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Statistical investigation of the effect of different damage conditions on the modal frequency value of a steel beam

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ABSTRACT

This study aimed to parametrically investigate the changes in modal frequency values on a steel beam caused by specified damaged schemes. In this context, the ANSYS Workbench software program was used to create a steel profile's finite-element model. A cantilever steel beam profile is created with a 60x60 mm cross-section and 3m length utilizing single-sided fixed support. In the finite-element model, the crack depth, width, and distance to the support were parametrically assigned as the damaged scheme to the steel profile. To investigate the effects of those damages on the modal frequency values of the steel profile, first of all, the modal frequency values for undamaged cases corresponding to the first ten-mode shapes were obtained. Then, the specified crack properties were determined parametrically, and the changes in frequency values for damaged cases were examined. In addition, a comparative evaluation of the effect of crack properties on the natural frequency of the steel element was performed by utilizing response surface and six sigma analysis. The analysis results demonstrated that specified crack schemes have different effects on different modal natural frequencies. The applied response surface and six sigma analysis provided important statistical data on the modal natural frequency values of the steel beam.

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1. Introduction

The methods of assessing and detecting structural damage by examining the dynamic behavior of structures were started in 1980 when fast Fourier transforms could be made with software programs. In general, this method has been used in mechanical engineering, fuel industry companies, companies that build spaceships, and civil engineering. In this direction, many studies have been carried out by these groups on the early detection of damage to structures. Studies have revealed different approaches, test methods, and analysis techniques. It is not an effective and correct approach to use a single method in all situations that may exist in all types of buildings (Montalvao et al. 2006; Choi et al. 2005).

Significant progress has been made in the determination of damage effects in various subjects in mechanical engineering. The greatest success in damage assessment has been achieved by studies on marine structures by the fuel industry between 1970 and 1980. Although significant progress has been made in making damage assessments based on changes in dynamic behavior, the application of these techniques in real structures has been very difficult and limited. The spaceship building industry achieved significant success in dynamic-based damage assessment studies in the early 1980s.

In parallel with these, civil engineers started to use similar techniques in the early 80s. Health monitoring of buildings has been the primary field of study in bridge assessments and has quickly spread to other building types. In the literature, there are many applications of this method in beam structures (concrete and steel), truss systems, frames, shells, and plates, buildings, bridges, and composite materials (Gounaris and Dimarogonas 1988; Ostachowicz and Kraawczuk 1991; Srinivasan and Kot 1992; Mohammad 1997).

The basis of structural health monitoring is that regional damages in structures cause a stiffness reduction

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in the relevant region. In this respect, updating the stiffness parameters of the finite-element (FE) model by taking into account several different damage conditions provides full and accurate information about the current state of the structure. Rytter (1993) stated the following 4 steps in determining the damage that may occur in an existing structure; 1) Determination of damage to the structure, 2) Evaluating the damage location in the structural geometry, 3) Determining the extent and significance of the damage, 4) Estimating how much time the structure will have to serve. The first three steps of these include studies on estimating the presence, location, and magnitude of damage. Although the first three steps are important, the last step is the most fundamental section of structural health monitoring and is also the most difficult step in terms of implementation.

By using the parameters obtained from the modal analysis techniques that can be applied to the structures, the presence, location, and size of existing damage can be determined. This approach is based on the fact that structural damage affects the entire behavior of the structure (mass, stiffness, or damping) and thus changes the dynamic characteristics of the structure (natural frequency, damping ratio, and mode shape) (Doebling et al. 1998). Therefore, the dynamic properties of any structure that can be obtained periodically play an important role in determining the current state of the structures. Based on the change values in these dynamic parameters that will occur in undamaged and damaged structures, it is possible to estimate the region and size of the structural damage.

Mermertas and Erol (2001) investigated the effect of mass attachment on the transverse vibration characteristics of a cracked cantilever beam by relative changes of the first three natural frequencies. They resulted in the study that the decreasing effects of the cracked beam on natural frequencies were more apparent with added mass to the beam in different cases. Owolabi et al. (2003) investigated the effects of cracks and damages on structural integrity by using two sets of aluminum beams as fixed and simple supports. As a result, it was emphasized that the vibration behavior of beams is very sensitive according to the amount, depth, and region of cracks. Gillich et al. (2012) presented a method to detect, locate and evaluate the damage severity of Euler-Bernoulli beams, based on the changes in the natural frequencies due to damage. The accuracy and reliability of the proposed method were validated by numerous experiments.

Dey et al. (2016) stated that crack parameters were revealed to a significant extent in their study to establish the relationship between the input parameters (crack zone and crack depth ratio) and the output parameter (natural frequencies) to estimate the crack parameters, region and depth ratio. Yendhe et al. (2016) investigated beam vibration behavior both experimentally and with the help of the FEM program ANSYS. Vibration analysis is carried out on several cantilever beams with transverse cracks and various boundary conditions in the current study. Altunışık et al. (2017a) performed a comprehensive study on the automated model updating of a cantilever beam with box cross-section including multiple cracks for damage detection experimentally and numerically. Modal Assurance Criterion and Coordinate Modal Assurance Criterion factors were employed to recognize and locate damaged elements within the beam.

Multiple damage schemes with different ranges of magnitudes such as crack depth, crack width and crack distance to the support point emerge as important parameters for vibration analysis of beams. This vibration analysis is generally carried out by examining the change of natural frequency values on the beam depending on the size and position of the damages. In the performed studies, parametric analyzes were made for one or two different damage cases at critical points for the types of damage that may occur on beam-type structures (Altunışık et al. 2017b). It is also important to investigate the effect of different crack types at different positions on natural frequency values at all points along the beam to ensure the safety and performance of beam-type structural elements. The scope of this study is aimed to investigate the relationships between input and output variables with a probabilistic approach, taking into account the uncertainty of various damages that may occur in structures. In this context, various crack schemes were chosen as random input variables and the effects of a steel beam on the modal natural frequency were evaluated using the response surface (RS) method and six sigma (SS) analysis. The FE model of the steel beam was created by ANSYS Workbench and modal parameters were obtained for the undamaged case. Then, the crack depth, width, and distance from the support point were defined as variable input parameters, and statistical analysis was performed by comparing the modal natural frequency values obtained for damaged cases with those values calculated in the undamaged condition. As a result of the analysis, valuable results were achieved to better understand the effects of damage schemes on the natural modal frequency values depending on the obtained mode shapes.

