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Research Article

The effect of slenderness on the lateral-torsional buckling and ultimate shear capacity of plate girders

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ABSTRACT

Lateral torsional buckling and shear buckling are two of the most significant structural responses that should be considered during the design process of plate girders. Particularly the importance of lateral torsional buckling was once again witnessed during the reconstruction process of a bridge in Edmonton, Alberta, Canada when the plate girders failed due to insufficient bracing. This current study aims to acquire a better understanding of the effect of geometric parameters such as the web slenderness, flange slenderness and span-to-depth ratio on the critical buckling moment and ultimate shear strength of plate girders. To achieve this goal the critical buckling moment and ultimate shear strength of a plate girder were parametrically studied for a large number of geometries using a load case from an experimental study. The results of this parametric study clarified the effects of web slenderness, flange slenderness and span-to-depth ratio on the structural performance of a plate girder. The visualization of the results was used to identify the ranges of these geometric parameters where the structural performance is most sensitive to changing them.

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1. Introduction

Lateral-torsional and shear buckling are known to be two of the major failure modes of slender structural members. While the web of the plate girders is the primary element resisting the shear buckling, the flanges of them are primarily carrying the bending and torsional loads. Both of these failure modes were investigated extensively in order to obtain analytical formulations that predict the load carrying capacity of structures subjected to shear forces and bending moments accurately. The research works done by Nethercot (1974), Fukumoto et al. (1980), and MacPhedran and Grondin (2011), can be mentioned among the notable works investigating lateral-torsional buckling. Particularly plate girders with doubly symmetric I-sections have been the subject of comprehensive research. The importance of having a sound understanding of the buckling behavior of plate girders was once again seen during the replacement project of the 102 Avenue Bridge over Groat Road in Edmonton, Alberta, Canada. The newly installed plate girders of this bridge failed in the lateral-torsional buckling mode under wind and construction loads due to insufficient bracing. This was one of many similar incidents of steel bridge girder failure due to lateral-torsional buckling which occurred at the construction/deconstruction stages or during the service life (Thiébaud et al., 2016).

In addition to bending moment plate girders are also expected to carry shear forces. The web part of the plate girders carries the shear forces and shear buckling is therefore a critical failure mode for these structural members. Frequently the out-of-plane shear buckling resistance of plate girders is increased through the application of transverse stiffeners or corrugated webs. Therefore, in addition to the lateral -torsional buckling analysis, the design process of plate girders also involves the shear buckling of the structural member. Fig. 1 shows an example of shear buckling during experiments carried out by Mamazizi et al. (2013).

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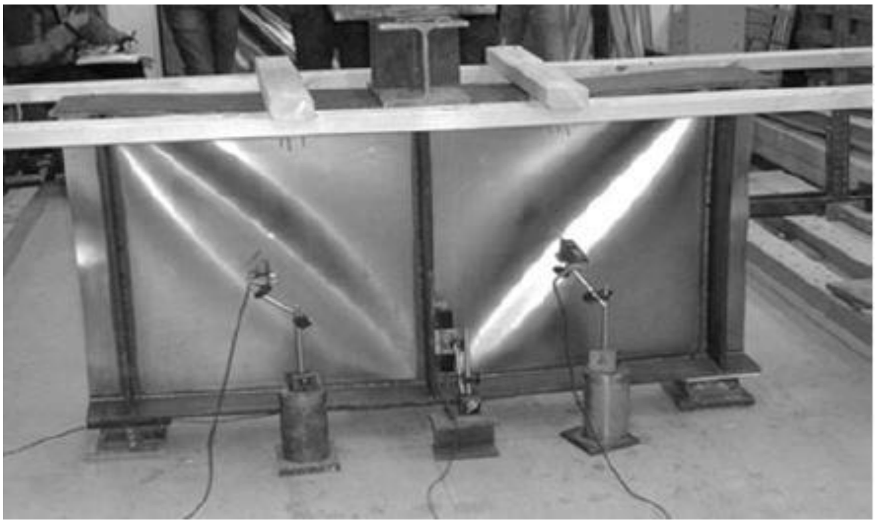

Fig. 1. Post-buckling deformation of stiffened plate girder under transverse loading (Mamazizi et al., 2013).

