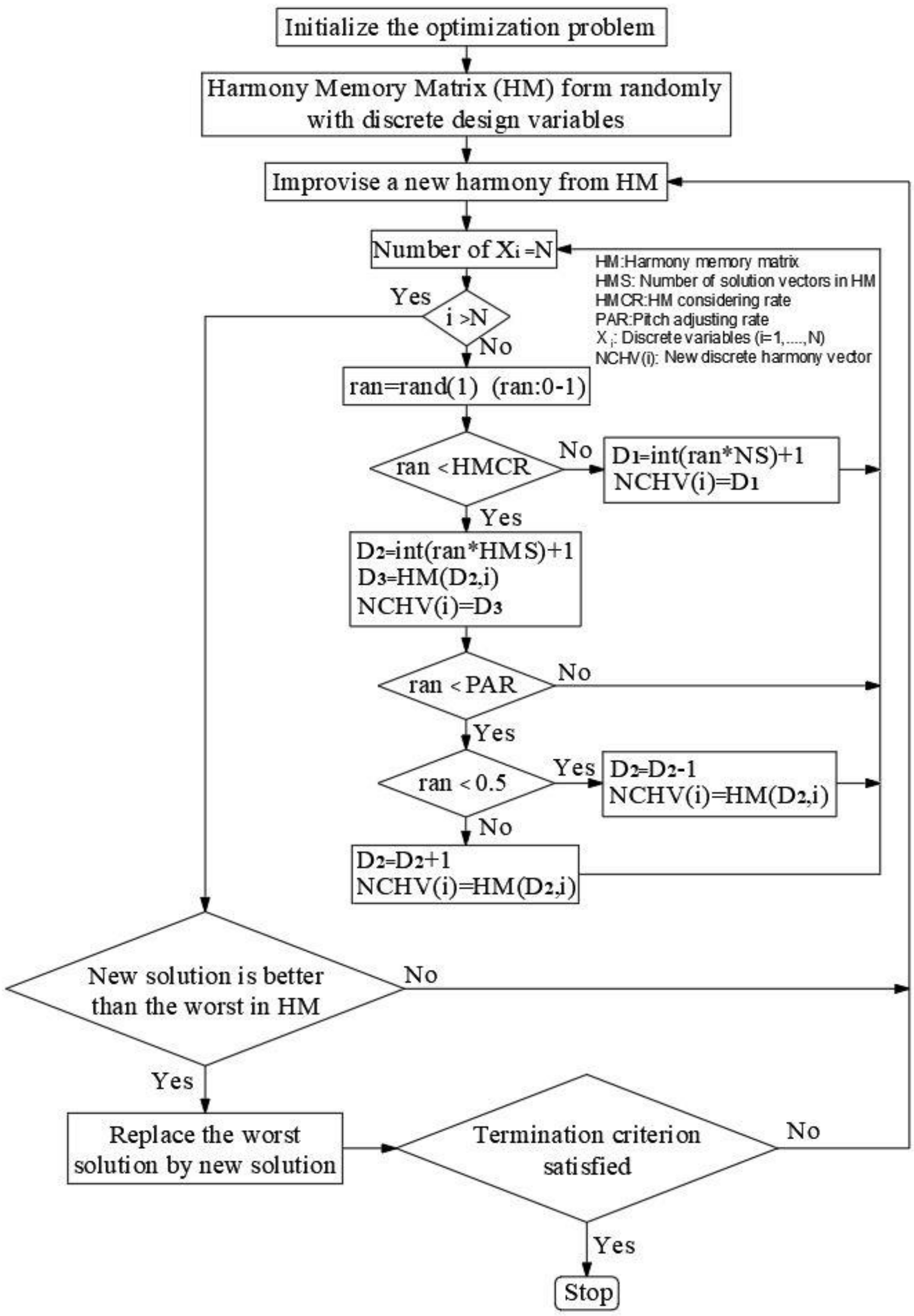

Fig. 2. Flowchart of harmony search algorithm for optimum design of a cantilever retaining wall.

