Challenge Journal of Structural Mechanics

**Research Article**

# The effect of dome properties on design of axial symmetric reinforced concrete cylindrical walls

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**ABSTRACT**

The usage of computer software in civil engineering has expanded in last decades. Many general-purpose and special-purpose commercial programs perform a very important function, especially at the design stage. In this study, a computer program is introduced for the analysis and design of the axial symmetric cylindrical wall considering the dome effects. Analysis processes are carried out according to Flexibility theory with long wall assumption and during the reinforced concrete (RC) design of the wall, ACI 318-Building code requirements for structural concrete are considered. In numerical investigation, the effects of the dome properties (thickness and height) on the analysis and design of the wall are investigated by performing a totally 72 case analyzes. These cases include different support condition at bottom of the wall, wall heights, dome thicknesses and heights. According to analysis results, it is concluded that effects of dome thickness and heights on the wall on the wall are very limited.

**ARTICLE INFO**

*Article history:*
Received 26 August 2020
Revised 9 October 2020
Accepted 23 October 2020

*Keywords:*
Cylindrical wall
Dome effect
Flexibility theory
Reinforced concrete design
ACI 318

## 1. Introduction

Shells are defined as structural members of which thickness is very small compared to other dimensions. These are several practical applications of these structural members such as places of worship, dams, water tanks etc.

When the equations used in the analysis of these structural elements are derived by writing equilibrium equations on a small part of the shell, it is seen that these equations contain four integration constants depending on the boundary conditions. These integration constants need to be determined to solve the equations. For this reason, many scientific studies have been conducted on the solutions of these equations until today. One of the methods used in these alternative solutions is the theory of beams on elastic foundation. The situation that enables this theory to be used in the cylindrical wall, is the similarity of the differential equations of the wall and beams. This situation has led to a rapid development in both scientific areas, as the analysis methods developed for the wall and can be used mutually. In this section, some important studies in the analysis methods of both structural members are summarized below.

A first model for the beams on elastic foundation was proposed by Winkler in 1867. According to this model, foundation is represented by infinite number of independent springs are used. Between the reaction forces on the soil and the displacement of the beams at that point, there is a ratio which is equal to the spring coefficient of the elastic springs. Also, it is assumed that the members that change shape under the applied load will return to their original state if the load is removed (Wang et al., 2005; Karaşin and Gülkan, 2008). Using this soil model, Winkler conducted stress analysis on railways. In the analysis, the rails were defined with beams and the displacements were obtained by solving the differential equation.

The integral coefficients in the displacement equation of the beam change with discontinuities in load distribution on the beam. This situation makes the determination of integral constants difficult. For that reason, various methods have been developed to overcome this problem. Initial parameter method developed by Umansky (1933) and the superposition method developed by Hetenyi (1936) are well known methods in scientific studies.

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ISSN: 2149-8024 / DOI: https://doi.org/10.20528/cjsmec.2021.01.002

The book written by Hetenyi in 1946 is one of the most important sources on this subject. Some other important books on this subject were written by Henry (1956), Jones (1997), Melerski (2000). Information on many solution methods can be found in these books. Apart from these, there are many methods published in various sources regarding the solution of the beams on elastic ground (Vianello, 1898; Cross, 1930; Levinton, 1947; Penzien, 1960; Miranda and Nair, 1966; Beaufait, 1977; Ting, 1982; Yankelevsky et al., 1989).

In the book called thin sell structures written by Billington in 1965, beam-wall similarity in the solution of the differential equation and flexibility theory in the analysis were used. Except this, the studies done by Bekdaş (2015; 2018; 2019) can be also given as examples for the analysis of the axial symmetric cylindrical walls.

In this study, a computer program has been developed for the design and analysis of the axial symmetric cylindrical wall considering the dome effects. Flexibility theory was used in the wall analyzes performed according to the long wall assumption. During the reinforced concrete design, rules of ACI 318 regulation are used. As numerical examples, totally 72 case analyses are done in order to observe effect of dome on design and analysis of the wall. In these cases, different support condition at the bottom and dome properties at the upper section for different wall height are investigated.