2. Methodology

2.1. Overview

In this study, the ANSYS DesignXplorer (ANSYS 2013) platform was utilized for the FE model of the steel profile and the SS Analysis for parametric studies to understand the effects of the damage cases on the modal behavior of the selected structure. The SS analysis system consists of a Design of Experiments (DoE) cell, an RS cell, and an SS analyses cell. The DoE cell allows to set up of the input parameters, which SS analyses refer to as uncertainty parameters, and generates the samples for the analysis. RS method is an approximate optimization method that seeks multiple design variables and their responses by employing the minimum number of design samples for the best experimental design, to determine the combination of design variables. It can be accomplished a good accuracy between the FE model-generated data and the experimental data by utilizing the RS method (Cheng et al. 2007; Landman et al. 2007).

2.2. Design of experiments

The design of experiments is a method for determining the location of sample points scientifically. In engineering research, there are several different designs of experiments algorithms or procedures, including star, complete factorial, central composite, and Box-Behnken designs. All of the strategies have one thing in common: they all strive to locate sampling points in the space of random input parameters in the most efficient way or with the fewest sample points possible to get the information they need. Sample points in effective locations not only lower the number of sampling points necessary but also improve the accuracy of the RS calculated from the sampling points' findings. Guo and Zhang (2004) revealed in their study that the central composite design and the D-optimal design give approximately the same results in the RS-based FE update approach. In addition, the efficiency of central composite design, Box-Behnken design, and D-optimal design for damage assessment of structures was evaluated by Umar et al. (2018) and it was stated that the central composite design is more effective in damage detection. The central composite design was chosen as a sampling strategy for three variables in this investigation.

There are different distribution types for input parameters in the DoE namely Triangular, Uniform Normal, Lognormal, Exponential Truncated, Weibull, and Beta. The goal of this study was to employ the Gaussian (Normal) Distribution, which is a fundamental and widely used statistical distribution. It's a term that's frequently used to describe the distribution of measurement data for a wide range of physical phenomena. If the random variable is a linear mixture of two or more other effects, and those effects also follow a Gaussian distribution, the Gaussian distribution is valid. The mean value and standard deviation of the random variable can be provided as input values (ANSYS 2013).

2.3. Response surface method

The genetic aggregation technique was employed to create RS models in this work. The method automates the process of selecting, configuring, and creating the RS. Due to various RS solutions and cross-validation methods, it automatically develops the most appropriate approach for each output and gives more dependable results than previous RS models. After their fitness has been evaluated, the fitted RS models should be utilized to update the FE model. For multi-RSM and sophisticated models, the R2 criterion and the root mean squared error (RMSE) criterion are commonly used (Box and Draper 1987; Fang and Perera 2009). The R-square statistic and the relative mean square error are presented in Eqs. (1) and (2), respectively, are used to assess the accuracy of fitted RS models.

$$R^2 = 1 - \frac{\sum_{j=1}^{N}[y_{RS}(j)-y(j)]^2}{\sum_{j=1}^{N}[y(j)-\bar{y}]^2} \quad (1)$$

$$RMSE = \frac{1}{N*\bar{y}}\sqrt{\sum[y_{RS}(j)-y(j)]^2} \quad (2)$$

The larger the value of R Square is, the more accurate the RS model is. The smaller the value of RSME is, the more accurate the RS model is.

2.4. Six sigma analysis

A technique known as SS analysis is used to assess the impact of unclear input parameters and assumptions on a model. It may be evaluated how much the model's uncertainties affect the analysis outcomes using an SS analyses system. Uncertainty (random quantity) is a parameter whose value is impossible to predict at a specific time or location (if it is time-dependent) (if it is location-dependent). The fundamental goal of an SS analysis is to determine the impact of uncertainties associated with the design's input parameter. Various statistical studies and post-processing analyses are used to attain this goal (ANSYS 2013).

2.4.1. Mean value

The average of a group of observations is measured by the mean. The following is the definition of the mean of a set of observations;

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} y_i \quad (3)$$

2.4.2. Standard deviation

The standard deviation is a measure of how far a group of data deviates from the mean. The following is how a set of observations' standard deviation is calculated;

$$\hat{\sigma} = \sqrt{\frac{1}{(n-1)}\sum_{i=1}^{n}(y_i-\hat{\mu})^2} \quad (4)$$

2.4.3. Sigma level

The inverse cumulative distribution function of a conventional Gaussian distribution in a particular percentile is used to compute the sigma level. To measure the data distribution from the mean, the sigma level is employed in conjunction with the standard deviation.

2.4.4. Skewness

The degree of asymmetry around the mean for a group of observations is measured by skewness. The observations are symmetrical if the distribution of the observations looks the same to the left and right of the mean. Negative skewness denotes a left-skewed distribution of observations. Positive skewness denotes a right-skewed distribution of observations. A collection of observations' skewness is defined as follows;

$$\hat{\gamma} = \frac{n}{(n-1)(n-2)}\sum_{i=1}^{n}\left(\frac{y_i-\hat{\mu}}{\hat{\sigma}}\right)^3 \quad (5)$$

2.4.5. Kurtosis

For a set of observations, kurtosis is a measure of the distribution's relative peak/flatness. When compared to the normal distribution, it's usually a relative comparison. The normal distribution is generally used as a reference point. As shown in Fig. 1, negative kurtosis indicates that the distribution of data is comparatively flat when compared to the normal distribution, whereas positive kurtosis indicates that the observations peaked substantially. As a result, calibration concerning the normal distribution defines the kurtosis of a set of observations as follows;

$$\hat{\kappa} = \left\{\frac{n(n+1)}{(n-1)(n-2)(-3)}\sum_{i=1}^{n}\left(\frac{y_i-\hat{\mu}}{\hat{\sigma}}\right)^4\right\} - \frac{3(n-1)^2}{(n-2)(n-3)} \tag{6}$$

where $\hat{\sigma}$ and $\hat{\mu}$ represent mean and standard deviation, respectively.