3.1. Design Examples

The HSA has been used in finding the optimal weight of the cantilever retaining wall, which is given in Fig. 1, and five design examples have been represented proving that proposed the algorithm is efficient and productive. In the optimization analysis, after the wall weight has reached minimum value, the optimal wall weight has remained constant, even if values of the angle of internal friction and the unit volume weight have changed. This design, in which the optimal wall weight does not change with continued analyses, has been presented as design examples for each wall height.

In Table 3, input values, which have been used in the analysis, and the optimal wall dimensions, which have been obtained as results of analyses, are tabulated. Furthermore, revised lower and upper borders of design variables are given in this table. While each value of the wall height is given, some angle of internal friction value and unit volume weight are given as input parameters. This is due to the fact that the change values of unit volume weights during the optimal design do not affect significantly the optimal weight values, thus, γ_s= 18 kN/m^3 is chosen as the mean value.

In the design examples, after the optimal wall weight is obtained, the angle of internal friction value is taken which does not show any change in the optimal wall weight with continuing analyses. When Table 3 is examined, it is obviously seen that the optimal values of the retaining wall satisfy the lower and upper bounds determined with reference to the provision of American Concrete Institute Building Code Requirements for Structural (ACI 318-08, 2008) and LRFD Bridge Design Specifications (AASHTO, 2010).

Table 3. Input and optimum values for design examples.

Example No		1	2	3	4	5
Input Values	H (m)	4	5	6	7	8
	Ø (°)	30	35	40	40	40
	γ_s (kN/m^3)	18	18	18	18	18
Optimum Values	X_1 (m)	1.20	1.50	1.80	2.10	2.40
	X_2 (m)	0.204	0.225	0.306	0.315	0.36
	X_3 (m)	0.24	0.30	0.36	0.42	0.48
	X_4 (%)	2	2	2	2	2
Lower-Upper Bounds	X_1 (m)	1.2-4	1.5-5	1.8-6	2.1-7	2.4-8
	X_2 (m)	0.18-2.20	0.225-2.75	0.27-3.30	0.315-3.85	0.36-4.40
	X_3 (m)	0.24-0.64	0.30-0.80	0.36-0.96	0.42-1.12	0.48-1.28
	X_4 (%)	2-7	2-7	2-7	2-7	2-7

In Table 4, the values of the optimal weight and the sliding and overturning safety factors correspond to those optimal weights are evinced. For each optimal analysis, the lower bounds of both the safety factors of sliding and overturning have been accepted as 1.3 to ensure the stability conditions of the wall. It has been seen that the sliding safety factor is greater than the overturning safety factor for design examples. This can be due to the coefficient of friction has taken as equal to the angle of internal friction in the analyses.

Table 4. The optimum wall weight and the safety factors of sliding-overturning.

Example No	W_{wall} (kN)	$F_{s\,(sliding)}$	$F_{s\,(overturning)}$
1	34.2344	2.22	1.32
2	46.1475	2.66	1.40
3	59.4024	3.40	1.60
4	73.9991	3.07	1.47
5	89.9376	2.83	1.40

The changes between the wall weights (W_{wall}) and iteration numbers (iteration) are stated in Fig. 3 In the harmony search optimization, the wall weight decreases with the increase in the iteration numbers. It is clear that the optimal wall weight has been reached with approximately 150 iterations. Although 5000 iterations have carried out for each design, the optimal wall weight and design variables have been obtained with 150 iterations and remained unchanged anymore.

3.2. A Parametric Study

In the design optimization of cantilever retaining wall, the wall height, the unit volume weight of backfill and the angle of internal friction are crucial parameters affecting the optimal design. In this section, a detail parametric study carried out by using variable values of those parameters has been given in Table 2. While there is no effect of the unit volume weight change is sight, the influence of the wall height and the angle of internal friction change on optimal wall weight, on optimal design variables values, and on sliding and overturning safety factors were observed. As mentioned before since the change of the unit volume weight during the optimal design do not affect the optimal wall design weight, significantly, it is selected as 18 kN/m^3 as a mean value among 16 kN/m^3 to 20 kN/m^3.

In Fig. 4, as the wall height increases, the minimum wall weight increases rapidly. Also, there is an inverse proportion between the wall height and the internal friction angle for same wall height.