## 2. Analysis and Design Process

Analysis and design process of the axial symmetric cylindrical walls can be summarized with two main stages. First stage is entering the data of structure. These data contain geometrical and sectional properties for wall and dome (thickness, height, radius, etc.), material properties (elasticity modulus, Poisson's ratio, etc), support conditions of the bottom section wall and unit material cost for concrete and steel. In the second stage wall with dome calculations are performed. These calculations are done in three steps. These steps are:

- Calculation of the wall displacements and internal forces;
- RC design of the wall in accordance with ACI 318 requirements;
- Determination of total wall material cost concrete and steel rebar.

This process is also given in the flowchart (Fig. 1).

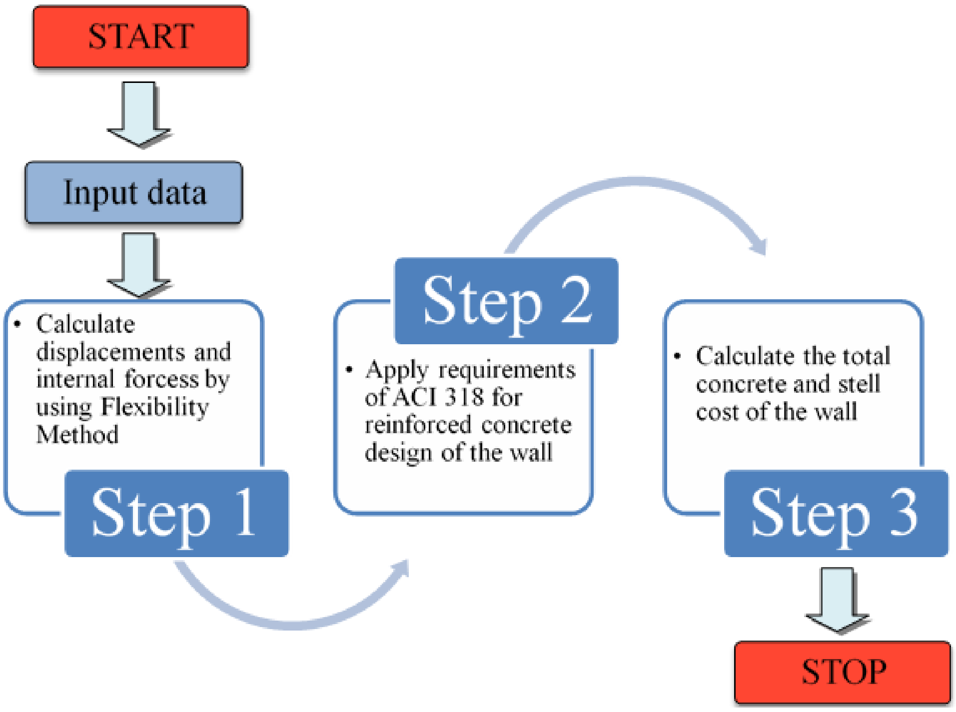


**Fig. 1.** Flowchart of the developed program.

### *2.1. Flexibility method for the analysis of the cylindrical walls with dome*

In this section, the application of the flexibility method to the axial symmetric cylindrical walls with dome will be introduced. Accordingly, unknown forces at the bottom and the upper section of the wall are calculated on the equivalent isostatic system. During calculations of the unknown forces, at the bottom section only wall with fixed (or simple) support, at the upper section wall and dome connection are taken consideration. The wall and circular dome are directly connected without using a stiffness ring beam in this study. In that case, the connection is provided as a continuous connection and the analysis were done according to this assumption. In future studies, the connection with using a stiffness ring beam can be also investigated.

In Fig. 2, fixed supported wall under the liquid loads and the equivalent isostatic demonstration of the wall is given. In the figure, the triangle distributed loads, $X_1$ and $X_2$ show the liquid loads, lateral and rotational redundant forces respectively.

