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Stress and displacement analysis of perforated circular plates

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ABSTRACT

Critical deflection and stress values of perforated circular plates under loads has an important role on the design criteria. For the perforated circular plates, the basic problem is determining how they have a perforation schema for the most suitable design. For this purpose, 10 different perforated circular plate models were presented and their static analysis was studied. All of the models have the same open area percentage but different number of holes. In this way, it was more convenient to compare the results. The circular plates were analyzed under their self-weight and uniformly distributed load with different nine thickness to diameter ratios obtained based on Classical Plate Theory. In addition, two set of analyses have been performed on the circular plates for fixed supported and simply supported boundary conditions. As an example, for the 6th model critical displacement and stress values under self-weight and under uniformly distributed load are investigated in detail. Designers of perforated circular plates can use the graphics presented in this study. The present study also purposes the shape optimization of thin circular perforated plates with round and staggered holes.

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1. Introduction

Circular plates are common in many structures. They are used such as nozzle covers, end closures in pressure vessels, and bulkheads in submarines and airplanes, reactors, heat exchangers, and distillation towers (Jawad, 2010).

There are many practical applications of perforated plates because of their advantages. They let flow of light and air. They have a resistant to high/low temperature and corrosion. They have light weight and they have a low cost. Also, in the architectural applications - office buildings, hospitals, educational establishments, airports, stations etc. - perforated plates are can be used as a ceilings, floors, dividers, walls etc. Many equipment as speakers, furniture, dryers, light fixtures etc. are made of perforated plates. Also daily use areas are very wide including electrical components, agricultural equipment, mining, food processing equipment, vehicles as well as other machines.

Studies about perforated circular plates with a serial solutions are not found in abundance in literature.

Timoshenko and Woinowsky-Krieger (1959), wrote the basic reference for plates and shells. Many authors have interested in bending of perforated plates. Harrop and Abdul-Karim (1967), investigated the deflections and stresses for circular plates with square pitch perforations. The plates are subjected to uniform lateral pressure. They also compare the results with experimental results obtained by other investigators on three plate models. Murakami and Konishi (1982), studied about an elastic-plastic constitutive equation for transversely isotropic materials and as an application of the resulting constitutive equation, elastic-plastic bending of perforated circular plates is analyzed by modelling them by equivalent homogeneous transversely isotropic plates. Albayrak and Saraçoğlu (2011, 2018) and, Saraçoğlu and Albayrak (2016, 2017, 2018), were interested in plates with multiple holes and published some research articles about their subject. Wu et al. (2003), present a mathematical

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model of axisymmetric elastic/plastic perforated circular plate bending and stretching is developed which accounts for through thickness yielding, through thickness variations in perforation geometry, elastic outer edge restraint, and moderately large deflections. Atanasiu and Sorohan (2016), studied the displacements and stresses distribution in bending of perforated circular plate in their work using the finite element analysis (FEA) and experimentally by holographic interferometry. Azelmad et al. (2018), proposed a 2D typical model for the numerical simulation of the circular clamped perforated thin plates behavior in the elastic and elastoplastic domains. Their problem model is based on the real geometry of plates. The model takes into account different perforation distribution patterns and ligament factors. Also it is then validated by experiments. Konieczny et al. (2020), presented an analysis of an isotropic circular axisymmetric perforated plate loaded with concentrated force applied in the geometric center of the plate using finite element software ANSYS. The results of numerical calculations were compared and verified with experimental results.

In addition to theoretical studies, there are also studies that carry out experimental studies about this subject takes place in the literature. Osweiller (1989), presented some curves about the effective elastic constants based on consistent theoretical and experimental results in his study. These curves allow one to determine accurate and confident effective elastic constants and they were some help to engineers concerned with the design of multi-perforated plates. Solar and Hill (1976), analyzed the perforated plates for tubesheet design in their study. And also a simple analytical expression is proposed to determinate the effective bending stiffness of a perforated plate in the paper. Achtelik et al. (2008), presented a paper about elaborating the methodology of empirical studies in perforated plates centrally loaded by a concentrated force. They also presented experimental results to verify the mathematical model proposed in a companion paper.

There are a considerable number of studies that perform dynamic analysis as well as static analysis. Civalek and Çatal (2003), studied numerical solution to static and free vibration analysis of thin circular and annular plates having various supports and load conditions are obtained by the method of Harmonic Differential Quadrature (HDQ). Mishra and Das (1971), studied about free vibrations of isotropic nonhomogeneous circular plates and discussed the transverse vibration of nonhomogeneous free circular plate in their paper. Jhung et al. (2006), investigated free vibration analysis of perforated plate and in their study the equivalent material properties of perforated plates are suggested by performing several finite element analysis with respect to the ligament efficiencies. Jhung and Jo (2006), suggested the equivalent material properties of a perforated plate by performing several analyses with respect to ligament efficiencies. Jhung et al. (2009), studied about equivalent material properties of perforated structures. Lee and Chen (2011), in their paper theoretically derived the natural frequencies and natural modes of a circular plate with multiple circular holes and numerically determined. Dinkar (2015), study about vibration analysis of perforated plates in his doctoral thesis. Senjanovic et al. (2017), investigate the vibration analysis of thin circular plates with multiple openings. They use the assumed mode method and evaluate the results through their comparison with an analytical and finite element method solution. Jeong and Jhung (2017), have presented free vibration analysis of partially perforated circular plates with a triangular hole pattern and clamped boundary condition. They proposed a theoretical method and compare the results with finite element method solutions.