In Fig. 5, the relation between the base width (X_1) and the wall height (H) is defined by the coefficient of "α_B". The change between α_B and the wall height (H) is given for various angle of internal friction in the figure. From this figure, it is also clear that the slope of curve decreases with the raise of the angle of internal friction and that slope of curve is zero for Ø=40-45°. In Fig. 5, economic retaining wall design has been obtained for Ø=30-45°. While the base width may be taken about between 40% to 60% of the overall wall height in American Concrete Institute Building Code Requirements for Structural (ACI 318-08, 2008), this value may be chosen between 70% and 75% of the stem height according to LRFD Bridge Design Specifications (AASHTO, 2010) without paying attention for change of angle of internal friction. In return for this, the optimal wall design has been obtained for α_B=0.30 with parametric study for Ø>30°.

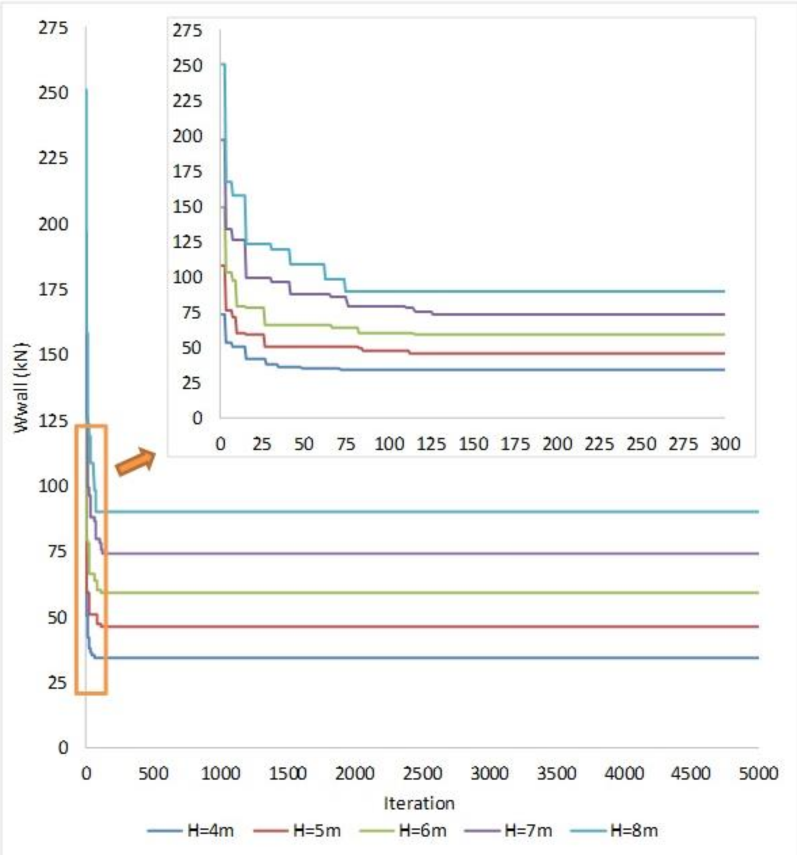

Fig. 3. Design histories of design examples.

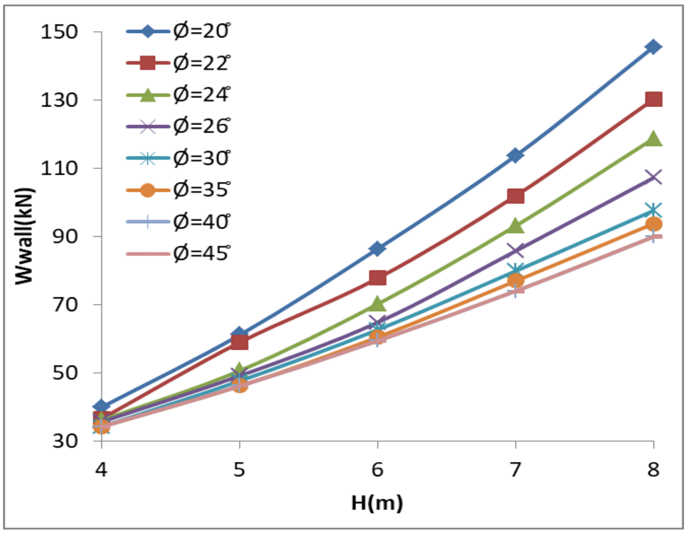

Fig. 4. The changes between the wall height and the minimum wall weight.