2.4.6. Shannon entropy

The term "entropy" was first used in classical thermodynamics. It is a measure of a system's dysfunction. Later, in the realm of information/communication, Claude Shannon extended and adopted the concept of entropy as a measure of uncertainty over the actual content of a message. Entropy is recast as a function of mass density in statistics as follows;

$$H = -\sum P_i lnP_i \tag{7}$$

where P_i represents mass density of a parameter. Entropy is a measure of a parameter's complexity and predictability in the context of statistics and probability. Entropy increases as a parameter becomes more complex and unpredictable, and vice versa.

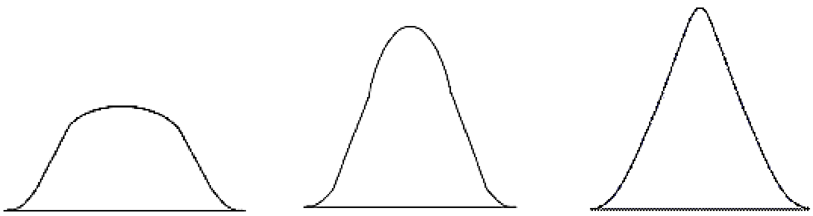


Fig. 1. Different types of kurtosis.

3. Application and Results

This study aimed to parametrically investigate the changes in modal frequency values on a steel profile caused by specified damaged schemes. For this, the ANSYS Workbench software program was performed to create a steel profile's FE model. The steel profile is modeled as a cantilever beam with a 60·60 mm cross-section as represented in Fig. 2 and 3m length utilizing single-sided fixed support. The material properties of the steel profile are E=2·10^{11} N/m^2, ρ= 7850 kg/m^3, and ν=0.3.

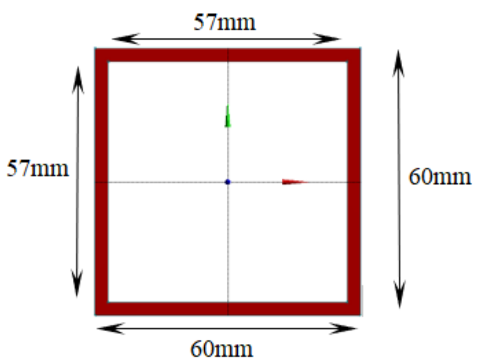
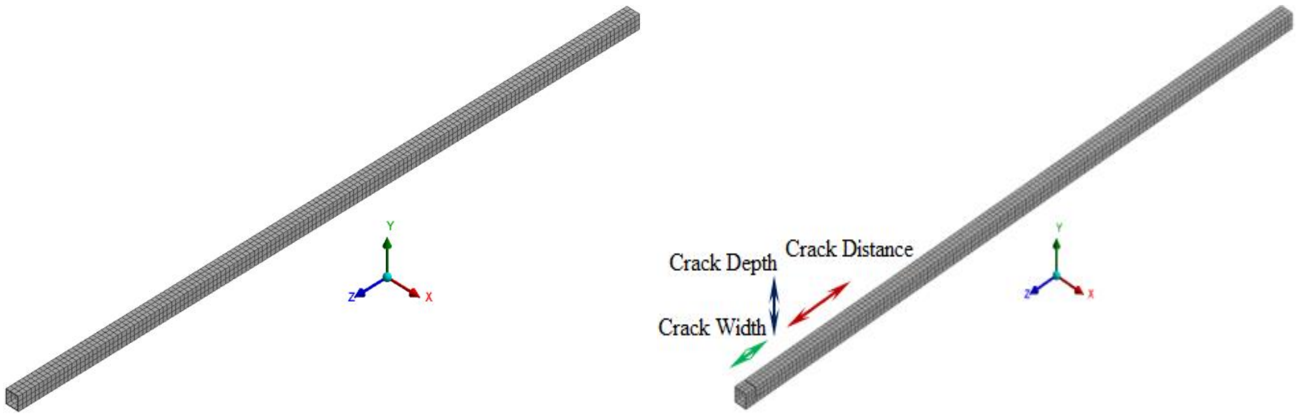


Fig. 2. Cross-section of the steel profile.

3.1. Finite element model

A 3D dimensional FE model of the steel profile as demonstrated in Fig. 3(a) has been created by using the ANSYS Workbench software program to obtain the dynamic characteristics such as natural frequency and mode shapes numerically. The FE model of the steel beam is assumed homogenous and isotropic elastic material. Since the effect of the mesh size of the structure on the calculated modal natural frequency values is important in modal analysis, the mesh convergence study is carried out to establish a satisfactory mesh size in this study. The initial mesh length was chosen as 0.5 m. The mesh size was reduced to such an amount that the corresponding mode shapes did not cause a significant change in the natural frequency values, and thus, the appropriate mesh size for the steel beam model was determined as 0.02m mesh size. The obtained modal natural frequency values of the undamaged steel beam are considered to compare the modal analysis results for damaged cases. In the FE model, the crack depth, width, and distance to the support were parametrically assigned as the damaged scheme to the steel profile as presented in Fig. 3(b). To investigate the effects of those effects on the modal frequency values of the steel profile, first of all, the modal frequency values for undamaged cases corresponding to the first ten-mode shapes were obtained as shown in Fig. 4. The first ten natural frequencies are calculated between 7.31 and 397.83 Hz for the undamaged beam. Also, the mode shapes of the steel beam appear as a bending mode in the x and y direction in all modes except the seventh mode which shows torsional modal behavior. The obtained undamaged case natural frequency values corresponding to ten-mode shapes are presented in Table 1. Also, the specified crack properties were determined parametrically, and the changes in frequency values for damaged cases were examined.



(a) (b)

Fig. 3. FE model of the steel profile: (a) Undamaged; and (b) damaged case.

