Challenge Journal of Structural Mechanics

Research Article

Buckling resistance of the cylindrical shells with two secondary stiffening rings under external pressure

Özer Zeybek[a,*], Yasin Onuralp Özkılıç[b]

[a] Department of Civil Engineering, Muğla Sıtkı Koçman University, 48000 Muğla, Turkey
[b] Department of Civil Engineering, Necmettin Erbakan University, 42090 Konya, Turkey

ABSTRACT

The common way to amplify strength and stiffness of the tank wall is to use the stiffening rings. These stiffening rings can be classified as the primary and secondary stiffening rings (PSRs and SSRs). PSR is placed around the top of the tank shell and it assists to avoid ovalization at the top when the open-top tank is exposed to the external pressure. PSR having small dimensions may be used for fixed roof tanks since the roof system attached to the top of the cylindrical shell provides a natural restraint at this location. On the other hand, one or more SSRs may be required to preclude local buckling in both fixed and open-top cylindrical steel tanks (CSTs) which are exposed to external pressure. The requirements of single secondary stiffening ring have been investigated in detail in previous studies. However, in same cases, a single SSR does not provide sufficient resistance to maintain stability over the entire shell height. Thus, buckling capacity of the CSTs with two identical secondary stiffening rings have been explored in this present work. Pursuant to this aim, the requirements for stiffness of the SSRs which are given in terms of minimum second moment of area (SMA) were evaluated by performing Linear Elastic Bifurcation Analysis (LBA) of CSTs under uniform external pressure. Analysis results show that minimum SMA expression proposed by Blackler underestimates critical buckling value for the tanks, having especially low height-to-diameter ratios and low radius-to-thickness ratios. Furthermore, to trace strength reduction in the tank due to geometrical imperfections, Geometrically Nonlinear Analysis including Imperfections (GNIA) was performed. Analysis findings affirm that thin-walled structures are very sensitive to geometrical imperfections.

ARTICLE INFO

Article history:
Received 16 June 2022
Revised 13 July 2022
Accepted 28 July 2022

Keywords:
Cylindrical steel tanks
External pressure
Buckling
Imperfections
Secondary stiffening rings

1. Introduction

Cylindrical shells (CSs) are commonly employed in many structural and engineering applications such as cooling towers, chimneys, silos, tanks and pipelines. In these types of the cylindrical structures, curved thin steel plates are commonly used to form shell courses of storage tank structures. Using welds or bolts, the courses are mounted on each other to form the entire tank shell wall (Shokrzadeh and Sohrabi 2016a; 2016b, Hu et al. 2019; Mehretehran and Meleki 2022). Since these structures have small wall thickness, they are vulnerable to buckling due to external or internal loads, which are two main type of loads that they are subjected to. Pressures from the stored product and vacuum pressure due to process can be classified as internal loads. On the other hand, seismic action and wind are considered external loads. Seismic effects on the tank structures have been extensively studied by many researchers (Haroun and Housner 1981; Çelik et al. 2018, 2019; Hamdan 2000; Maheri and Abdollahi 2013; Güray and Yazici 2015; Spritzer and Guzey 2017a, 2017b; Çelik and Köse 2020; Çelik 2022). Effect of external pressure such as wind on the cylindrical shell structures was studied by Esslinger

* *Corresponding author*. Tel.: +90-252-211-1931 ; Fax: +90-252-211-1912 ; E-mail address: ozerzeybek@mu.edu.tr (Ö. Zeybek)
ISSN: 2149-8024 / DOI: https://doi.org/10.20528/cjsmec.2022.03.003

Nomenclature

t	thickness of the shell
H	height of the cylindrical shell
r	the radius of the cylindrical shell
D	the diameter of the cylindrical shell
E	modulus of elasticity
I_x	minimum second moment of area expression
$I_{x,\text{Blackler}}$	minimum second moment of area expression proposed by Blackler
ν	Poisson's ratio
q_{cr}	circumferential classical elastic buckling pressure
q_{LBA}	critical buckling pressure obtained from LBA
q_{GNIA}	critical buckling pressure obtained from GNIA
u_r	radial displacement

and Geier (1976), Guggenburger (1995), Greiner (2001), Sosa and Godoy (2010), Cao et al. (2018), Zeybek et al. (2019, 2021), Zeybek (2021), Zdravkov (2022), and Zeybek and Topkaya (2022). Either uniform or non-uniform external pressure was considered in these studies. Furthermore, the thin walls make the tank more vulnerable to instability under external pressure when empty or at low levels (Ansourian 1992; Flores and Godoy 1998; Maraveas 2015). Improving strength and stiffness of the tank can be performed with an increase in the wall thickness of it. However, this solution may not be economical in some cases. Another alternative is to use stiffening rings. Two main stiffening ring types for CSs are illustrated in Figs. 1-3, which are primary stiffening ring (PSR) and the secondary stiffening ring (SSR) (Zeybek 2022).