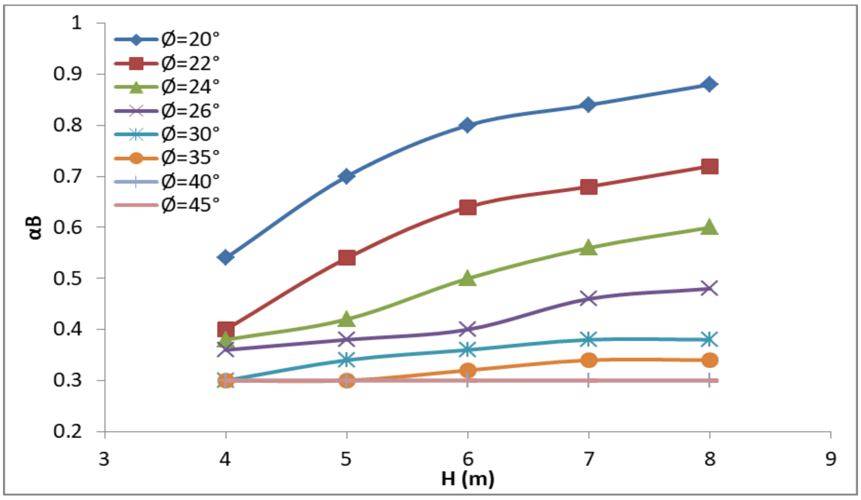

Fig. 5. The change between α_B and the wall heights.

The change of the toe width (X_2) according to the base width (X_1) is defined by the coefficient of "α_T". The values of α_T are given in Table 5 for the different values of the angle of internal friction and the wall heights. In Table 5, a non-proportional change between the base width (X_1) and the toe width (X_2) is evident. This non-proportional change is due to the randomization characteristics of the algorithm and the random search solution. For the toe width, 30% of length of base has been suggested according to above mentioned provisions and obtained α_T values less than this value. In analyses, it has seen that the coefficient of the thickness of base slab "α_D" has same ratio as 0.06 for all values of the wall heights and the angle of internal friction. Value of the angle of front face has been obtained as %2 (minimum value) like suggested in American Concrete Institute Building Code Requirements for Structural (ACI 318-08, 2008) and LRFD Bridge Design Specifications (AASHTO, 2010) in all analyses.

Table 5. The values of α_T.

Ø (°)	H=4m	H=5m	H=6m	H=7m	H=8m
20	0.15	0.15	0.15	0.15	0.15
22	0.15-0.21	0.15	0.15	0.15-0.17	0.15
24	0.15	0.15-0.17	0.15	0.15-0.17	0.15
26	0.15-0.21	0.17	0.15	0.15-0.17	0.15-0.17
30	0.15-0.17	0.19	0.15-0.21	0.19	0.15
35	0.25	0.15-0.23	0.17-0.21	0.17	0.15-0.17
40	0.25	0.23	0.17	0.15-0.17	0.15
45	0.25	0.23	0.17	0.17	0.17

In Table 6, for each wall height from 4m to 8m, the acquired the best and the worst design weights are designated for different values of the unit volume weight and the angle of internal friction. It is worthy to mention from this table that the worst design weights obtained for each wall height and each unit volume weight is reached at same angle of internal friction, which is Ø=20°. But, the best design weights for wall heights of 4m and 5m under all unit volume weights is achieved at Ø=30° and 35° the angle of internal friction values, respectively.

The most striking result can be deducted from this table is that the best design weights, for wall heights of 6m, 7m, and 8m under all unit volume weights, come into the view at the same angle of internal friction, which is 40°. This proves that the change of the angle of internal friction is more effective than change of the unit volume weight on the optimal wall weight. And also, it can be educed from this table that the higher the wall height, the more wall weight needed to attain the optimal design weight under each unit volume weight.

In the weight optimization of a cantilever retaining wall, the most desirable design is that the sliding and overturning constraints are close to limit safety factor and each other. So, if the safety factors of a design being close to limit value (1.3), the non-economical wall design is avoided.