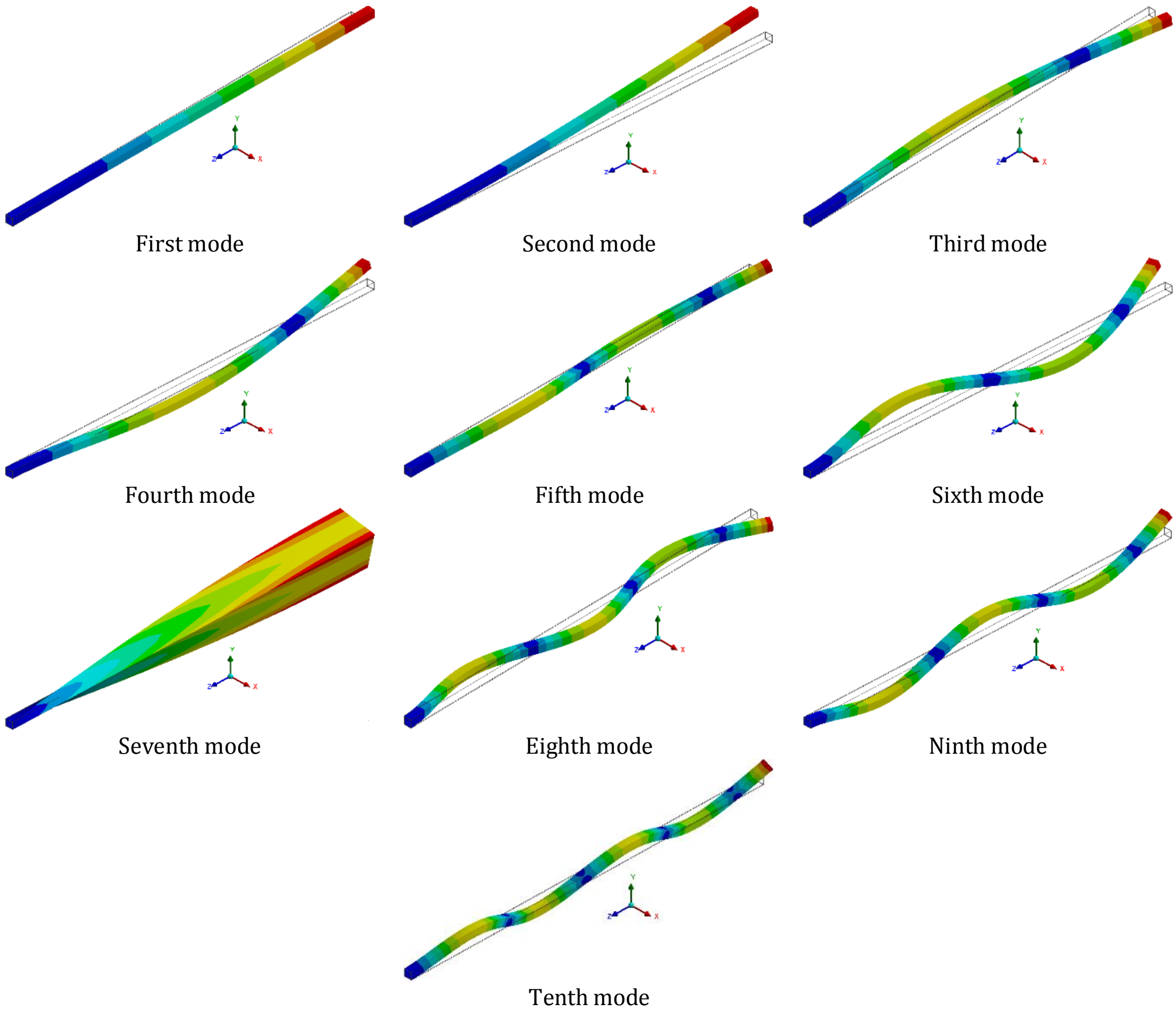


Fig. 4. Mode shapes of the steel beam.

Table 1. Undamaged beam modal frequencies.

Mode number	1	2	3	4	5	6	7	8	9	10
Frequencies (Hz)	7.31	7.31	45.56	45.56	126.45	126.45	229.93	244.64	244.64	397.83

3.2. Creating design of experiments

To achieve satisfactory design space for a good representation of an underlying physical problem and not able to miss capturing an unexpected change in some regions of the design space, an enhanced Face-Centered design type was used (Wagner et al. 2014). For an enhanced design of the experiment, to be able to catch any drastic changes in the design space, a mini-CCD is added to a standard CCD design, where a second alpha value is added and arranged to half the alpha value of the standard CCD. Therefore, in this study, the central composite design is selected as a sampling method (Ren and Chen 2010).

The lower and upper limits of the damage scenario including crack depth, width, and distance, as input parameters of the steel beam model for experimental design, are shown in Table 2. In the experimental design, a total of 29 design points (one center point, 2x (2·3=6 axis point, and 2^3=8 (factorial points)) were evaluated using the enhanced face-centered central composite design approach at these upper and lower limits of the input parameters. The natural frequency values of the corresponding ten-mode shapes were selected as output parameters. In the experimental design calculated min frequency values depending on the upper and lower limits of input varying parameters are represented in Table 2.

Table 2. Upper and lower limits of input parameters and obtained corresponding min natural frequency values.