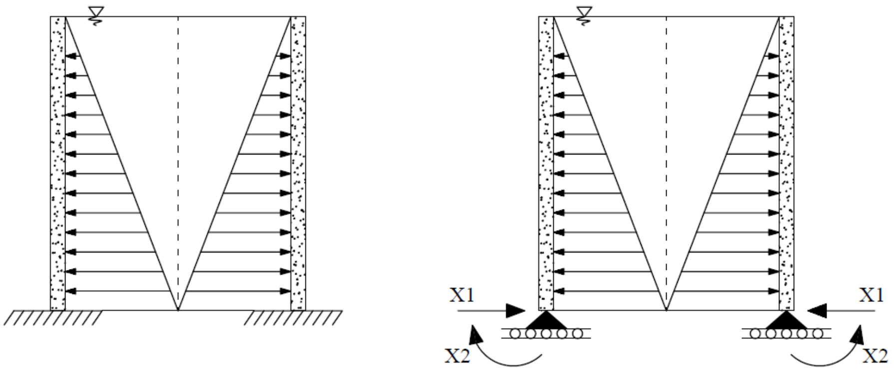


**Fig. 2.** Fixed supported cylindrical wall and equivalent isostatic system of the wall.

The redundant forces can be easily found by compatibility equations of system. For the fixed supported cylindrical walls, since there is no relative displacement at the support point, compatibility equations can be written as in Eq. (1).

$$D_{10} + F_{11}X_1 + F_{12}X_2 = 0 \quad \text{and}$$

$$D_{20} + F_{22}X_2 + F_{21}X_2 = 0 \tag{1}$$

In Eq. (1), $D_{10}$ and $D_{20}$ represent the displacements due to external loads in the freedoms corresponding to $X_1$ and $X_2$, respectively. $F_{11}$, $F_{12}$, $F_{21}$ and $F_{22}$ show flexibility coefficients. These flexibility terms indicate the displacement in the freedom indicated by the first subscript in case of a unit force applied to the freedom indicated by the second subscript.

The lateral ($D_{10}$) and rotational displacements ($D_{20}$) can be found by;

$$D_{10} = -\frac{\gamma r^2 H}{Eh} \quad and \quad D_{20} = \frac{\gamma r^2}{Eh} \tag{2}$$

where $\gamma$, $r$, $H$, $E$ and $h$ are density of liquid, radius of the wall, wall height, elasticity modulus and wall thickness respectively.

Flexibility coefficients can be calculated as in Eq. (3).

$$F_{11} = \frac{1}{2\beta^3 D} \quad , \quad F_{22} = \frac{1}{\beta D} \quad and \quad F_{12} = F_{21} = -\frac{1}{2\beta^2 D} \tag{3}$$

In Eq. (3), $\beta$ and $D$ are parameters (Eq. 4), including rigidity and geometrical properties, and flexural rigidity (Eq. 5) respectively.

$$\beta = \sqrt[4]{\frac{Eh}{4r^2 D}} = \sqrt[4]{\frac{3(1-\nu^2)}{r^2 h^2}} \tag{4}$$

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{5}$$

In Eqs. (4-5), $\nu$ represent Possion's ratio. If Eqs. (2-5) are implemented in Eq. (1), redundant forces will be;

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = F^{-1} \begin{bmatrix} -\frac{\gamma \cdot r^2 \cdot H}{E \cdot h} \\ \frac{\gamma \cdot r^2}{E \cdot h} \end{bmatrix} \tag{6}$$

As mentioned before for the calculations at the upper section, wall and dome are considered together. The displacement and flexibility terms for the dome written as;

$$D_{10d} = \frac{r_d^2 q}{E_d h_d}\left(\frac{1+\nu_d}{1+\cos\alpha} - \cos\alpha\right)\sin\alpha \quad and$$

$$D_{20d} = \frac{r_d q}{E_d h_d}(2+\nu_d)\sin\alpha \tag{7}$$

$$F_{11d} = \frac{2 r_d \lambda_d \sin^2\alpha}{E_d h_d} \quad , \quad F_{12d} = F_{21d} = \frac{2\lambda_d^2 \sin\alpha}{E_d h_d} \quad and$$

$$F_{22d} = \frac{4\lambda_d^3}{E_d h_d r_d} \tag{8}$$

where the terms represent $r_d$, radius; $\nu_d$, Poisson ratio; $\alpha$, starting angle; $E_d$, modulus of elasticity; $h_d$, thickness of the dome; $\lambda_d$, a constant parameter including rigidity, geometrical properties; and $q$, distributed load on the dome.