In Table 7, among all optimal designs, those are presented whose sliding and overturning safety factors constraints are very close to each other. Thus, by use of those solutions, the most economical design can be obtained. This table may be useful in determining of approximate cost of a particular wall design with known wall dimensions and soil properties of backfill.

Table 6. Weight values of optimum design for different design parameters.

	γ_s =16 kN/m^3				γ_s =18 kN/m^3				γ_s =20 kN/m^3			
	Worst		Best		Worst		Best		Worst		Best	
H (m)	Ø (°)	W_{wall} (kN)	Ø (°)	W_{wall} (kN)	Ø (°)	W_{wall} (kN)	Ø (°)	W_{wall} (kN)	Ø (°)	W_{wall} (kN)	Ø (°)	W_{wall} (kN)
4	20	39.514	30	34.234	20	39.994	30	34.234	20	40.474	30	34.234
5	20	60.397	35	46.147	20	61.147	35	46.147	20	61.897	35	46.147
6	20	85.322	40	59.402	20	86.402	40	59.402	20	86.402	40	59.402
7	20	112.219	40	73.999	20	113.681	40	73.999	20	115.159	40	73.999
8	20	143.697	40	89.937	20	145.617	40	89.937	20	147.537	40	89.937

Table 7. Optimum wall design values in which sliding and overturning safety factor constraints are very close to each other.

H (m)	Ø (°)	γ_s (kN/m^3)	X_1 (m)	X_2 (m)	X_3 (m)	X_4 (%)	$F_{s\,(sliding)}$	$F_{s\,(overturning)}$	W_{wall} (kN)
4	22	16	1.60	0.336	0.24	2	1.3254	1.3559	36.6344
4	22	20	1.60	0.24	0.24	2	1.3146	1.3190	36.6344
5	24	16	2.00	0.3	0.30	2	1.3077	1.3402	49.8975
5	26	18	1.90	0.323	0.30	2	1.4423	1.3072	49.1475
6	26	18	2.40	0.36	0.36	2	1.3474	1.3776	64.8024
6	30	20	2.16	0.324	0.36	2	1.6634	1.3094	62.6424
7	26	18	3.22	0.483	0.42	2	1.3343	1.7091	85.7591
7	30	16	2.66	0.505	0.42	2	1.5912	1.4201	79.8791
8	26	16	3.84	0.653	0.48	2	1.3102	1.8498	107.2176
8	30	20	3.04	0.456	0.48	2	1.4694	1.3557	97.6176

In Fig. 6, soil types of compactness is studied by Peck (1974) have been given according to values of the angle of internal friction. By examining the graph plotted in Fig. 6, it is obvious that the optimal wall weight decreases rapidly for H=6-7-8 m. In increasing values of the angle of internal friction, it shows an approximately linear behavior for the other H values. It is obvious that the optimal design weights of the wall have been obtained for Ø=30° according to figure, which corresponds to the angle of internal friction of the medium dense sand.

Fig. 7 shows that the base width of the wall with increase the angle of internal friction decreases till Ø=30° for different wall height. Observation of a linear behavior after Ø=30° means that optimal values are obtained for approximately this angle of internal friction. According to figure, when there is a reduction of 66% from Ø=20° to Ø=30°, a decrease of 21% is observed after Ø=30° for H=8 m.

In Fig. 8, nearly linear behavior in increase of the values of the angle of internal friction has been seen after especially Ø=30°. Since the toe with has been defined as depend on the base width given in Table 1, curve of change the toe width is similar to curve of change the base width in varied wall height.

Change of the thickness of base slab depend on the wall height has been dedicated with the angle of internal friction in Fig. 9. As regards figure, increasing of the angle of internal friction has not affected the thickness of base slab. for different values of all wall height, fixed values of X_3 have been achieved, even if the angle of internal friction changes; for instance, X_3 is equal to 0.24 m for H=4 m or 0.30 m for H=5 m. However, same angle of front face of the wall has been obtained as X_4=2% for each different wall design during the parametric study. Because, values of the angle of front face have remained constant for diversified values of design parameters like the wall height, the unit volume weight and the angle of internal friction, it is concluded that X_4 is not effective on wall design.