Input parameters	Parameter name	Lower upper limit (mm)	Mean/standard deviation
P1	Crack depth	1-40	20.5/6.31
P2	Crack width	1-20	10.5/3.075
P3	Crack distance	50-2950	1500/469.22
Output parameters		Min (Hz)	
P5	Mode 1	4.02	
P6	Mode 2	6.58	
P7	Mode 3	33.66	
P8	Mode 4	42.25	
P9	Mode 5	102.30	
P10	Mode 6	117.82	
P11	Mode 7	159.00	
P12	Mode 8	200.68	
P13	Mode 9	231.05	
P14	Mode 10	360.83	

3.3. Generating response surfaces

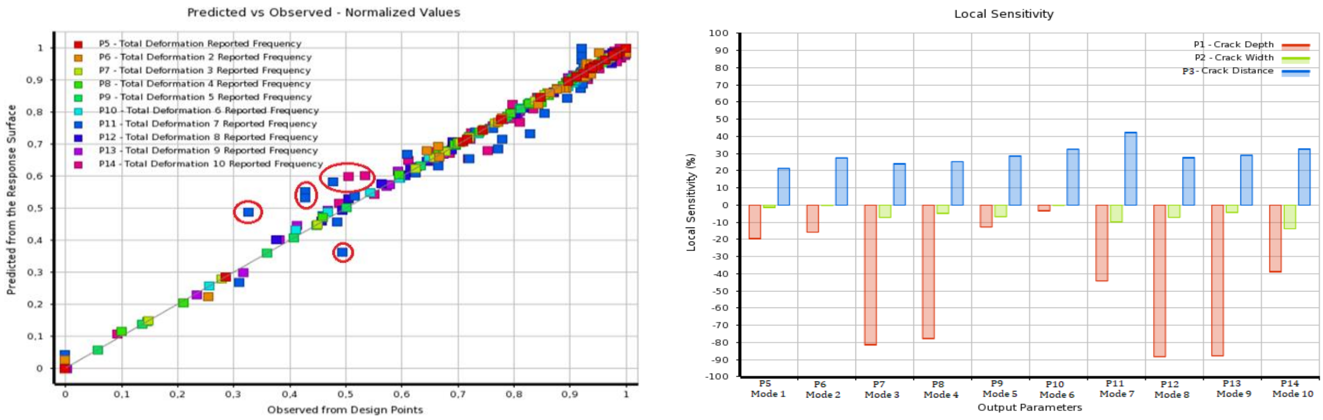
The RS models are created using a genetic aggregation approach. The R2 and RMSE metrics are used to assess the correctness of the RS models. Table 3 shows the R2 and RMSE values obtained for each related mode. It is generally shown that R2 and RMSE values are close to one and zero, respectively, which means that the RS models obtained have a high regression accuracy. Although minor deviations in RMSE values occur only in the seventh and tenth modal frequency values as marked in Fig. 5(a) the relevant values are still within an acceptable range.

Another indicator that shows the accuracy of the generated RS is the predicted-observed graph. This graph demonstrates the predicted values from the RS and the observed values from the design points. This provides a quick understanding of the compatibility of the RS to the experimental design points. The closer the points are to the diagonal line, the more the RS is aligned with the points. Otherwise, it cannot adequately capture the parametric behavior of the RS. Therefore, it is necessary to reconstruct the RS. Fig. 5(a) shows the predicted-observed graph created at this point. All output parameters and output values are normalized in the graph. As can be seen, the values on the RS are in good agreement with the values at the experimental design points.

Table 3: Accuracy check for response surfaces.

Modes	1	2	3	4	5	6	7	8	9	10
R2	1	0.99	1	0.99	1	0.99	0.92	0.99	0.99	0.98
RMSE	4.3E-7	0.013	1.9E-5	0.027	8.2E-6	0.081	5.57	0.46	0.18	1.35

The local sensitivity plots of RS models according to the average value of the input parameters as a response point are shown in Fig. 5(b). In general, crack width seems to have the lowest effect compared to the other two damage conditions, considering all-natural frequency values. The effects of crack depth and distance on natural frequency values vary in each mode. It is observed that the crack distance has the most important effect on the first, second, fifth, and sixth modal natural frequency values, while the crack depth has the most influence on the natural frequency values for the other mode shapes. It can also be seen that there is no significant effect on the fifth and sixth natural frequency values in the case of crack depth compared to those in other modal natural frequency values. The local sensitivity of the crack depth ranges from approximately 3% to 90%, with the greatest sensitivity occurring in the eighth mode shapes. On the other hand, the local sensitivity range of the crack distance is not as variable as the crack depth and it seems to be between 20 and 45%. The mode in which the crack distance is most effective corresponds to the seventh mode which is the torsional mode.



(a) (b)

Fig. 5. (a) Predicted vs. observed points; (b) Sensitivity analysis of input-output parameters.

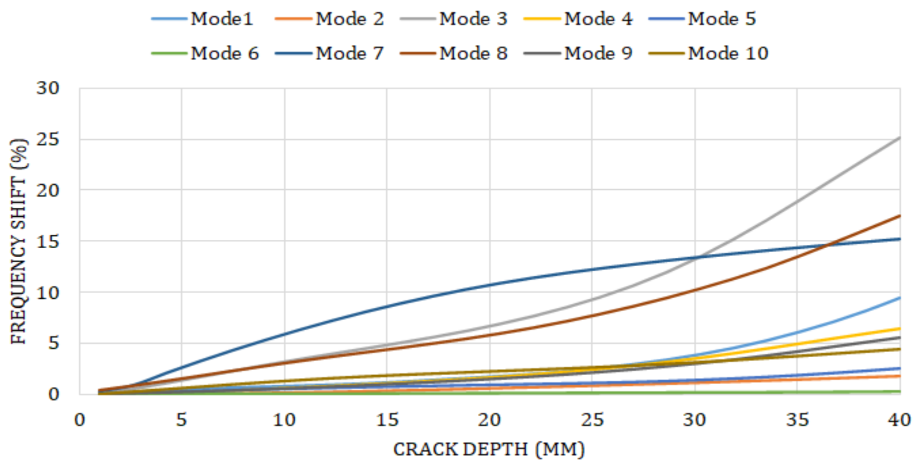
In the result of the RS analysis, another significant view to understanding the effects of damage scenarios on the steel beam is the min search matrix of output parameters as depicted in Table 4. The importance of the min search matrix of the output parameters is valuable in terms of determining the convergence and divergence of the min modal natural frequency values in the lower and upper limit values given in damage scenarios.

The min values for all modal natural frequencies occur when the crack depth converges to the upper limit value. Also, in general, crack width shows the same behavior as crack depth, just the last two natural frequencies are close to the upper limit values. On the other hand, the min values of the first, second, fourth, sixth, seventh, and tenth modal natural frequencies are revealed when the crack distance converges to the lower limit value. Furthermore, in third, eighth, and ninth mode shapes, natural frequencies converge towards the mean value of lower and upper limit values in the crack distance.