The primary job of PSR (Figs. 1-3) is restraining ovalization at the top of CSTs when the external pressure exposed to tank. The full tank height is stabilized by PSR in open-top tanks (EN 14015 2004). It is placed around the top of CSTs (EN 14015 2004; EN 1993-4-2 2007; API 650 2013). PSR can sometimes be called as the wind girder (Zeybek et al. 2021; Zeybek and Topkaya 2022). On the other hand, PSR having small dimensions may be used for fixed roof tanks since the roof system attached to the top of the cylindrical shell provides a natural restraint at this location (Zeybek 2022).

Zeybek et al. (2019, 2021), and Zeybek and Topkaya (2022) investigated both the requirements for strength and stiffness of the PSRs. Practical expressions for the stress resultants and required second moment area of the PSR were developed by considering interaction between CS and PSR. Furthermore, buckling strength curves were proposed for the open-top tanks by considering different imperfection amplitudes.

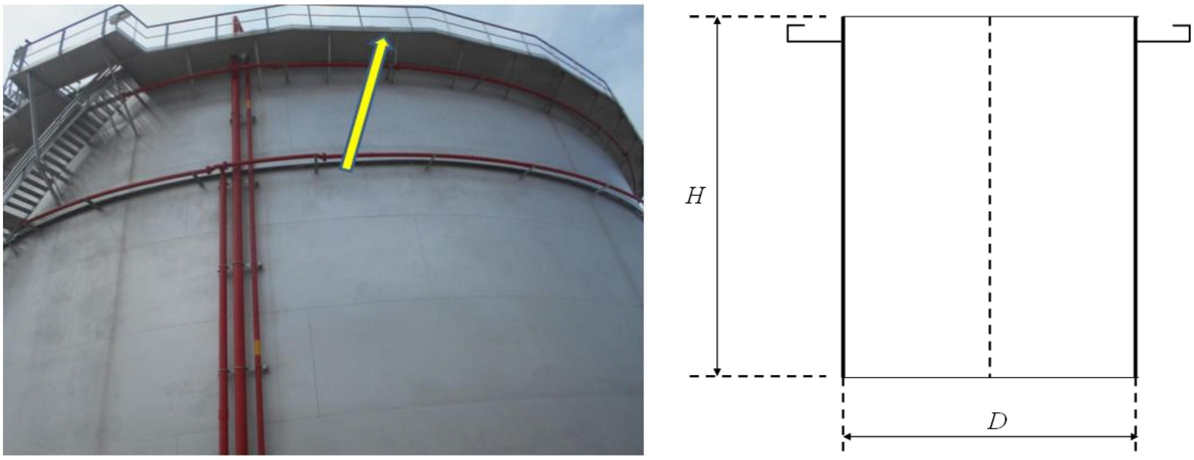


Fig. 1. A cylindrical steel tank with a PSR (https://epcmholdings.com/introduction-to-storage-tanks/).