The design process of plate girders with the maximum load carrying capacity within cost and material constraints includes finding the optimal combination of plate thicknesses, web plate slenderness and stiffener spacing (Ziemian, 2010; Lee and Yoo, 1998; Gupta et al., 2006). While obtaining the plate girder profile that delivers the maximum structural performance with the minimum material usage is a challenging task the variation of the structural cost and performance with respect to flange and web slenderness values is not clear and can be at times counterintuitive. In order to gain a better understanding of these variations parametric studies of the buckling load and cross sectional area of a plate girder are carried out in this study. The main objective of this study is to clarify the impacts of changing the web plate slenderness, flange slenderness and the span-to-depth ratio of a plate girder on the structural performance. The critical buckling moment that leads to lateral torsional buckling and the ultimate shear stress are used as measures of structural performance.

1.1. Slenderness of girder plates

Plate girder components subject to flexure can be classified as compact, noncompact or slender according to their slenderness. Compact sections are those that have a small enough slenderness such that a local buckling of the compression flange or the web would not occur before the entire section reaches its yield strength and the section is able to attain its full plastic moment (Williams, 2011). The slenderness of a flange is quantified as $\lambda_f = b_f/2t_f$ where b_f and t_f are the width and the thickness of the flange respectively. Similarly, the slenderness of the web plate is quantified as $\lambda_w = D/t_w$ where D is the height of the plate girder excluding the flange thicknesses and t_w is the web plate thickness. In order to be classified as compact the flange and the web of a plate girder must satisfy the inequalities in Eqs. (1) and (2) respectively as per (AISC, 2016).

$$\lambda_f = \frac{b_f}{2t_f} \leq 0.38\sqrt{\frac{E}{\sigma_y}} \tag{1}$$

$$\lambda_w = \frac{D}{t_w} \leq 3.76\sqrt{\frac{E}{\sigma_y}} \tag{2}$$

In the above equations σ_y is the yield stress and E is the Young's modulus of steel. Noncompact structural members are those that are susceptible to local buckling before the section attains its full plastic moment. In order for the flange or the web of a plate girder to be classified as noncompact the slenderness values of these members must fall into the intervalls given in Eqs. (3) and (4) respectively (AISC, 2016).

$$0.38\sqrt{\frac{E}{\sigma_y}} < \lambda_f = \frac{b_f}{2t_f} < \sqrt{\frac{E}{\sigma_y}} \tag{3}$$

$$3.76\sqrt{\frac{E}{\sigma_y}} < \lambda_w = \frac{D}{t_w} < 5.70\sqrt{\frac{E}{\sigma_y}} \tag{4}$$

Slender sections are those that are susceptible to local buckling prior to reaching the yield stress anywhere on the section. In case of slender members, the slenderness parameter λ is greater than all limit values given in Eqs. (1) to (4). The classification of structural members with respect to their slenderness values is also visualized in Fig. 2.

1.2. Lateral-torsional buckling

Lateral Torsional Buckling (LTB) can be defined as a combination of lateral displacement and twisting due to the application of transverse forces on a beam type structure in the absence of sufficient lateral bracing (Kabir and Bhowmick, 2016). The research in the field of beam and plate girder buckling resulted in various equations for the prediction of the lateral torsional buckling load. The solutions available in the (AISC, 2016) and (CSA, 2009) codes for the prediction of the lateral torsional buckling capacity provide the buckling capacity for the case of uniform bending moment distribution together with a moment gradient factor C_b for the adjustment of the predicted capacities to the case of a non-uniform bending moment distribution. (Wong and Driver,

2010) developed Eq. (5) in order to incorporate the variation of the bending moment along unbraced sections of the plate girder into the buckling load capacity.

$$C_b = \frac{4M_{max}}{\sqrt{M_{max}^2+4M_A^2+7M_B^2+4M_C^2}} \leq 2.5 \quad (5)$$