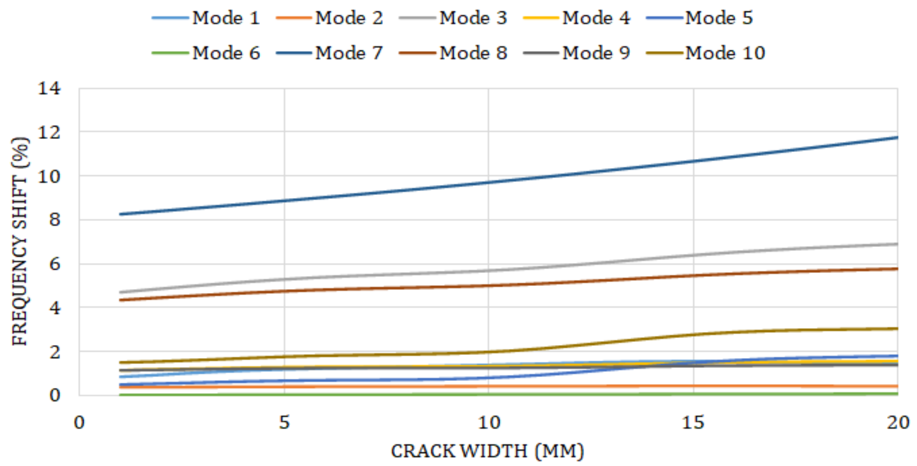
Table 4. Min search of output parameters.

	Input parameters (mm)			Output parameters (Hz)									
Name	P1	P2	P3	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14
Output parameter minimums													
P5 - Mode 1	40.00	20.00	50.00	4.02	6.58	35.87	42.25	109.67	117.82	159.00	222.02	234.56	360.85
P6 - Mode 2	40.00	20.00	50.00	4.02	6.58	35.87	42.25	109.67	117.82	159.00	222.02	234.56	360.85
P7 - Mode 3	40.00	20.00	1499.36	6.32	7.17	33.66	42.62	122.01	125.96	187.61	200.68	231.22	377.97
P8 - Mode 4	40.00	20.00	50.00	4.02	6.58	35.87	42.25	109.67	117.82	159.00	222.02	234.56	360.85
P9 - Mode 5	40.00	20.00	2224.63	6.96	7.28	40.41	44.37	102.30	119.83	206.53	217.23	235.80	381.68
P10 - Mode 6	40.00	20.00	50.00	4.02	6.58	35.87	42.25	109.67	117.82	159.00	222.02	234.56	360.85
P11 - Mode 7	40.00	20.00	50.00	4.02	6.58	35.87	42.25	109.67	117.82	159.00	222.02	234.56	360.85
P12 - Mode 8	40.00	20.00	1498.31	6.32	7.17	33.66	42.62	122.01	125.96	187.58	200.68	231.22	377.97
P13 - Mode 9	40.00	15.15	1497.28	6.52	7.18	33.84	42.61	122.46	126.05	191.92	201.09	231.05	378.82
P14 - Mode 10	40.00	18.73	50.00	4.11	6.59	35.94	42.28	109.72	117.96	160.49	222.06	234.57	360.83

The last investigation in this section is to present the created RS that allows understanding the effect of each input parameter on the output parameters. Each damage scenario is examined for all ten modal natural frequency values corresponding to mode shapes as illustrated in Fig. 6.



(a)



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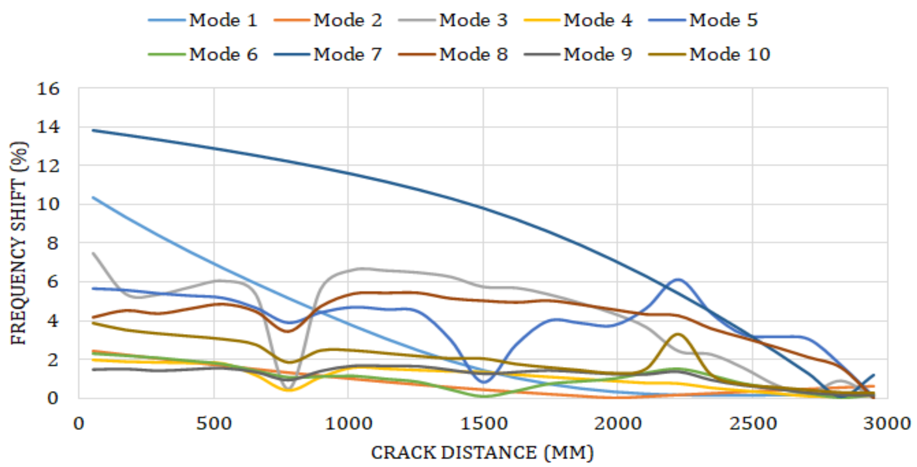

(c)

Fig. 6. Obtained response surfaces of input-output parameters: (a) Crack depth; (b) Crack width; (c) Crack distance.

The increase in crack depth and crack width causes an increase in the all-modal natural frequency values of the steel beam model. The frequency changes for corresponding modes caused by the crack depth and width vary between about 0.25%-25% and 0.02%-3.5%, respectively, depending on the lower and upper limits. In the case of crack depth, the max shift in natural frequency occurs in the third mode with an approximate increase of 25%, and the modal frequency values in torsional mode 7 follow it with a raise of about 15%, while the results show that percentage of increase in modal natural frequency is the minimum at mod 6 with a value of 0.25%. When the percentage increase of crack width on the modal frequency values is examined, it is observed that the lowest percentage increases are 0.015% in the sixth mode and 0.02% in the second mode. On the other hand, it turns out that the highest percentage increase is approximately 4% in the torsional mode, which is the seventh mode shape. In addition, when the crack width is analyzed as percentage increase values, it is revealed that it increases less than other crack parameters. Finally, for the crack distance, it is observed that the modal natural frequency increase values vary in the range of approximately 1.4%-10% in the corresponding mode shapes. It is seen that the percentage increase of the crack distance decreases as it moves away from the support area. The highest percentage increase is observed at the first natural frequency, and the lowest percentage increase is at the ninth natural frequency.