The wall terms previously calculated for the bottom section is also valid for upper section. After by superposing of the wall and dome terms, the redundant forces can be calculated;

$$\begin{bmatrix} X_3 \\ X_4 \end{bmatrix} = F_s^{-1} \begin{bmatrix} D_{10s} \\ D_{20s} \end{bmatrix} \tag{9}$$

For further detailed information can be found in reference book of Billington (1965).

### *2.2. Reinforced concrete design of the cylindrical wall*

In this section, design constants of the problem are summarized. According to the regulation of ACI 318-Building code requirements for structural concrete, these constraints are listed in the Table 1. As seen from the table, mainly these requirements are related to sections capacities and limits for area and orientation of reinforcement design.

## 3. Numerical Investigation

In this section, several numerical cases including different wall support condition and wall height, dome thickness and height are presented in order to investigate the effect of dome properties on cylindrical wall analyses and design. Totally 72 cases are considered. In these cases, fixed and simple support at the bottom of the wall, 5-7 m for wall height, 2.5 m and 4.5 m dome heights and 0.2-0.3m dome thickness (0.02 m increment) are used. The other design parameters of the problem are presented in the Table 2.

**Table 1.** Design constraints.

| Description | Constraints |
|---|---|
| Flexural strength capacity, $M_d$ | $M_d \geq M_u$ |
| Shear strength capacity, $V_d$ | $V_d \geq V_u$ |
| Minimum steel ratio, $\rho_{min}$ | $A_s \geq A_{smin}$ |
| Maximum steel bars spacing, $S_{max}$ | $S \leq S_{max}$ |
| Minimum steel bars spacing, $S_{min}$ | $S \geq S_{min}$ |
| Minimum concrete cover, $c_{cmin}$ | $c_{cmin} \geq$ 40 mm |

**Table 2.** Design parameters for cases.

| Definition | Value |
|---|---|
| Radius of wall, $R$ (m) | 8 |
| Height of the wall, $H$ (m) | 5-7 |
| Yield strength of steel, $f_y$ (MPa) | 420 |
| Concrete cover, $c_c$ (mm) | 50 |
| Compressive strength of concrete, $f'_c$ (MPa) | 30 |
| Elasticity modulus of concrete, $E_c$ (MPa) | $4700(f'_c)^{1/2}$ |
| Poisson ratio of concrete, $\nu$ | 0.2 |
| Density of liquid, $\gamma$ (kN/m$^3$) | 7 |
| Minimum reinforcement ratio, $\rho_{min}$ | 0.008 |
| Unit concrete cost, $C_c$ (TL/m$^3$) | 200 |
| Unit steel cost, $C_s$, TL/ton | 5000 |

### *3.1. Simple support cases*

This section presents the effects of the thickness ($h_d$) and height ($H_d$) of the dome on longitudinal moments and design for the simple supported cylindrical wall. For this purpose, the analyses of 36 different cases are conducted. An iterative way is followed during determining the wall thicknesses used in the analyzes. The minimum wall thickness, which provides all design constraints, is used in the design by repeating the analyzes for each wall height ($H$). In this way, it is aimed to find more economical results in terms of cost. Accordingly, the thickness values for the heights of 5m, 6m and 7m were obtained as 0.25, 0.3 and 0.35 m, respectively.

In Fig. 3, each graph corresponds the longitudinal moment distribution according to six different dome thicknesses for a constant wall and dome height.

When each wall height cases are evaluated within itself, by considering different dome thicknesses it is seen that the differences between the maximum moment on the wall and the moment at the junction point of the wall-dome are relatively small. At points with maximum moments it is understood that this difference is below 0.07% and this difference also does not change the wall design as well as material cost of the wall. A similar situation is valid for the height of the dome. The maximum difference depending on the height of the dome was found to be 0.22%.

Therefore, it is concluded that the effect of both dome thickness and height on the analysis and design is quite limited for the analyzed cases.