Results from the optimization analyses show that change of the angle of internal friction has an important influence on safety factors of sliding and overturning. The changes between the angle of internal friction and sliding and overturning safety factors have been tabulated in Figs. 10 and 11, respectively.

In Fig. 10, as the angle of internal friction increase, the safety factor has gone up especially after Ø=30°. The sliding safety factor has increased by an average of 52% with higher values of angle of internal friction for H=4 m and Ø=35°, 40°, 45°. In this case, since the optimal wall weight has been attained at Ø=30° and not to alter for other angle of internal friction, the optimized-economic wall design cannot be achieved after Ø=30°.

When the Fig. 11 is investigated, the effect of the angle of internal friction change on the overturning safety factor has been seen clearly. As the safety factor decreases till Ø=30°, it increases after this value for each wall height. Since this behavior has not any impact on the optimal wall weight, it is obvious that economic wall design cannot be achieved for higher the angle of internal friction like 35°, 40°, 45°.

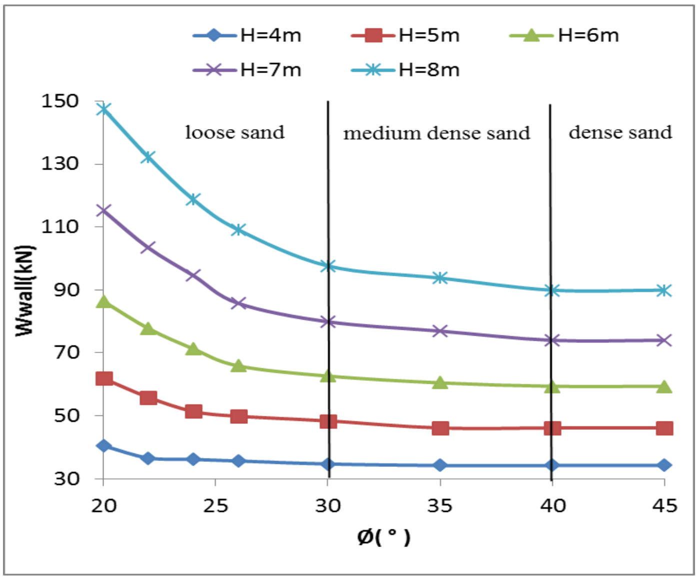

Fig. 6. The changes between the wall weight and the angle of internal friction.

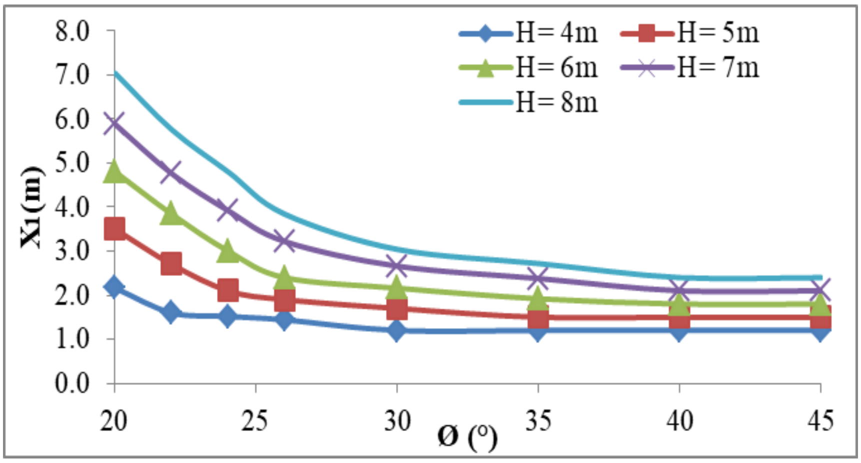

Fig. 7. The change between the base width and the angle of internal friction.