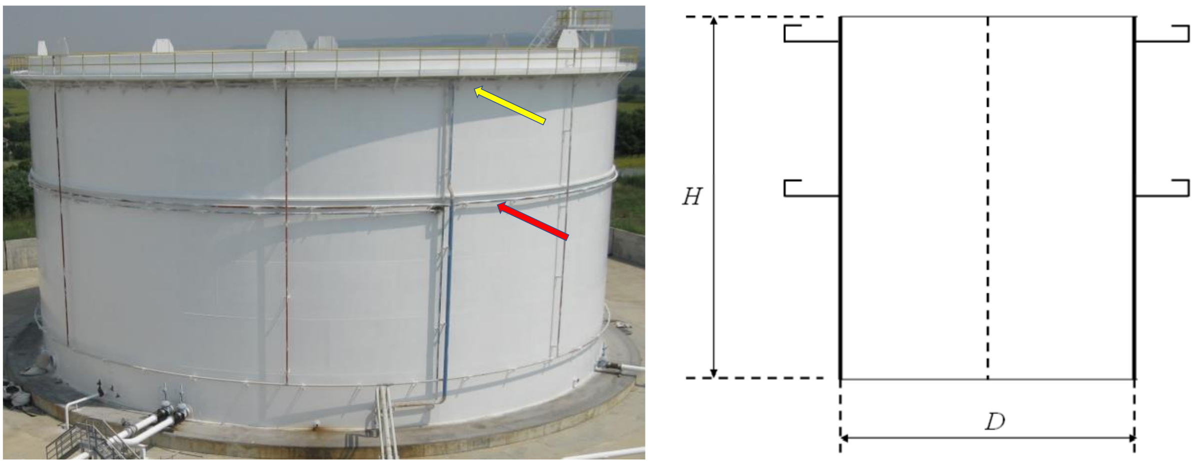


Fig. 2. A cylindrical steel tank with a PSR and a single SSR (courtesy of Dr. Zdravkov).

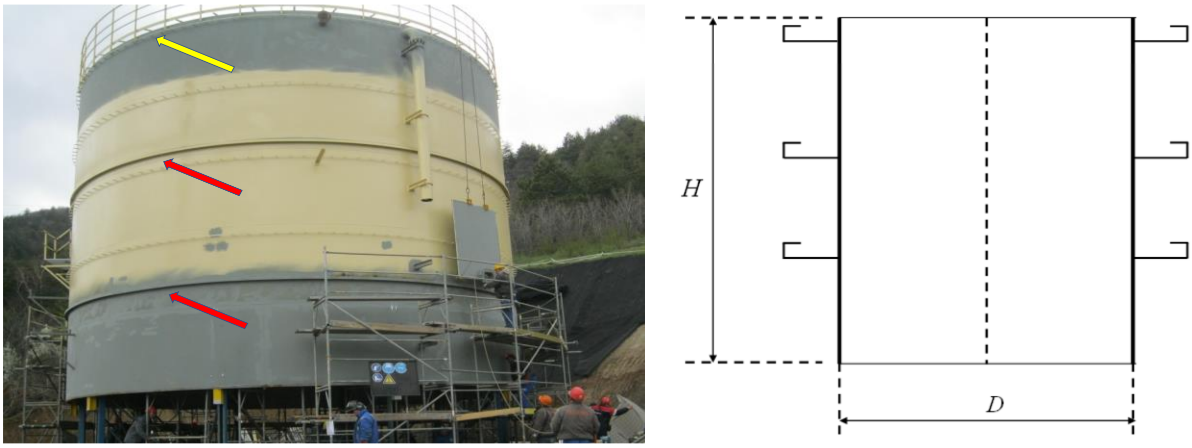


Fig. 3. A cylindrical steel tank with a PSR and a two SSRs (courtesy of Dr. Zdravkov).

When the PSR does not have enough strength for buckling resistance, it is expected that a global buckling can be formed, i.e., the edges of CSTs and the PSR buckle together (Schmidt et al. 1998; Sun et al. 2018). Both PSR and SSR are designed to not buckle even though the shell wall buckles. For both open-top and fixed roof CSTs, one or more SSRs may be necessary to prevent local buckling of the CSTs under external pressure. Sun et al. (2018) examined the open-topped CTSs with PSR and one SSR exposed to wind loads. The possibility of reducing sizes of stiffening rings was investigated by making use of numerical studies.

Zeybek (2022) developed the design expressions for anchored open top tanks with a single SSR. Both required stiffness and strength of the single SSR were obtained from proposed shell-ring stiffness ratio. A new criterion for the stability design of the tanks with one SSR was developed by considering shell-ring interaction.