In this study, the effect of perforation pattern in perforated circular plates is investigated. For this purpose, ten perforated plate models were produced and statically analyzed under their self-weight. The finite element software ANSYS workbench was used to analyze the deflection behavior of thin circular plates with open holes in bending, and put forth a suitable perforation pattern for a perforated circular plate was proposed.

2. Material and Methods

Deformations and stresses in a plates are calculated by using a plate theory. Various plate theories have been developed since 19th century. In engineering two of them are widely used. One of them is named Kirchhoff–Love plate theory and also named as Classical Plate Theory (CPT). The other is named as Mindlin–Reissner plate theory and also named as First-order Shear Deformation Plate Theory (FSDPT). Every theory has different assumptions. In this study perforated circular plates are analyzed using CPT. In this theory the following kinematic assumptions are made:

- Straight lines normal to the mid-surface remain straight after deformation.
- Straight lines normal to the mid-surface remain normal to the mid-surface after deformation.
- The thickness of the plate does not change during a deformation.

In CPT, thickness to width ratio of a plate structure is have to be between 1/150 and 1/20.

The equations developed with reference to Cartesian coordinates are not convenient for the analysis of circular plates (Bhaskar et al., 2014). Therefore, the differential equations for the bending of a rectangular plate have to be transform to the polar coordinates for circular plates.

For the analysis of stress and displacement distribution of circular plates under loads, the derived differential equations of circular plates must have to be solved. These equilibrium equations of plates was made around 1900 by Love. These investigations were made for various boundary conditions and loadings.

When the load is symmetrically on the axis perpendicular to the circular plate through its center, the deflection surface is bent also symmetrically. So that, all points which have equal distance from the center of the plate the deflections will be the same as can be seen at Fig. 1.

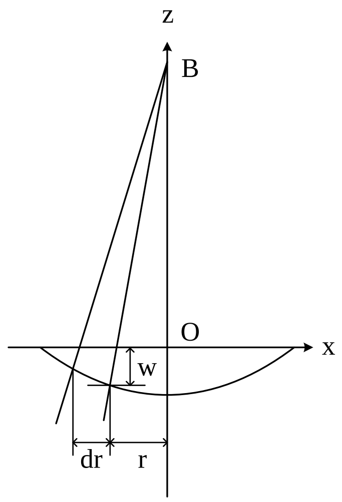


Fig. 1. Deflection of a circular plate.

The governing differential equation of a circular plate in cylindrical coordinates is given in Eq. (1):

$$\frac{1}{r}\frac{d}{dr}\left\{r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]\right\}=\frac{q}{D} \tag{1}$$

In this equation w is the deflection of the plate, r is the radius coordinate of the circular plate, q is the intensity of loading and D is the flexural rigidity of the plate formulated at Eq. (2) dependent to E modulus of elasticity, υ Poisson's ratio and h plate thickness.

$$D=\frac{Eh^3}{12(1-\nu^2)} \tag{2}$$

Considered coordinates for circular plates with fixed supported and simply supported boundary conditions are shown in Fig. 2.

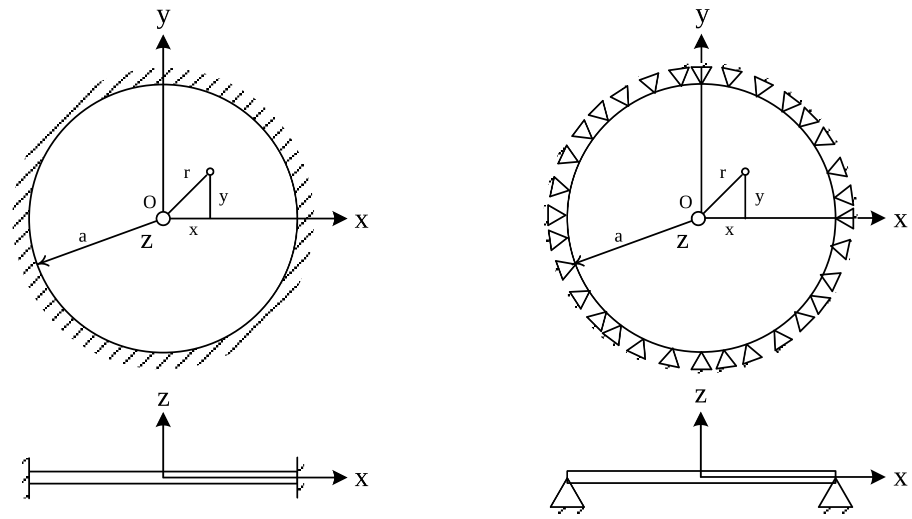


Fig. 2. Coordinates of a circular plate.

3. Numerical Applications

For demonstrating the differences between perforation patterns ten models were produced systematically as shown in Fig. 3. Firstly, a non-perforated circular plate with no holes is taken as a reference plate. These models were subjected to bending analysis under their self weights.