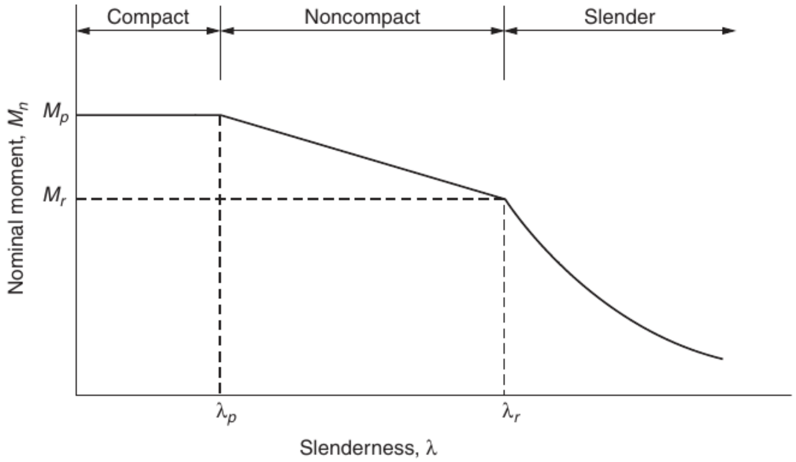

Fig. 2. Classification of structural members according to slenderness (Williams, 2011).

In Eq. (5) M_{max} is the absolute value of the maximum bending moment and M_A, M_B, M_C are the absolute values of the bending moments at a / 4, a / 2 and $3a$ / 4 length along the unbraced span of the plate girder respectively where a denotes the total length of the unbraced span. In this study the moment distribution between two stiffeners is assumed linear corresponding to an unbraced span of the girder beam shown in Fig. 1. Once C_b is known, the critical bending moment M_{cr} for lateral torsional buckling can be calculated using Eq. (6).

$$M_{cr} = C_b M_{0cr} \quad (6)$$

In Eq. (6), M_{0cr} is the critical bending moment of an unbraced span under uniform bending moment. The solution for M_{0cr} is given in Eq. (7) (Galambos and Surovek, 2008).

$$M_{0cr} = \frac{\pi}{a}\sqrt{EI_y\left(GJ + \frac{\pi^2 EC_w}{a^2}\right)} \quad (7)$$

In Eq. (7), G is the shear modulus, E is the modulus of elasticity, J is the St. Venant torsion constant, I_y is the moment of inertia with respect to the minor axis of the I-section and C_w is the warping constant.

1.3. Ultimate shear strength

The shear forces acting on girder plates are largely carried by the web plates. After the onset of local buckling of the web these steel plates under shear force are known to exhibit a significant amount of load carrying capacity in the post-buckling regime (Glassman and Moreyra Garlock, 2016). This structural behavior is thoroughly investigated in the literature and attributed to the existence of tensile stresses acting in the diagonal direction of the plates after the onset of shear buckling (Basler, 1961; White and Barker, 2008; Ziemian, 2010). These areas along the diagonal of the web plate where tensile stresses are acting can also be seen in Fig. 1. The research led to the development of various models for the prediction of the post-buckling shear capacity of web plates based on the concept of a tension field along the plate diagonal. "Tension field theory" is often used as a concept that includes all of these models. This theory is based on the observation that the stiffeners of a plate girder take up the compressive stresses resulting from the shear forces and the web plate resists buckling due to shear forces through tensile stresses forming along the plate diagonal (Wagner, 1931; Wilson, 1886). Among the models of the tension field theory the one which gained the most widespread acceptance in the research community is the model developed by (Basler, 1961) which is also included in (AISC, 2016).

Fig. 3 illustrates the concept of tension field on a web plate surrounded by flanges and transverse stiffeners. Here τ_u denotes the ultimate post buckling shear strength. The plate in Fig. 3 represents the unstiffened part of a plate girder web and it is assumed to be simply supported (SS) at all edges. An equation that predicts τ_u was first developed by (Basler, 1961) and later modified by (Fujii, 1968; Gaylord, 1963; Selberg, 1974) as given in Eq. (8).

$$\tau_u = \tau_{cr} + \sigma_y\left(1 - \frac{\tau_{cr}}{\tau_y}\right)\left(\frac{sin\theta_d}{2+cos\theta_d}\right) \quad (8)$$

Once τ_u is known, the ultimate postbuckling shear force V_u can be obtained through $V_u = \tau_u\, Dt_w$ where D is the depth of the plate as seen in Fig. 3 and t_w is the thickness of the web plate.