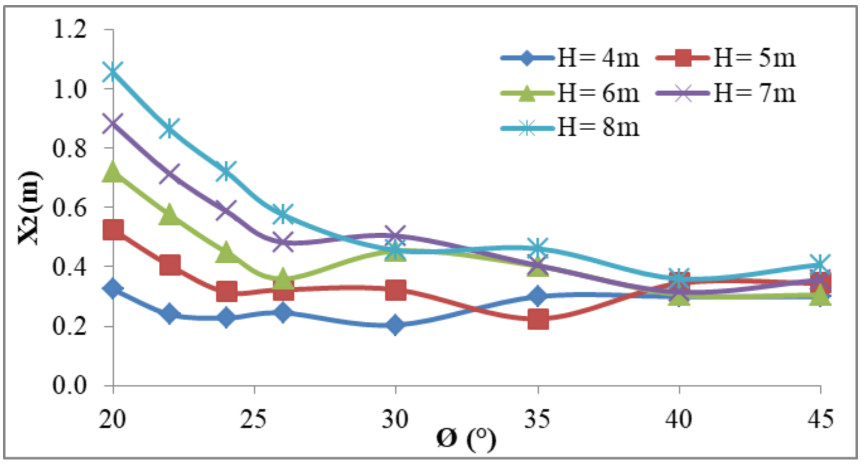

Fig. 8. The change between the toe width and the angle of internal friction.

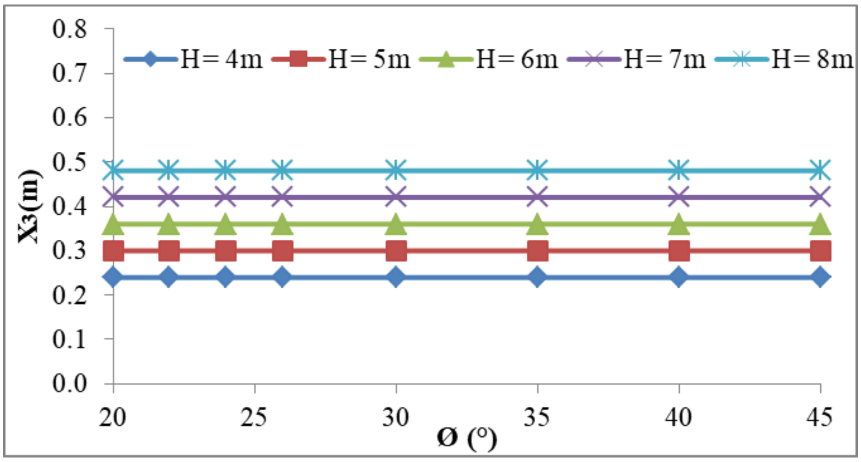

Fig. 9. The change between the thickness of the base slab and angle of internal friction.

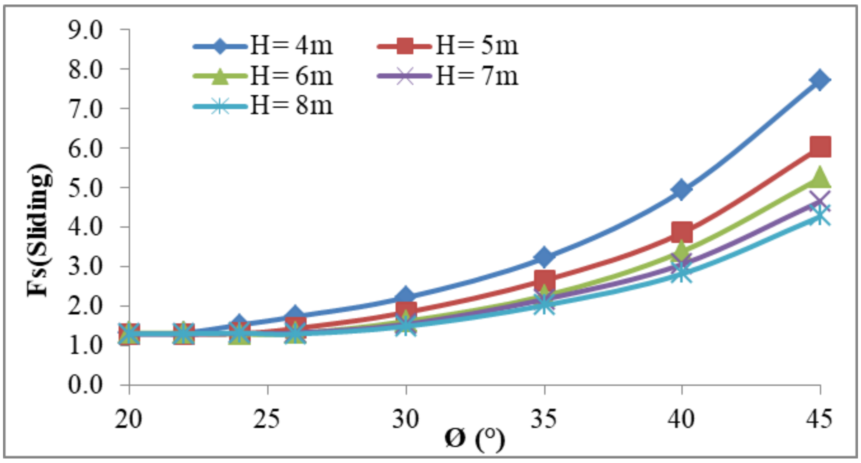

Fig. 10. The change between sliding safety factors and angle of internal friction.