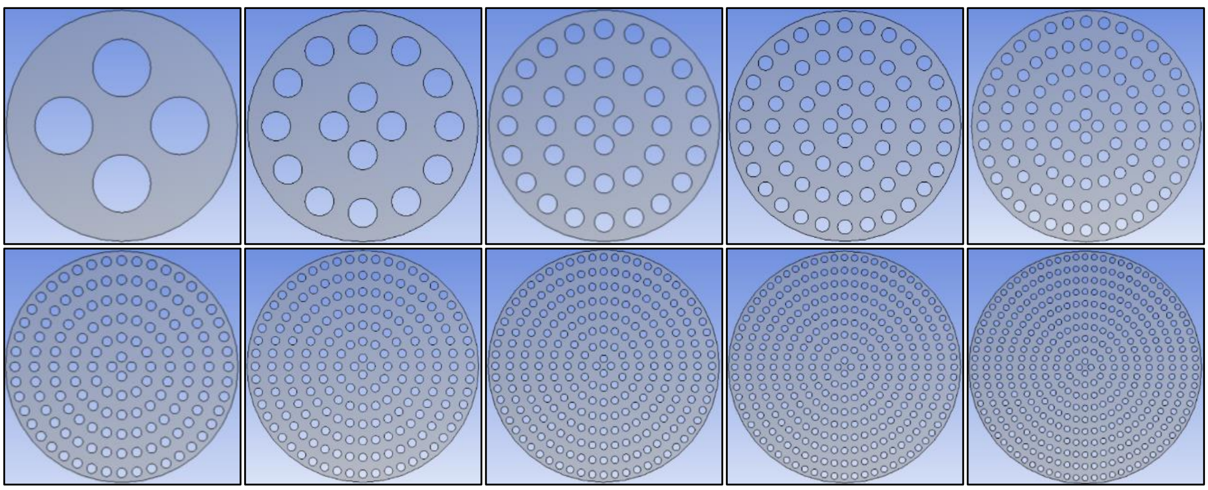


Fig. 3. Models of perforated circular plates.

Stress and displacement analysis of these models investigated and mid-point deflections and stresses are calculated. Circular plates are considered as they are thin plates. Nine different thickness to diameter ratio of these thin plate structures is taken into account as 2/300, 3/300, 4/300, 5/300, 6/300, 8/300, 10/300, 11/300 and 14/300 respectively. The solutions were developed for boundary conditions of simply supported and fixed supported.

The circular plate models are assumed to have the radius of 150 mm and made of steel material. The material parameters of the steel plates are assumed as shown in Table 1.

Table 1. Material properties of the perforated steel plate.

Property	Value
Young's modulus, E (GPa)	200
Poisson's ratio, υ	0.3
Mass density, ρ (kg/m^3)	7850

Shell181 has a capability for analyzing thin to moderately-thick shell structures. It is defined by four nodes. There are six degrees of freedom at each node as: rotations about the x, y and z axes, and translations in the x, y and z directions.

Triangular free mesh is used in free meshing operations. In the example models 1mm finite element mesh size is used.

After preprocessing the problem in the program, static analysis was performed and the results of deformations and stresses were obtained from the program.

Stress output for SHELL181 element is as follows:

σ_x is normal stress due to X axis (SX)

σ_y is normal stress due to Y axis (SY)

τ_{xy} is shear stress (SXY)

Number of holes, radius of the circular holes and total hole area is dependent to the model number n. In all of the perforated circular plate models % open area is the same as shown in Table 2.

Analyses of these models ANSYS software was used. This finite element software has various elements in the element library for static and dynamic analyses. For this study Shell181 is the most suitable element for the problem.

Table 2. Total hole areas for the models.

Model No	Number of holes	Radius	Total hole area
0	0	0.0000	0*π*0^2
1	4	37.5000	4*π*37.5000^2
2	16	18.7500	16*π*18.7500^2
3	36	12.5000	36*π*12.5000^2
4	64	9.3750	64*π*9.3750^2
5	100	7.5000	100*π*7.5000^2
6	144	6.2500	144*π*6.2500^2
7	196	5.3571	196*π*5.3571^2
8	256	4.6875	256*π*4.6875^2
9	324	4.1667	324*π*4.1667^2
10	400	3.7500	400*π*3.7500^2
.	.	.	.
.	.	.	.
.	.	.	.
n	(2*n)^2	R/(4*n)	(π*R^2)/4

4. Results and Discussion

Although the plate models in this study have the same arrangement they have different number of holes. The circular plates examined to different number of holes as 4, 16, 36, 64, 100, 144, 196, 256, 324, 400, respectively and radius and locations are also different. As seen from the Table 2, % open areas of the models are exactly same. Analyze results evaluate how the perforated circular would perform under self-weight. The results can be examined under two headings, namely displacements and stresses.

4.1. Displacements under self-weight

Fig. 4 shows comparison of mid-point deflections for fixed supported and simply supported non-perforated and perforated circular plates according to ten models with different nine thickness to diameter ratios obtained by ANSYS with solutions based on CPT.

It can be seen from the Fig. 4 that mid-point deflections of the circular plates have almost the same value for the models greater than 6th model.

Critical displacements for non-perforated and perforated (6th model) circular plates are given in Table 3.