2. Linear Elastic Bifurcation Analyses (LBA) to Evaluate the Current Approach to SSR Size

The buckling behavior of unstiffened CSs which were exposed to external pressure was examined by many researchers (von Mises 1914; Southwell 1913a, 1913b, 1915; Flugge 1932; Batdorf 1947; Ebner 1952; Donnell 1956, 1958). Based on these comprehensive studies, circumferential classical elastic buckling pressure was proposed as follows:

$$q_{cr} = 0.92E\left(\frac{t}{r}\right)^2\left(\frac{\sqrt{rt}}{H}\right) \quad (1)$$

As mentioned before, the most economical path of increasing the buckling resistance of the CS is to use SSRs (Greiner 2001). However, the main challenge for many designers is determining the required ring size. Many researches were conducted to obtain proper size of the ring. Biezeno and Koch (1939) showed that usage of a stiffening ring may increase load carrying capacity of the CS and it may assist to prevent out-of roundness of the shell under external pressure. Blackler (1986) investigated buckling resistance of the CS structures, which were uniformly exposed to external pressure. He proposed a stiffness requirement, which is related to minimum SMA, by considering classical eigenvalue analysis. Based on the simple boundary conditions and constant shell wall thickness, the minimum SMA expression for two identical stiffening rings was obtained as follows:

$$I_{x,\mathrm{Blackler}} = 0.105Ht^3 \quad (2)$$

Abovementioned expression proposed by Blackler (1986) is assessed by considering comprehensive finite element analyses in this part of the study. According to the studies conducted by Godoy (2016) and Rotter et al. (2015), tanks are thinner and squatter structures having r/t ranging between 500 and 3000 ($500<r/t<3000$) and $H/D<1.0$. Thus, the geometrical properties of the tanks used in the numerical study are given in Table 1.

For this purpose, H/D *ratio*, r/t *ratio* and the dimension of the SSRs were selected for prime parameters. H/D ratios of 1.0, 0.75, 0.50, and 0.25 and r/t ratios of 500, 1000, 1500, 2000, 2500 and 3000 were utilized. In the numerical study, a total of 720 analyzes were performed by taking into account the different SSR ring sizes. Finite element software of ANSYS v12.1 (2010) was utilized to assess the effectiveness of current design approach. As demonstrated in Fig. 4, four-node shell elements (shell63) and two-node beam elements (beam4) were utilized to mesh tank shell and two identical SSRs, respectively. All freedoms were restrained at the base of the CS to represent the bottom edge of the CSTs. On the other hand, radial and tangential translational restraints ($U_r = U_\theta = 0$) were performed to the top of the CST to mimic the effects of the steel roof or PSR (Zhao and Lin 2014). E of 200 GPa and ν of 0.3 were utilized. Two identical SSRs were placed at the $H/3$ and $2H/3$ of the CSTs in all numerical models as depicted in Fig. 4. A uniform external pressure was applied around the circumference of the cylindrical shell in the models.

The critical buckling pressure (q_{LBA}) was determined by performing LBA. The requirement of SMA recommended by Blackler (1986) was assessed by taking into account of the entire dataset. Non-dimensional findings are depicted in Figs. 5-8. In this figure, the SMA of the annular plate rings (I_x) is normalized by the SMA recommended by Blackler (1986) ($I_{x,\mathrm{Blackler}}$). q_{LBA} is normalized by the circumferential classical buckling pressure (q_{cr}) given in Eq. (1). It should be noted that normalized values ($I_x/I_{x,\mathrm{Blackler}}$) are plotted on the logarithmic scale.

Table 1. Geometrical properties of the cylindrical steel tanks.

r	t	r/t	H	H/D
6000	12	500	12000	1
6000	6	1000	12000	1
12000	8	1500	24000	1
12000	6	2000	24000	1
12000	4.8	2500	24000	1
15000	5	3000	30000	1
8000	16	500	12000	0.75
8000	8	1000	12000	0.75
12000	8	1500	18000	0.75
12000	6	2000	18000	0.75
12000	4.8	2500	18000	0.75
15000	5	3000	22500	0.75
6000	12	500	6000	0.5
6000	6	1000	6000	0.5
12000	8	1500	12000	0.5
12000	6	2000	12000	0.5
12000	4.8	2500	12000	0.5
15000	5	3000	15000	0.5
12000	24	500	6000	0.25
12000	12	1000	6000	0.25
12000	8	1500	6000	0.25
12000	6	2000	6000	0.25
12000	4.8	2500	6000	0.25
15000	5	3000	6000	0.25