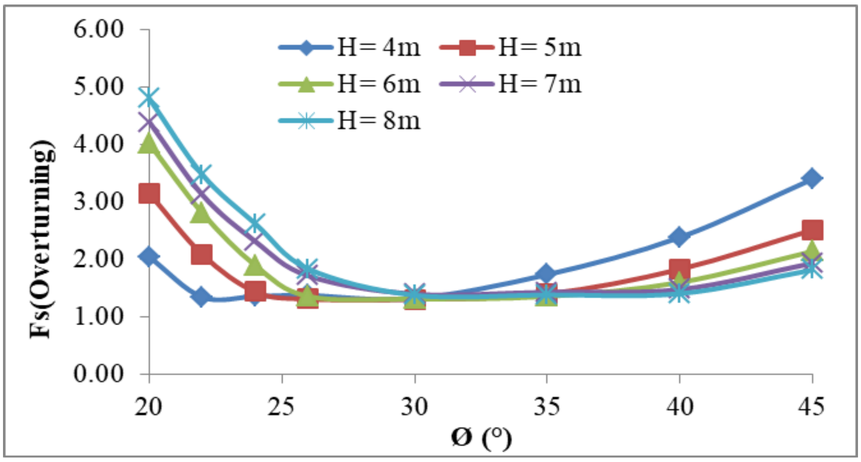

Fig. 11. The change between overturning safety factors and angle of internal friction.

4. Conclusions

In the current paper, the optimization study and a detail parametric study has been presented for the optimal design of a cantilever retaining wall by using the harmony search algorithm, which is one of the recently improved and successful optimization methods. The optimal wall dimensions leading the minimum wall weight have been found. The wall weight is treated as the objective function. In the design problem, due to the fact that safety factors against sliding and overturning are taken as constraints, the stability of the cantilever retaining wall provides for the determined optimal weights. In addition, geometric constraints due to the wall geometry have been considered in the optimal design of cantilever retaining wall. The optimal wall dimensions obtained have been satisfied the upper and the lower limits given for the design variables. The optimal weight of the wall has been obtained in less time and with less iteration compared with the traditional design of the cantilever retaining walls.

In the parametric study, the wall height and the angle of internal friction have found to be quite effective in the design of cantilever retaining walls. Generally, the unit volume weight values vary in a very limited range according to type of soils, for instance, value of the unit volume weight may be considered as 20 kN/m^3 for gravel and as 23 kN/m^3 for silty sand and gravel (Das, 2008). Therefore, obtained results show that unit volume weight change does not affect effectively design in this study which values of the unit volume weight have been taken between 16-20 kN/m^3.

In analyses, because soil type used in optimization analyses have been considered as sand, cohesion value of soil has been taken zero for the optimal cantilever retaining wall design. In general, the optimal wall weights have been attained for Ø=30° which correspond to the angle of internal friction of medium dense sand. In this point, because the compactness of soil change, from loose to medium density, it is reasonable to obtain optimal values in this value. In other words, while the weight of wall rapidly declines (34%) in the range of loose-medium dense sand (Ø<30°), it is seen that a less weight decrease of wall (8%) in the range of medium dense-dense sand (30°<Ø<40°) for H=8 m.

In this study, selection of wall dimensions has been conducted according to provisions of American Concrete Institute Building Code Requirements for Structural (ACI 318-08, 2008) and LRFD Bridge Design Specifications (AASHTO, 2010). On the other hand, the optimum wall dimensions obtained in the optimization analyzes differ from suggested wall sizes according to the ACI 318-08 (2008) and AASHTO (2010) which are independent of variable value of internal friction. This result shows that, it is necessary to take into consideration the soil property such as the internal friction angle in the design of cantilever wall.

Eventually, this study has shown that such heuristic methods may be used in finding the optimal solutions for geotechnical engineering problems. The harmony search algorithm and its improved versions may be applied easily used in any similar future research. In this way, it is possible to obtain pre-dimension guides which help to determine the optimal wall dimensions and to provide safe and economic design. Because the proposed optimization algorithm is simple mathematically, application of the algorithm is easier than other traditional optimization methods. As a future work, the algorithm may be used effectively and reliably in the design of the cantilever retaining wall for different cases such as sloping filling, surcharge load, groundwater, multilayer soil and cohesive soil.

Acknowledgements

In this study, results of a part of continuing doctoral thesis has been submitted and also it is supported by Scientific Research Projects Coordination Units of Selçuk University Research Funding (BAP-17401107) which are gratefully acknowledged.

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