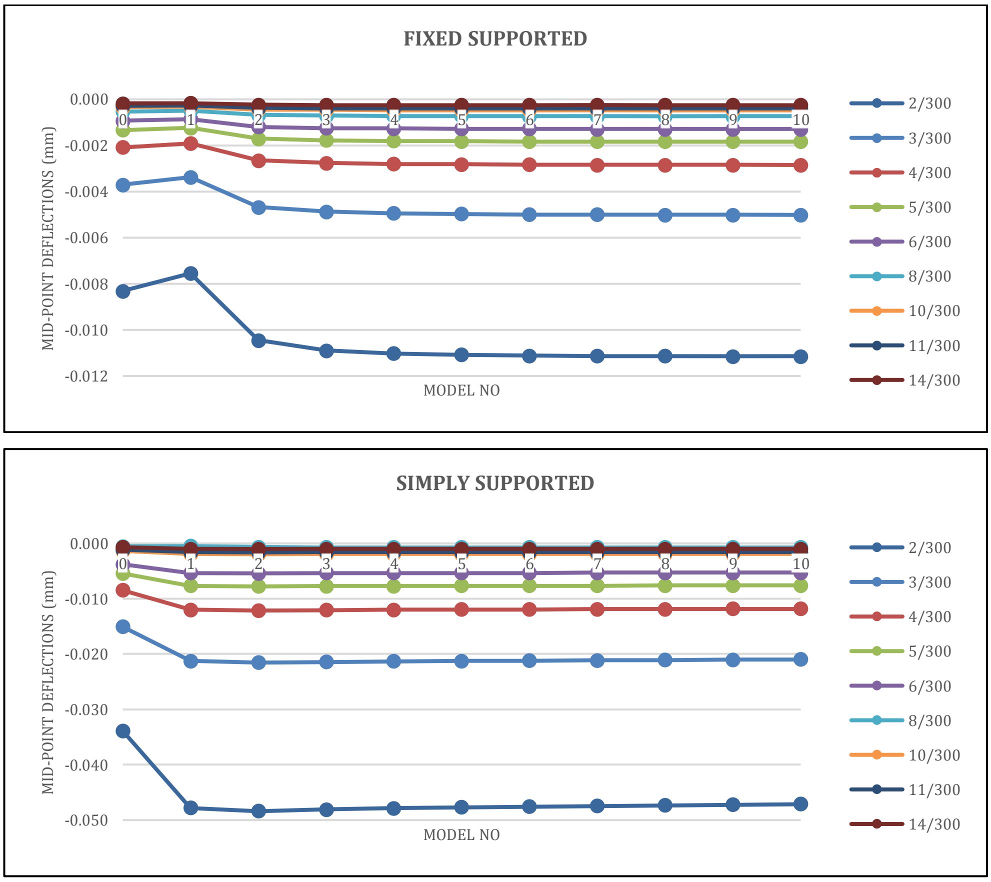


Fig. 4. Midpoint deflections of fixed supported and simply supported non-perforated and 10 perforated circular plate models according to thickness to diameter ratios.

Table 3. Critical displacements of the plates under self-weights (mm).

h/D	Fixed Supported		Simply Supported	
	non-perforated	perforated	non-perforated	perforated
2/300	-0.008318	-0.011111	-0.033893	-0.047570
3/300	-0.003701	-0.004989	-0.015067	-0.021177
4/300	-0.002085	-0.002833	-0.008478	-0.011930
5/300	-0.001337	-0.001828	-0.005429	-0.007647
6/300	-0.000930	-0.001280	-0.003772	-0.005318
8/300	-0.000526	-0.000730	-0.002125	-0.003000
10/300	-0.000339	-0.000473	-0.001362	-0.001925
11/300	-0.000282	-0.000393	-0.001127	-0.001593
14/300	-0.000176	-0.000247	-0.000698	-0.000988

Displacement values for all points of fixed and simply supported perforated (6th model) circular plate for D/h=150 under self-weights is shown in Fig. 5.

4.2. Displacements under uniformly distributed load

Displacements under uniformly distributed q=1kN/m^2 load has been performed on the two set of plates for fixed supported and simply supported non-perforated and 6th model perforated circular plates.

The critical displacements of the circular plates loaded with uniformly distributed load are shown in Table 4 with two support conditions. The maximum deflection is occurs at the midpoint of the circular plate and smaller than the thickness of the plate.

4.3. Stresses under self-weight

Stress distribution for the perforated circular holes under self-weights is more complex than the deflection distributions.

In Fig. 6, minimumSX and maximumSX stress values of non-perforated circular plate and ten models of fixed and simply supported perforated circular plates are shown.

MinimumSY and maximumSY stress values of non-perforated circular plate and ten models of perforated circular plates are shown in Fig. 7.

MinimumSXY and maximumSXY stress values of non-perforated circular plate and ten models of perforated circular plates are shown in Fig. 8.

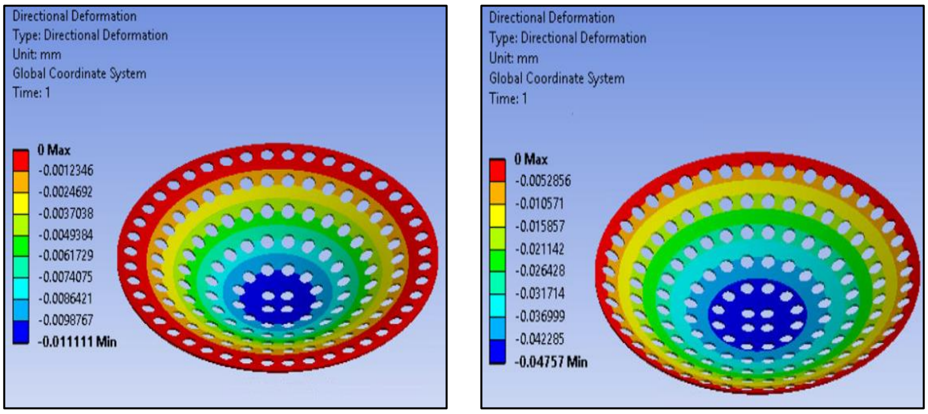
As an example stress distribution of reference (non-perforated) and 6th model (perforated) plate for D/h=150 is shown in Fig. 9. In the example variation of stresses along line from point (0,0,0) to point (150,0,0) for perforated and non-perforated reference plates under their self-weights are given.