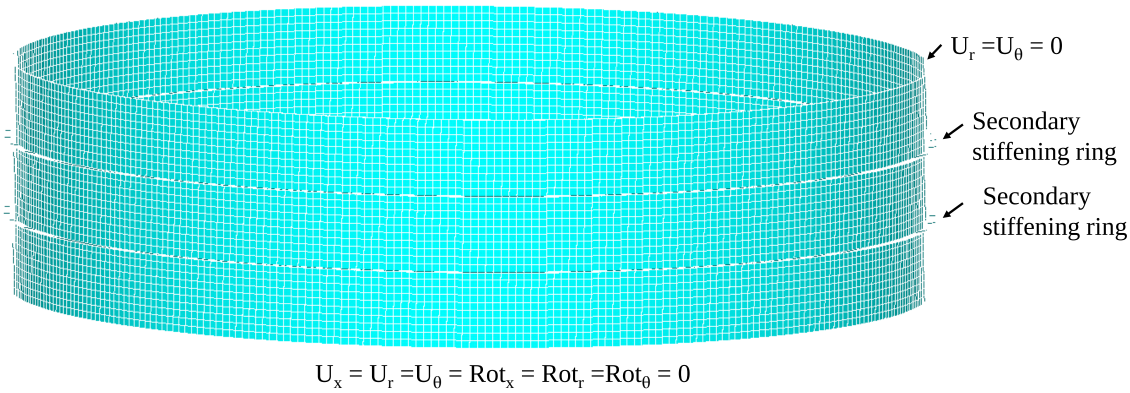


Fig. 4. Finite element modelling of a representative tank with two SSRs.

As shown in Fig. 5, proposal of Blackler (1986) provides sufficient stiffness for the secondary stiffening rings in the cylindrical shells having radius-to-thickness (r/t) ratio ranging between 500-3000. Two bounds (unstiffened and fully stiffened) are also shown in the same figures. In unstiffened case, there is no secondary stiffening ring in the cylindrical shells; however, relatively stiff two SSRs are installed in the fully stiffened case. For the unstiffened case, the normalized buckling pressures (q_{LBA}/q_{cr}) converge to the value of 1.25. On the other hand, normalized buckling pressures are 3.37, 3.28, 3.24, 3.23, 3.20, 3.19 for r/t of 500, 1000, 1500, 2000, 2500 and 3000 respectively, taking into account fully stiffened case considered.

In the same manner, Fig. 6 shows that Blackler's (1986) recommendation is almost sufficient for the all cases in the H/D of 0.75. For the unstiffened case, the normalized buckling pressures (q_{LBA}/q_{cr}) converge to the value of 1.25. On the other hand, normalized buckling pressures are 3.45, 3.35, 3.29, 3.27, 3.25, 3.24 for r/t of 500, 1000, 1500, 2000, 2500 and 3000 respectively, taking into account fully stiffened case considered.

Fig. 7 shows that Blackler's (1986) recommendation provides sufficient stiffness for the r/t of 2000, 2500 and

3000 in the H/D of 0.5. However, proposal of Blackler (1986) slightly underestimates required ring sizes for the r/t of 500, 1000 and 1500. For the unstiffened case, the normalized buckling pressures (q_{LBA}/q_{cr}) converge to the value of 1.25. On the other hand, normalized buckling pressures are 3.66, 3.5, 3.4, 3.35, 3.33, 3.31 for r/t of 500, 1000, 1500, 2000, 2500 and 3000 respectively, taking into account fully stiffened case considered.