In Eq. (8), θ_d is the angle of the web panel diagonal. σ_y is the yield strength of the plate material from which the shear yield strength τ_y can be obtained through $\tau_y = 0.6\sigma_y$. The elastic shear buckling strength τ_{cr} in Eq. (8) is calculated through Eq. (9) (Timoshenko, 2009).

$$\tau_{cr} = \frac{k\pi^2 E}{12(1-\nu^2)\left(\frac{D}{t_w}\right)^2} \quad (9)$$

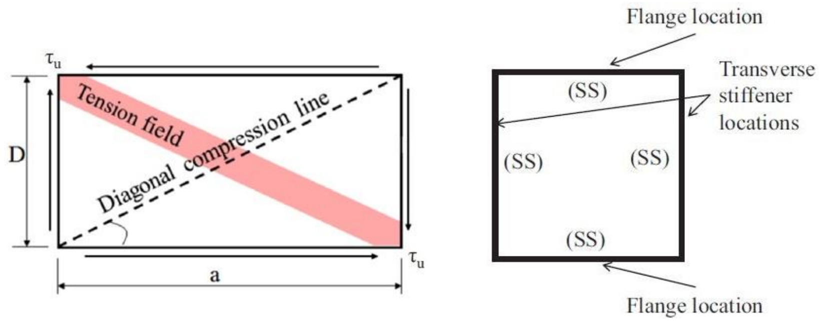

Fig. 3. Schematic of Basler's tension field theory (Glassman and Moreyra Garlock, 2016).

In Eq. (9), E is the modulus of elasticity, ν is the Poisson's ratio, $D\ /\ t_w$ is the slenderness ratio and k is the shear buckling coefficient which can be calculated as a function of $a\ /\ D$(span-to-depth ratio) and the assumed boundary conditions of the web plate. The equation for k is given in Eq. (10) for simply supported boundary conditions where a is the clear distance between transverse stiffeners (AISC, 2016).

$$k = 5.0 + \frac{5.0}{(a/D)^2} \tag{10}$$

2. Methodology and Results

Even though the lateral-torsional buckling load and the ultimate shear strength of plate girders can be predicted for given profile dimensions reliably, the dimensioning of a profile for the best combination of economic design and structural performance often necessitates the application of optimization procedures (Bekdaş and Nigdeli, 2013). Due to the computational overhead that many optimization techniques entail this dimensioning is often done in practice through trial and error. In this process it is crucial to have an intuition about the effect of slenderness of the web and the flange on the structural performance. However, from Eqs. (5) to (8) the variation of M_{cr} and τ_u with respect to the slenderness values of the web and the flange cannot be easily discerned. In order to gain a better understanding of the relationship between slenderness and structural performance, a parametric study of the slenderness is carried out in this study. As the base model the girder plate tested by (Mamazizi et al., 2013) is used. The material properties and dimensions of this model are given in Table 1. In this table f_{yf} and f_{yw} denote the yield strengths of the flange and the web respectively. The parametric study resulted in 917826 different combinations of the web thickness, web plate depth, flange thickness, flange width and corresponding critical LTB moment and ultimate shear strength values. The analysis was carried out using a custom program developed in the Python programming language. The parameter ranges of the parametric study are selected in such a way that they are in the same order of magnitude as the plate girder dimensions listed in Table 1. The upper and lower bounds of these parameters are determined as listed in Table 2 considering fabrication constraints such that web plate thicknesses less than 2mm are not included in the parametric study.

Table 1. The geometric and material properties used in the experimental study (Mamazizi et al., 2013).

$t_f\ [mm]$	$b_f\ [mm]$	$t_w\ [mm]$	$D\ [mm]$	$a\ [mm]$	$f_{yf}\ [MPa]$	$f_{yw}\ [MPa]$
15	250	2	800	750	235	210

Table 2. Parameter ranges and increment sizes used in the parametric study.

	$t_f\ [mm]$	$t_w\ [mm]$	$b_f\ [mm]$	$D\ [mm]$
Upper bound	30	10	350	1200
Lower bound	5	2	150	400
Increment size	1	0.2	10	20

2.1. Effects of slenderness on the lateral-torsional buckling load

Even though the critical buckling moment can be computed using Eqs. (5) to (7), it is hard for the practicing engineer to develop an intuition about how the flange and web slenderness ratios effect the lateral-torsional buckling moment from these equations. To clarify these effects, M_{cr} has been calculated using Eqs. (5) to (7) and its variation has been plotted with respect to λ_f. Fig. 4 shows this variation for five different values of λ_w. Fig. 4 shows that increasing the flange slenderness has a favorable effect on the critical moment for all values of the web slenderness. Furthermore, this favorable effect becomes more pronounced as the web slenderness increases.