Similar to fixed supported plates, variation of stresses for simply supported circular plates are also given in Fig. 9.

From the graphs it can be inferred that there are large stress jumps on the perimeters of the holes.

The critical stresses of the circular plates under self-weights are shown in Table 5 with two support conditions. Circular plates are taken as non-perforated and as an example for 6th perforated model.

Stress values for all points of fixed and simply supported perforated (6th model) circular plate for D/h=150 under self-weights is shown in Fig. 10.



fixed supported simply supported

Fig. 5. Displacement values of fixed and simply supported perforated (6th model) circular plate for D/h=150 under self-weights.

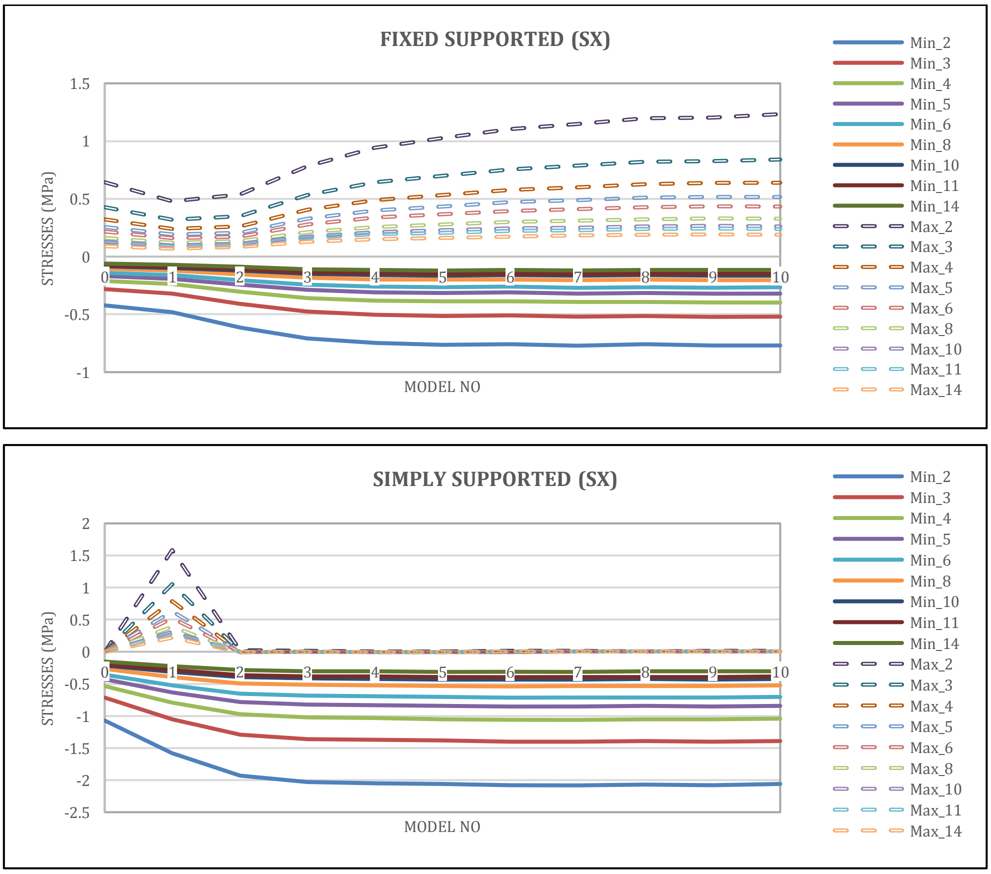


Fig. 6. MinimumSX and MaximumSX stress values of fixed supported and simply supported non-perforated and 10 perforated circular plate models.

Table 4. Critical displacements of the plates loaded with uniformly distributed load (mm).

h/D	Fixed Supported		Simply Supported	
	non-perforated	perforated	non-perforated	perforated
2/300	-0.054028	-0.072168	-0.220140	-0.308970
3/300	-0.016025	-0.021603	-0.065242	-0.091695
4/300	-0.006770	-0.009199	-0.027534	-0.038743
5/300	-0.003473	-0.004750	-0.014104	-0.019866
6/300	-0.002014	-0.002771	-0.008166	-0.011513
8/300	-0.000854	-0.001185	-0.003450	-0.004871
10/300	-0.000441	-0.000615	-0.001770	-0.002501
11/300	-0.000332	-0.000465	-0.001331	-0.001882
14/300	-0.000164	-0.000230	-0.000648	-0.000917