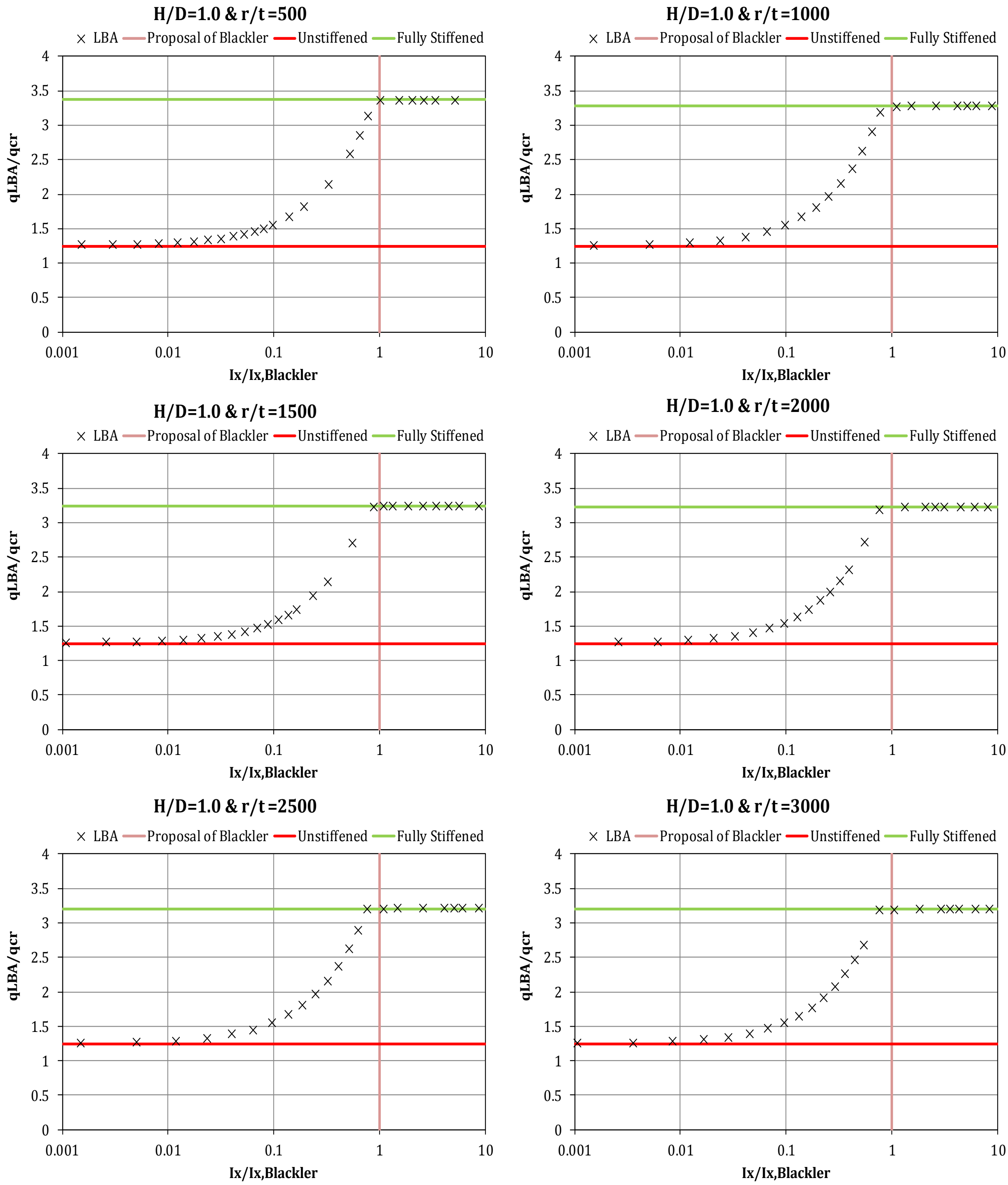


Fig. 5. Evaluation of SMA requirement proposed by Blackler for H/D=1.0.

As shown in Fig. 8, proposal of Blackler (1986) does not provide sufficient stiffness for all cases in the H/D of 0.25. For the unstiffened case, the normalized buckling pressures (q_{LBA}/q_{cr}) converge to approximately the value of 1.25. On the other hand, normalized buckling pressures are 4.56, 3.96, 3.76, 3.67, 3.6, 3.56 for r/t of 500, 1000, 1500, 2000, 2500 and 3000 respectively, taking into account fully stiffened case considered.

As already mentioned, two bounds representing unrestrained shell (no SSRs) and fully restrained with two identical stiff SSRs are also reported in Figs. 5-8. The data points fall in between these two bounds. Furthermore, the proposed SMA by Blackler (1986) underestimates the critical size of the SSRs, especially for low height-to-diameter ratios and low radius-to-thickness ratios. For instance; ring size should be at least 1.88 times to proposal of Blackler to reach the critical size of the ring for H/D=0.5 with r/t=500 case. On the other hand, for H/D=0.25 with r/t=500 case, ring size should be at least 3.0 times to proposal of Blackler to reach the critical size of the ring.

A typical CST was chosen to investigate modes of buckling of the tank which is uniformly exposed to external pressure by considering different normalized SMA values ($I_x/I_{x,\text{Blackler}}$). For this purpose, a CS having diameter of 6000mm, height of 6000mm, and uniform wall thickness of 12mm was selected (H/D=0.5, r/t=500).