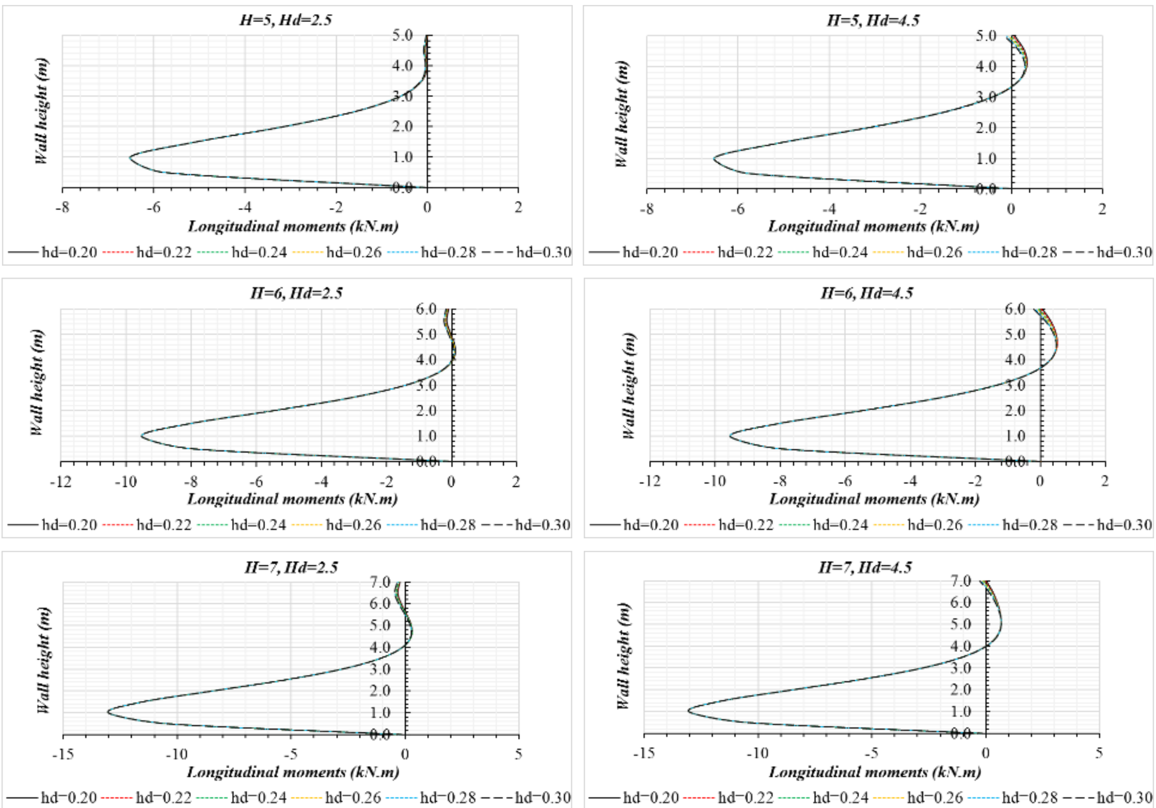


**Fig. 3.** Simple supported wall height-longitudinal moment graphs for different cases.

### 3.2. Fixed support cases

The same investigations are also done for fixed supported cylindrical wall. In this section, the results of these investigations are presented. Similar 36 cases are handled for fixed supported wall. According to the iterative process for obtaining minimum thickness, the wall thickness values are found 0.3, 0.35 and 0.45 m for 5m, 6m and 7m wall heights, respectively. In Fig. 4, longitudinal moment along wall height can be seen. Each graph corresponds six different dome thicknesses for a constant wall and dome height.

The analysis results show that the effect of dome thickness and height on longitudinal moment is quite limited, as in simple supported cases. For that reason, the wall design as well as material cost of the wall are same for the cases presented in each graph. When each plot is evaluated separately, it is concluded that the maximum differences are below 0.01% and 0.28% for the moments in fixed support, maximum moments along the wall respectively. If a similar comparison is done according to the dome height, the differences are seen to be 0.11% and 0.37%.