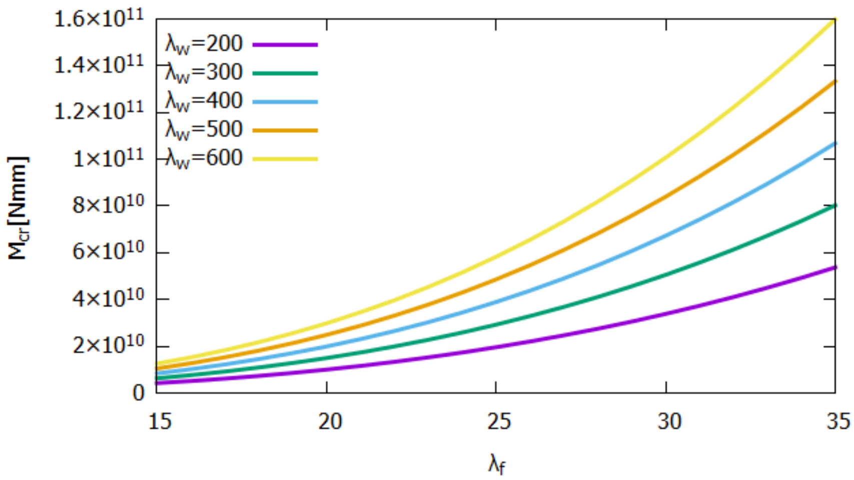

Fig. 4. Variation of the critical bending moment with respect to flange slenderness.

Fig. 5 shows that a similar variation of the critical moment can also be observed with respect to the web slenderness. Also, the rate of increase of M_{cr} seems to be proportional to the flange slenderness. The major difference between the two variations in Fig. 4 and Fig. 5 is that M_{cr} has a nonlinear variation with respect to λ_f and a linear variation with respect to λ_w. A combination of the effects of λ_f and λ_w on M_{cr} can be seen in Fig. 6.

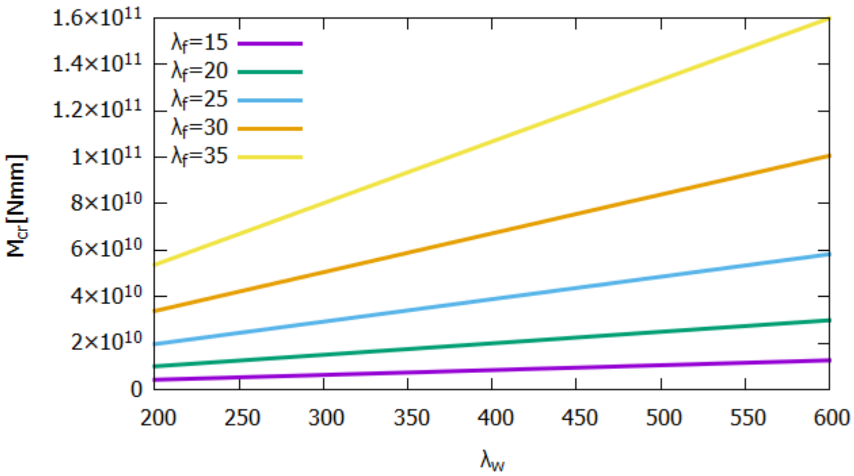

Fig. 5. Variation of the critical bending moment with respect to web slenderness.

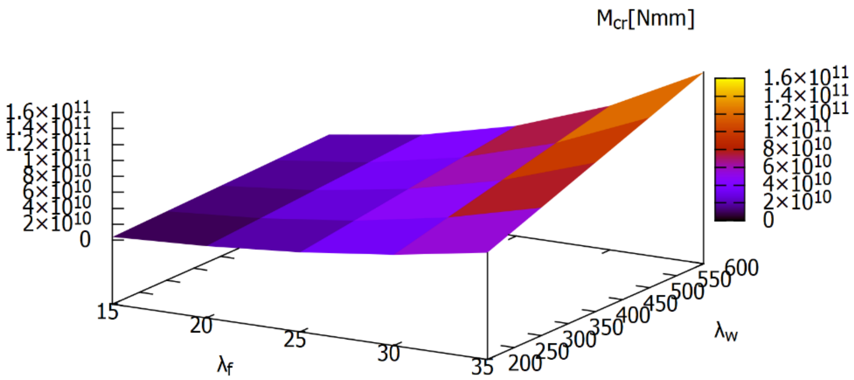

Fig. 6. Variation of the critical buckling moment with respect to the flange and web slenderness ratios.