3.4. Six sigma analysis

The Latin Hypercube Sampling (LHS) approach is used by default in the ANSYS Workbench platform for SS assessments. With LHS, points are created at random over the design space in a square grid, but no two points have the same value input parameters. This means that no point in the grid shares a row or column with another point.

The statistical features described in previous sections of the input and output parameters are depicted in Tables 5 and 6, respectively. Although the standard deviations of the lower and upper limit values seem to be a little bit higher amount, especially in the crack distance due to a better understanding of the effects of cracks schemes in the input parameters, the standard deviation values obtained in the output parameters were not that high. Considering the numerical magnitudes of the modal frequency values obtained from the modal analysis, it can be said that the standard deviation values obtained in the output parameters are suitable for statistical analysis. This can be also said for the sigma level and skewness which shows enough convergence of both input and output parameters. In the kurtosis of input and output parameters, all of the input parameters demonstrate normal distributions, whereas all of the output parameters illustrate peak distribution except the seventh and tenth modes which are close to normal distribution. Finally, The Shannon entropy which presents a measure of the complexity and predictability of input and output parameters is calculated as enough small amount that demonstrates less complex, and more predictable distributions of the corresponding parameters.

Also, probability density function histograms and cumulative density functions of the random variable distribution after 10,000 simulations of the 3 input and 10 output parameters are represented in Figs. 7 and 8, respectively. The histogram of random variable sample distribution can describe the distribution of random variables. As it can be seen from the probability density distributions of the parameters, the input and output parameter distribution curves are smooth and without any gaps, presenting that enough input simulations are converging the statistical requirements.

Table 5. Description of input parameters.

Properties	Crack depth	Crack width	Crack distance
Mean	20.50	10.50	1500
Standard deviation	6.31	3.07	469.41
Skewness	-5.71E-04	4.22E-04	1.59E-03
Kurtosis	-3.54E-03	-2.15E-03	1.93E-02
Shannon entropy	3.26	2.54	7.57
Sigma minimum	-3.81	-3.81	-3.81
Sigma maximum	3.81	3.81	3.81

Table 6. Description of output parameters.

Modes	1	2	3	4	5	6	7	8	9	10
Mean	7.12	7.26	42.64	44.89	121.12	125.20	206.67	230.66	240.54	388.80
Standard deviation	0.20	0.05	1.33	0.34	2.26	0.87	8.18	4.63	1.56	3.59
Skewness	-2.12	-1.32	-0.42	-0.95	-0.65	-0.97	0.18	-0.50	-0.51	-0.10
Kurtosis	7.72	3.75	1.43	2.38	2.34	1.33	0.28	1.52	1.51	0.27
Shannon entropy	-0.48	-1.68	1.69	0.28	2.17	1.16	3.51	2.94	1.84	2.70
Sigma minimum	-3.81	-3.81	-3.81	-3.81	-3.81	-3.81	-3.81	-3.81	-3.81	-3.81
Sigma maximum	3.81	3.81	3.81	3.81	3.81	3.81	3.81	3.81	3.81	3.81

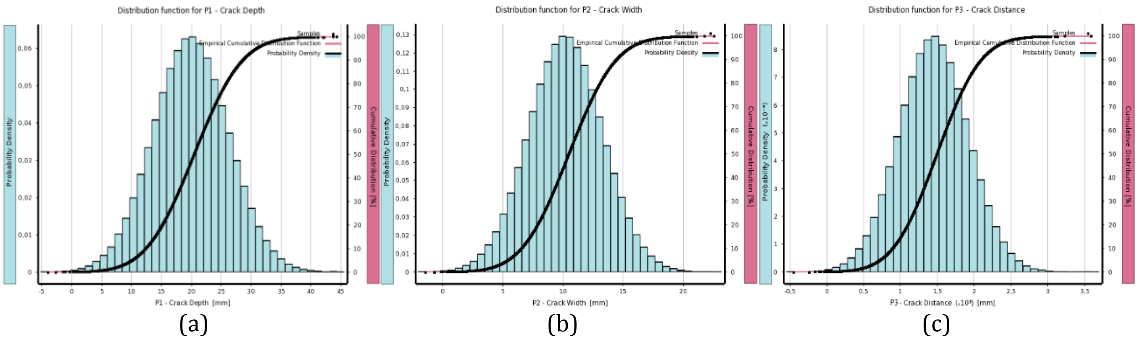

Fig. 7. Probability density of parameters: (a) Crack depth, P1; (b) Crack width, P2; (c) Crack distance, P3.