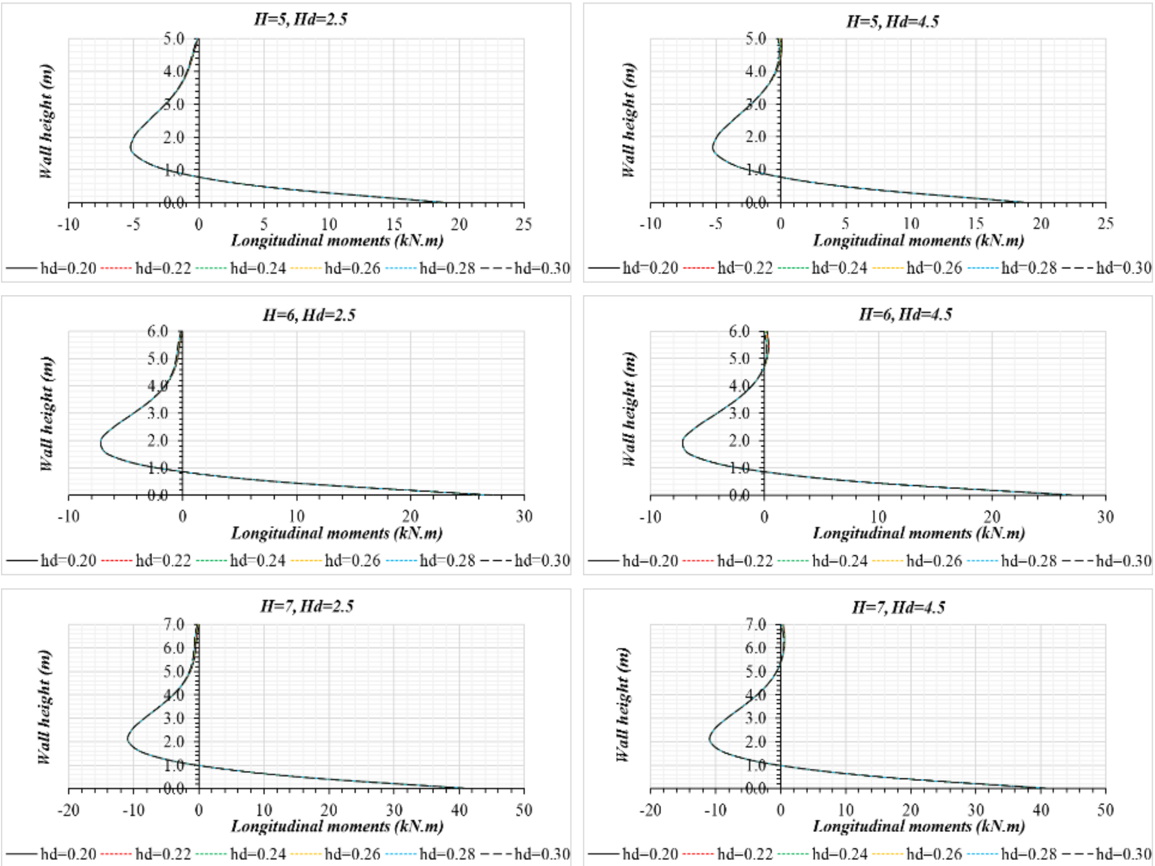


**Fig. 4.** Fixed supported wall height-longitudinal moment graphs for different cases.

## 4. Conclusions

In this study, a computer program has been developed to perform the analysis and design of axial symmetrical walls with a dome. In the analyzes, flexibility theory was considered, and reinforced concrete design was done according to ACI 318-Building code requirements for structural concrete. Total of 72 cases including different support conditions (fixed and simple) at the bottom of the wall, wall heights, dome heights and thicknesses were considered to investigate effect of dome properties on the longitudinal moment distribution and design of the wall.

For a constant wall height, wall thicknesses were determined according to the minimum value that provides design constraints by using 0.5 m increments. According to analyzes results, the effects of dome thickness and heights are quite limited in the simple support cases. When this effect is defined by the maximum moments on the wall for a specific wall and dome height, differences are below %0.07 and %0.22 for different dome thicknesses and heights respectively. When the similar evaluations were done for fixed support cases, it was seen that the maximum differences between the moments were found to be 0.28% and 37% for dome thickness and height, respectively. It was understood that these differences did not have any effect on the cost of the wall design for the solved 36 cases.

For future studies in order to verify these results, analyzes can be done for different loading conditions and dimensions for wall and dome). In addition, by adding an optimization module to the program, the lowest cost designs can be obtained. Scientific data to be obtained from these studies may guide engineers in practical applications as well as in design.

## Publication Note

This research has previously been presented at the 6th International Conference on Harmony Search, Soft Computing and Applications (ICHSA 2020) held in İstanbul, Turkey, on July 16-17, 2020. Extended version of the research has been submitted to Challenge Journal of Structural Mechanics and has been peer-reviewed prior to the publication.

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