2.2. Effects of slenderness on the ultimate shear strength

From Eqs.(8) to (10) the parameters that τ_u depends on can be summarized in function format as in Eq. (11).

$$\tau_u = f(\theta_d, \sigma_y, 4\,\lambda_w, \nu, E) \tag{11}$$

From these five parameters σ_y, ν and E are material constants and the effect of the remaining two design variables are investigated in this study. According to Eqs.

(8) and (9) the flange slenderness has no effect on the ultimate shear strength. Therefore, the variation of τ_u with respect to the web slenderness is the same for any flange slenderness. This variation can be seen in Fig. 7 for $\lambda_f = 15$ while this variation is independent of the value that λ_f takes. From Fig. 7 it can be observed that the effect of web slenderness on τ_u is directly related to the web plate thickness. For relatively small values of t_w such as 2mm or 3mm, increasing the web slenderness seems to have a favorable effect on the ultimate shear strength. On the other hand, for greater values of the web plate thickness, increasing the web plate slenderness has the opposite effect on τ_u.

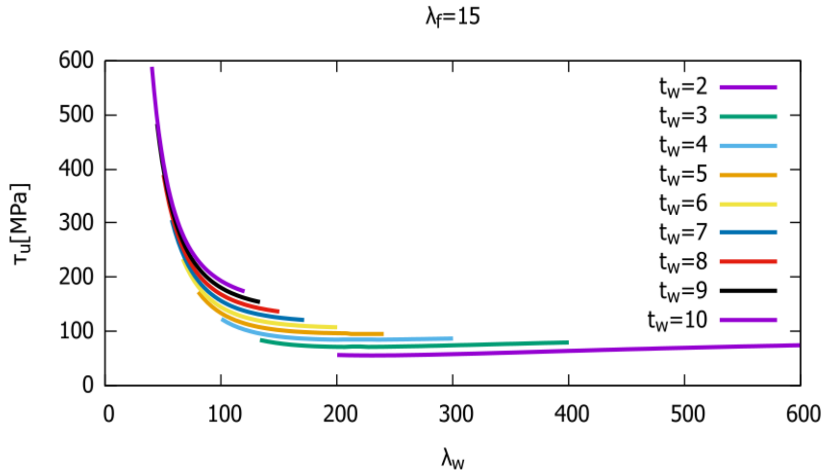

Fig. 7. Variation of τ_u with respect to web plate slenderness.

2.3. Effects of plate diagonal angle on the ultimate shear strength

The effect of the angle θ_d that appears in Eq. (8) is visualized in Figs. 8 and 9. Since this angle is a function of the span-to-depth ratio (a/D), understanding the dependence of τ_u on θ_d also gives insights about the relationship between τ_u and the a/D ratio. According to (Williams, 2011), the a/D ratio is recommended to be in the range from 10 to 12 which corresponds to a range from 4.76° to 5.71° for θ_d. The variation of τ_u for the recommended range of θ_d is plotted in Fig. 8 for five different web plate thicknesses. For all of these t_w values increasing θ_d seems to have an adverse effect on τ_u. Increasing the value of t_w leads to greater ultimate shear stresses as expected.

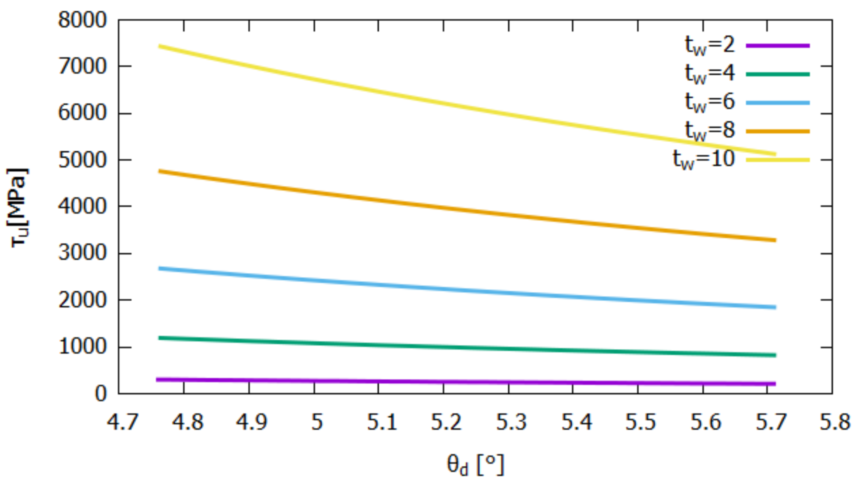

Fig. 8. The variation of τ_u with respect to θ_d in the recommended range for θ_d.