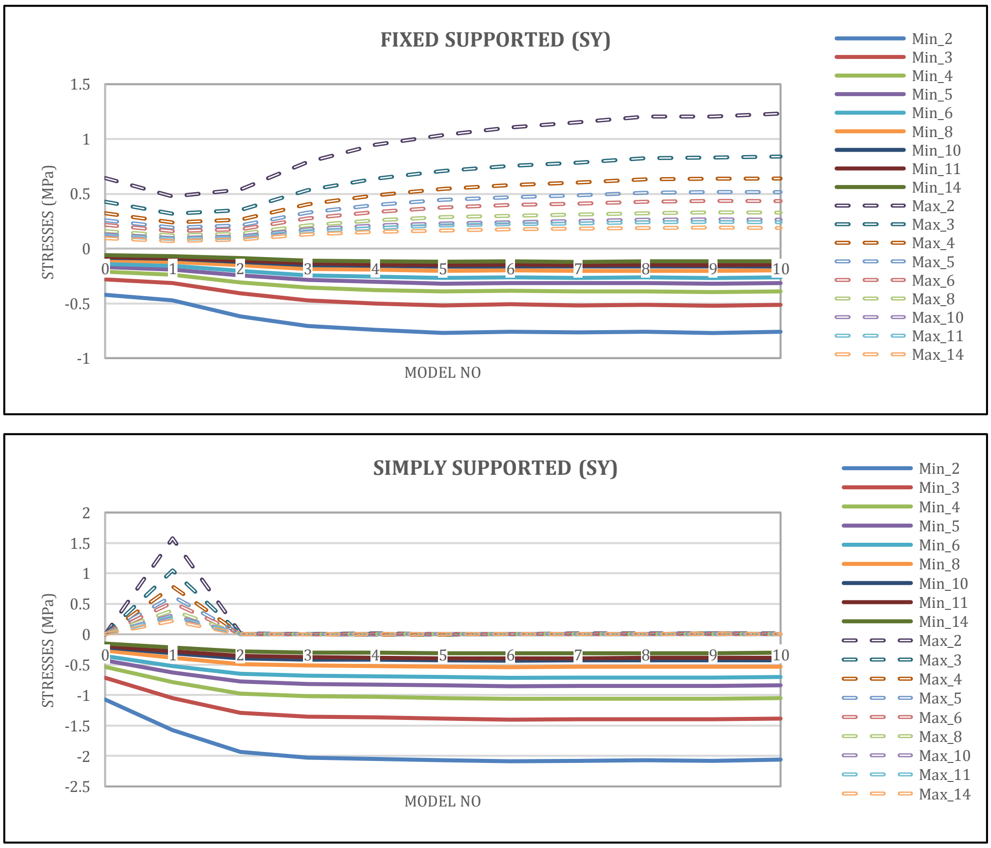


Fig. 7. MinimumSY and MaximumSY stress values of fixed supported and simply supported non-perforated and 10 perforated circular plate models.

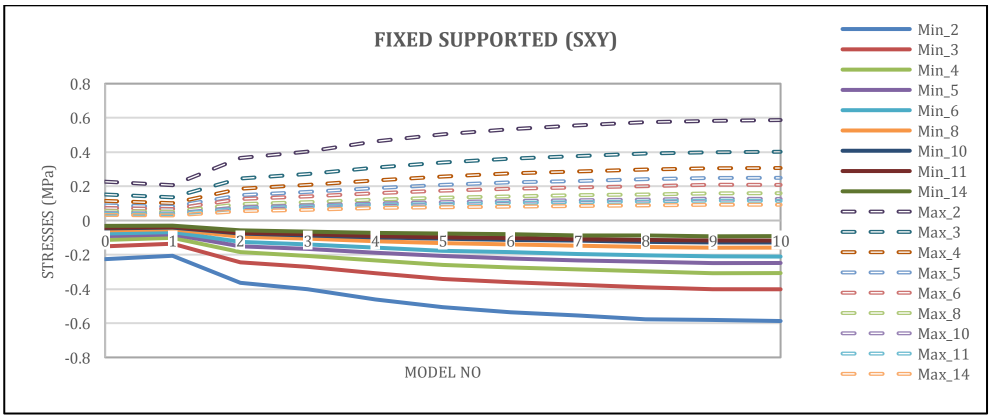


Fig. 8. (continued)

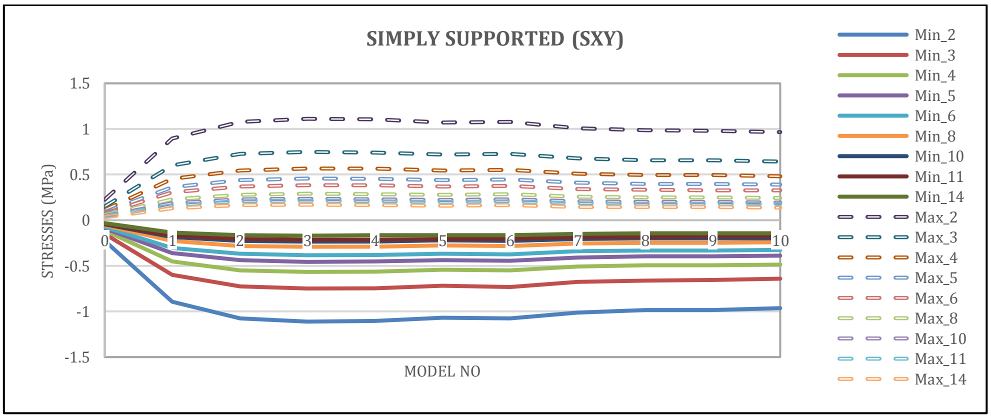


Fig. 8. MinimumSXY and MaximumSXY stress values of fixed supported and simply supported non-perforated and 10 perforated circular plate models.