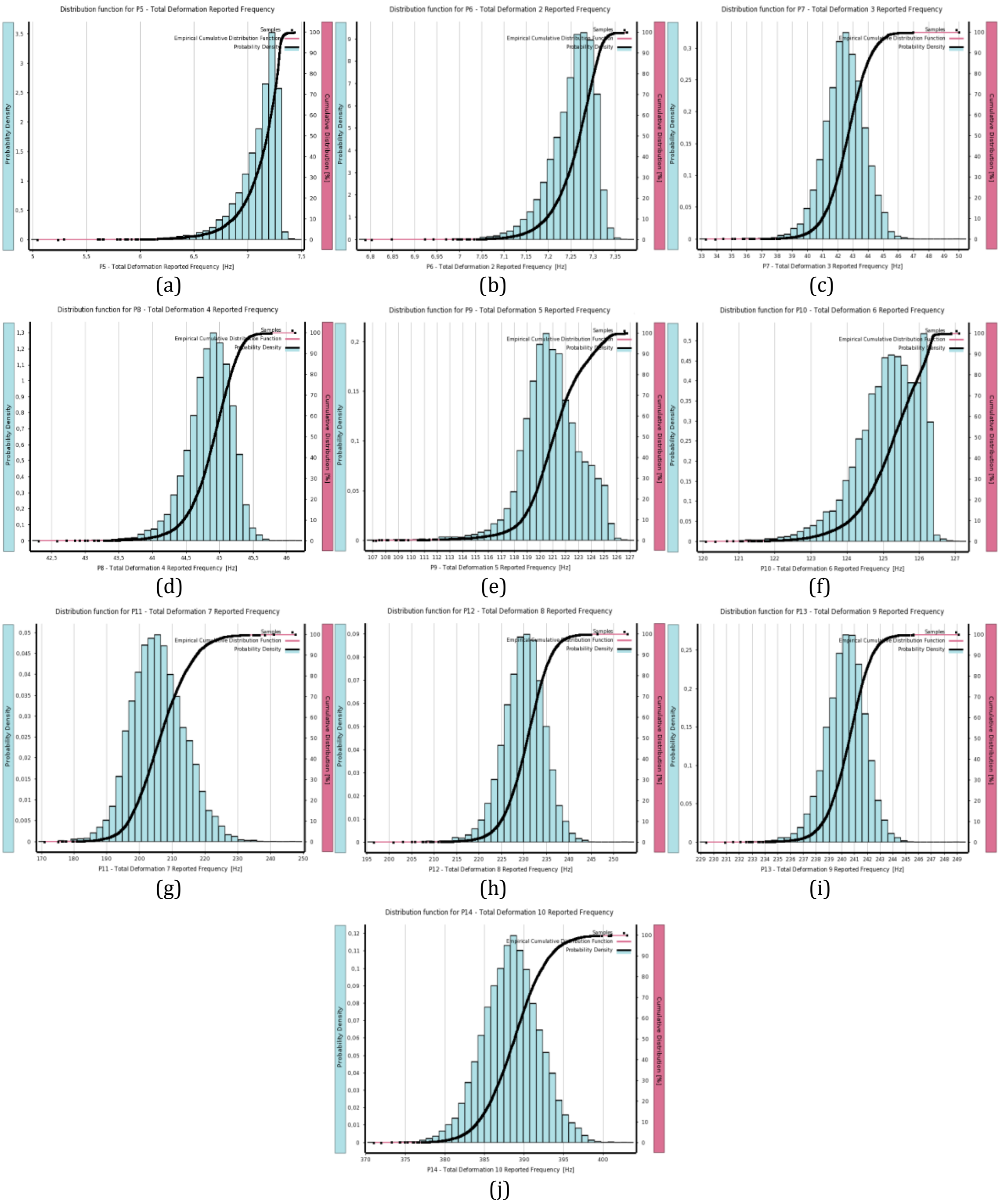

Fig. 8. Probability density of parameters: (a) Mode 1, P5; (b) Mode 2, P6; (c) Mode 3, P7;(d) Mode 4, P8; (e) Mode 5, P9; (f) Mode 6, P10; (g) Mode 7, P11; (h) Mode 8, P12; (i) Mode 9, P13; (j) Mode 10, P14.

A cumulative distribution function plot is a useful tool for calculating the probability that a project's design meets or fails to meet reliability standards, as well as assessing the project's reliability and failure probability. Reliability is defined as the probability that no failure occurs. The probability that an output parameter will remain below a given level as shown by the values on the X-axis of the plot is represented by the value of a cumulative distribution function of that output parameter. In the context of this study, the eigenfrequencies can be chosen to be above a specific permissible limit, and if an eigenfrequency falls below this limit, a failure event happens. As a result, the cumulative distribution function is understood as the steel beam's failure probability curve.

It can be seen from Fig. 8 that the distributions for first, second, fourth, and sixth modal natural frequency values increase as those get closer to the natural frequency value obtained in the undamaged condition. It is observed that the natural frequency values of the third, fifth, seventh, eighth, ninth, and tenth mode shapes are concentrated at about 7, 5, 10, 6, 2, and 3% of undamaged modal natural frequency values, respectively.

4. Conclusions

This study aimed to examine the effect of damage schemes specified on the steel beam model on the modal frequency value of the steel beam, by using the natural frequency values representing the undamaged state obtained by creating a FE model of a box section steel profile modeled as a fixed beam. For damage schemes, crack depth, width, and distance from the support point were selected and were defined as the input variable parameters. The effects of the generated crack schemes on the modal natural frequencies of the steel beam were investigated statistically with the help of RS and SS analyses. The following conclusions can be drawn from the present study;

- The first ten modal natural frequency values were evaluated between 7.31 Hz and 397.83 Hz for the undamaged steel beam model. For the damage cases in the steel beam, the max decrease in those frequency values for the corresponding mode shapes was calculated as 4 Hz and 360.8 Hz.
- Considering the three damage conditions, crack width seems to be the least sensitive effect on the natural frequency change corresponding to all mode shapes when compared with crack depth and distance.
- Crack depth has the most influence on first, second, fifth, sixth, and tenth bending modes and torsion modes, whereas, the crack distance presents the most sensitive behavior in third, fourth, eighth, and ninth mode shapes.
- The max decrease in modal natural frequency values is approximately 25, 3.5, and 10% for crack depth, width, and distance, respectively compared to those with undamaged cases in the steel beam.
- SS analyses demonstrate that there is enough convergence of both input and output parameters from the statistical analysis point of view and less complex, and more predictable distributions are achieved.
- The first, second, fourth, and sixth modal natural frequency distributions obtained in the damaged condition converge to the frequency values calculated by the natural frequency values in the undamaged condition. The natural frequency values of the other mode shapes are concentrated in approximately 7, 5, 10, 6, 2, and 3% of the frequency values obtained in the undamaged condition, respectively.

The fundamental of structural health monitoring is that the damage in the structures will cause a general decrease in stiffness, which will cause a change in the general dynamic behavior of the structures. This decrease in stiffness causes a decrease in the modal frequency values of the structure. As a result of the analyses performed in this study, the fact that the damages to the steel beam model caused a significant decrease in the modal frequency values of the structure shows parallelism with this issue. The employed analysis will enable which type of damage plays a more critical role in the modal natural frequency values of the structure, thus determining the resonance frequency values of the structure and taking the necessary precautions beforehand. In addition, in cases where existing damage to the structure is known, the progression of the damage will provide statistically significant data on the modal behavior of the structure.

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Conflict of Interest

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