In order to better understand the variation of τ_u for a wider range of θ_d, Figure 9 was plotted for θ_d values up to 45°. From Fig. 9 it can be seen that the effect of θ_d on τ_u is most significant for angles less than about 20°. Overall increasing θ_d has an adverse effect on the ultimate shear strength even though this effect becomes negligible for angles close to 45°.

3. Conclusions

The (AISC, 2016) and (CSA, 2009) codes include equations for the prediction of the critical buckling load and the ultimate shear stress of plate girders. However due to the formulation of these equations it is not easy to identify the effect of slenderness and span-to-depth ratio on the structural performance even though the information about the effect of these parameters is indirectly included in these equations. The aim of this study is to clarify the effect of these geometric variables on the structural performance. To this end, a parametric study was carried out that calculated the critical buckling load and the ultimate shear stress for a wide range of web and flange slenderness as well as span-to-depth ratio values. As a result, increasing both the flange and the web slenderness ratios were found to have a favorable effect on the critical buckling moment. The effect of the web plate slenderness on the ultimate shear stress was also investigated. The visualization of the results showed that the effect of the web plate slenderness on the ultimate shear stress depends on the web plate thickness. For small web plate thicknesses an increase in web plate slenderness is observed to be favorable whereas for larger thicknesses increasing the web plate slenderness adversely affects the ultimate shear strength. Finally, the effect of the plate diagonal angle on the ultimate shear stress was visualized and this variable was found to have a significant effect on the structural performance only for angle values less than about 25°. Also, this variable is observed to have a greater effect on the structural performance for larger web plate thicknesses.

This study was concerned with the sensitivity of the structural performance to changing cross sectional properties of plate girders related to slenderness and span-to-depth ratio. Knowing the effect these parameters have on the structural performance can be a great advantage for practicing design engineers. In the absence of clearly documented information about the relationship between geometric properties such as slenderness and the structural performance more sophisticated optimization techniques need to be used in the structural dimensioning process. Further research in this field can be carried out to investigate the effect of material properties such as yield stress and elasticity modulus as well as other cross-sectional properties such as the warping constant and the St. Venant torsion constant on the structural performance.

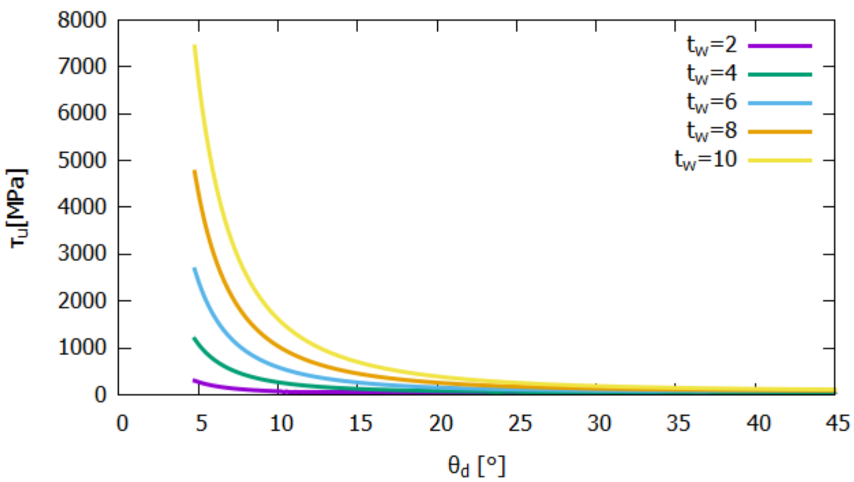

Fig. 9. The variation of τ_u with respect to θ_d.

Appendix

Torsional section properties of a doubly symmetric I-section (Kulak and Grondin, 2002).

St. Venant's torsion constant:

$$J = \frac{2b_f t_f^3 + (D+t_f)t_w^3}{3}$$

Warping constant:

$$C_w = \frac{(D+t_f)^2 b_f^3 t_f}{24}$$

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