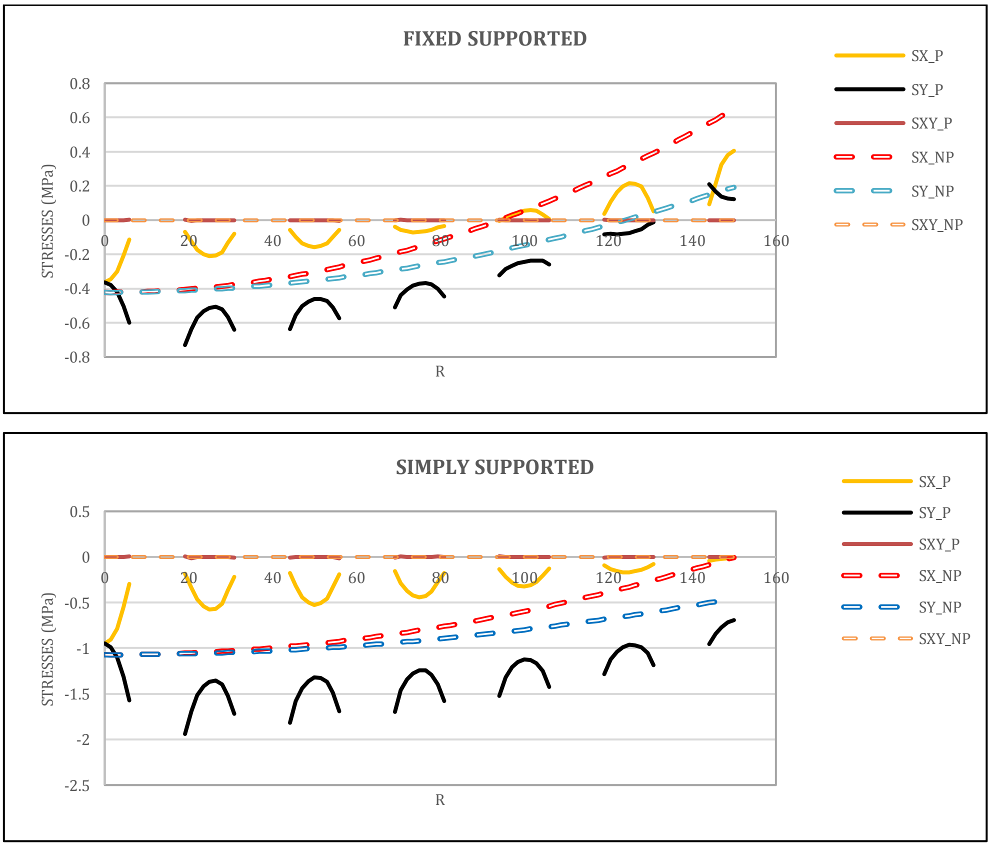


Fig. 9. Stress values of fixed supported and simply supported non-perforated and perforated (6th model) circular plate for D/h=150.

Table 5. Critical stresses of the plates under self-weights (MPa).

	Fixed Supported				Simply Supported			
	Minimum		Maximum		Minimum		Maximum	
	non-perforated	perforated	non-perforated	perforated	non-perforated	perforated	non-perforated	perforated
S_X	-0.422180	-0.756420	0.644680	1.104500	-0.004581	0.012353	-1.071700	-2.080700
S_Y	-0.422170	-0.757280	0.644880	1.106200	-0.004164	0.008235	-1.071700	-2.086300
S_{XY}	-0.226080	-0.535380	0.226080	0.535380	0.226120	1.077800	-0.226120	-1.077800

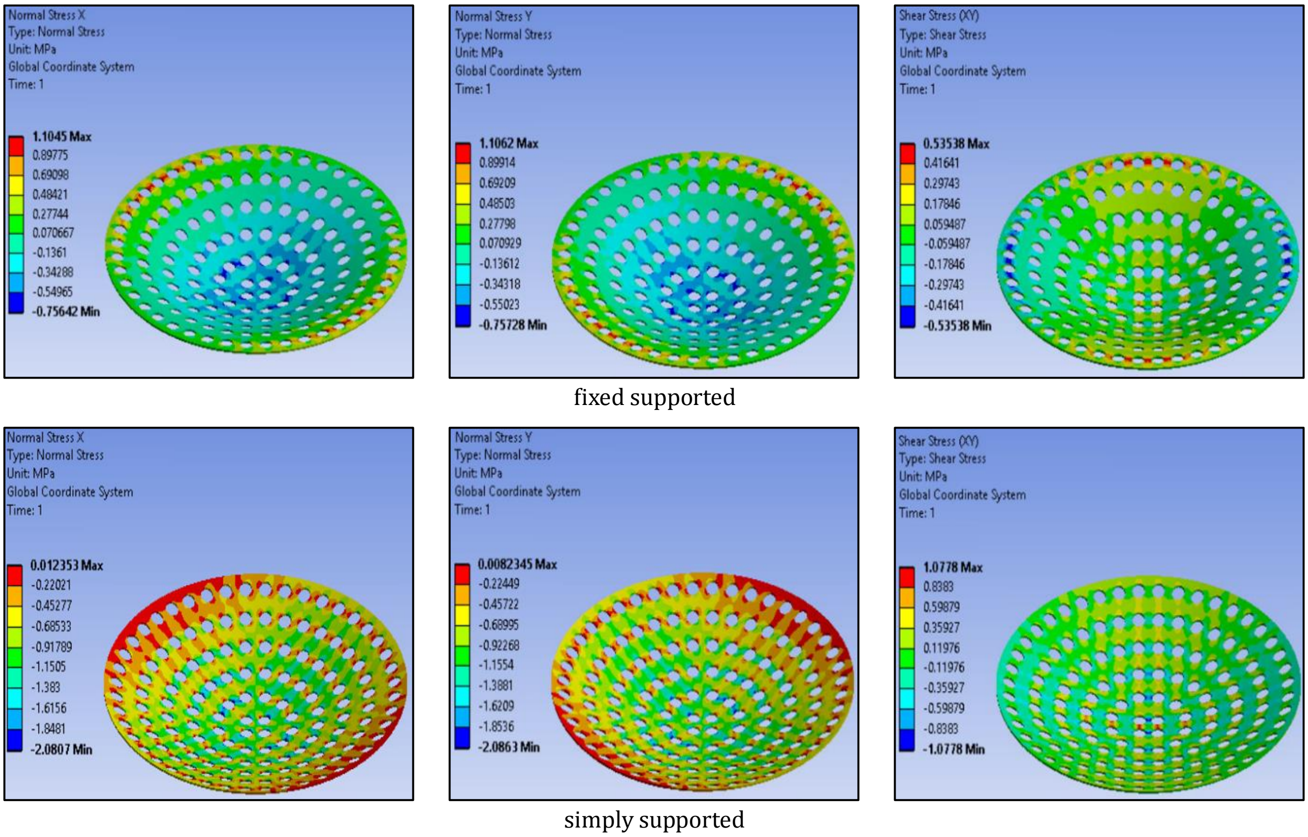


Fig. 10. Stress values of fixed and simply supported perforated (6th model) circular plate for D/h=150 under self-weights.

4.4. Stresses under uniformly distributed load

Stresses under uniformly distributed q=1kN/m^2 load has been performed on the two set of plates for fixed supported and simply supported non-perforated and 6th model perforated circular plates.

The critical stresses of the plates loaded with uniformly distributed load can be seen in Table 6.

Table 6. Critical stresses of the plates loaded with uniformly distributed load (MPa).

	Fixed Supported				Simply Supported			
	Minimum		Maximum		Minimum		Maximum	
	non-perforated	perforated	non-perforated	perforated	non-perforated	perforated	non-perforated	perforated
S_X	-2.742	-4.9129	4.1872	7.1739	-0.029753	0.080234	-6.9607	-13.514
S_Y	-2.742	-4.9186	4.1885	7.1847	-0.027043	0.053483	-6.9607	-13.551
S_{XY}	-1.4684	-3.4773	1.4684	3.4773	1.4686	7.0004	-1.4686	-7.0004

5. Conclusions

In this paper, the mid-point deflection and stress values of the perforated circular plate under self-weights and under uniformly distributed load with the influence of holes was calculated and analyzed. Series of models have been produced for investigating the stresses and displacements of perforated circular plates.

The results obtained can be listed as follows:

- In all of the perforated circular plate models % open area is the same as shown in Table 2.
- Problems were analyzed by using finite element analysis software ANSYS Workbench.
- Displacement and stress distribution of perforated circular plate under self-weights and under distributed q=1kN/m^2 load was separately analyzed and results are presented as tables and graphics.
- Critical deflection and stress values has an important role on the design of perforated circular plates.
- The deflections for the simply supported perforated circular plates is approximately 4.3 times higher than those fixed supported perforated circular plates.

- The results obtained for nine different diameter/thickness values within the limits of thin plate acceptance showed approximately the same behavior.
- The variation of the deflection and stress values are getting asymptotic after the 6^{th} model.
- Critical displacement values of perforated circular plate under self-weights for fixed supported is 0.011111 mm and for simply supported is 0.047570 mm.
- Critical SX values of perforated circular plate under self-weights for fixed supported is 1.104500 MPa and for simply supported is 2.080700 MPa.
- Critical SY values of perforated circular plate under self-weights for fixed supported is 1.106200MPa and for simply supported is 2.086300 MPa.
- Critical SXY values of perforated circular plate under self-weights for fixed supported is 0.535380 MPa and for simply supported is 1.077800 MPa.
- Critical displacement values of perforated circular plate under distributed load for fixed supported is 0.072168 mm and for simply supported is 0.308970 mm.
- Critical SX values of perforated circular plate under distributed load for fixed supported is 7.173900 MPa and for simply supported is 13.514000 MPa.
- Critical SY values of perforated circular plate under distributed load for fixed supported is 7.184700 MPa and for simply supported is 13.551000 MPa.
- Critical SXY values of perforated circular plate under distributed load for fixed supported is 3.477300 MPa and for simply supported is 7.000400 MPa.
- When the number of holes increases, deflection and stress values are getting asymptotic.
- Designers of perforated circular plates can take into account these midpoint deflections by using these graphics.

As a result, the present study purposes the shape optimization of thin circular perforated plates with round and staggered holes. And perforation schemas can also be developed by different optimization techniques.

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