The buckled shapes of CSTs which is uniformly exposed to external pressure are presented for the $I_x/I_{x,\text{Blackler}}$=0.01, $I_x/I_{x,\text{Blackler}}$=0.2, $I_x/I_{x,\text{Blackler}}$=1.0, $I_x/I_{x,\text{Blackler}}$=1.88 in Fig. 9.

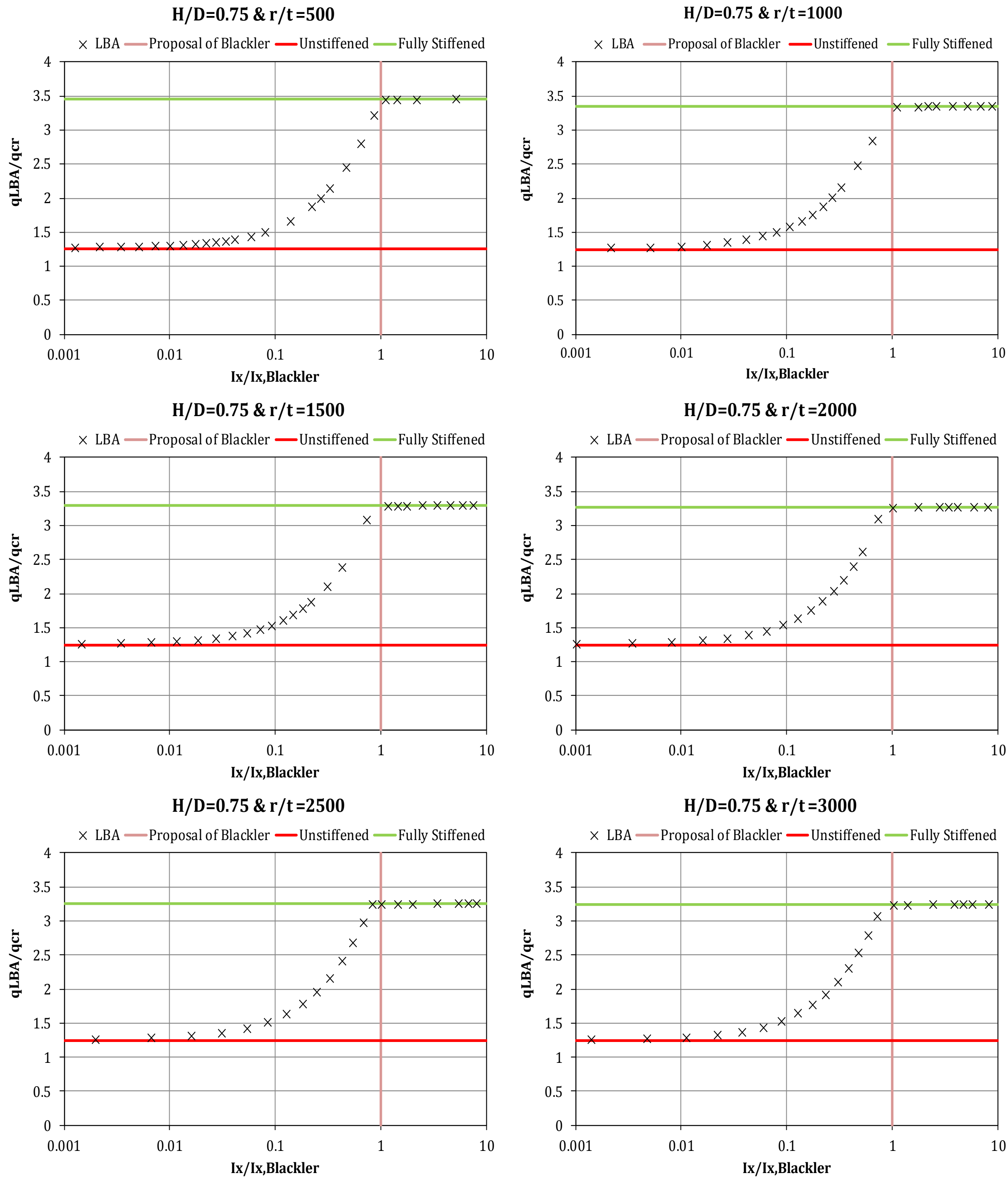


Fig. 6. Evaluation of SMA requirement proposed by Blackler for H/D=0.75.

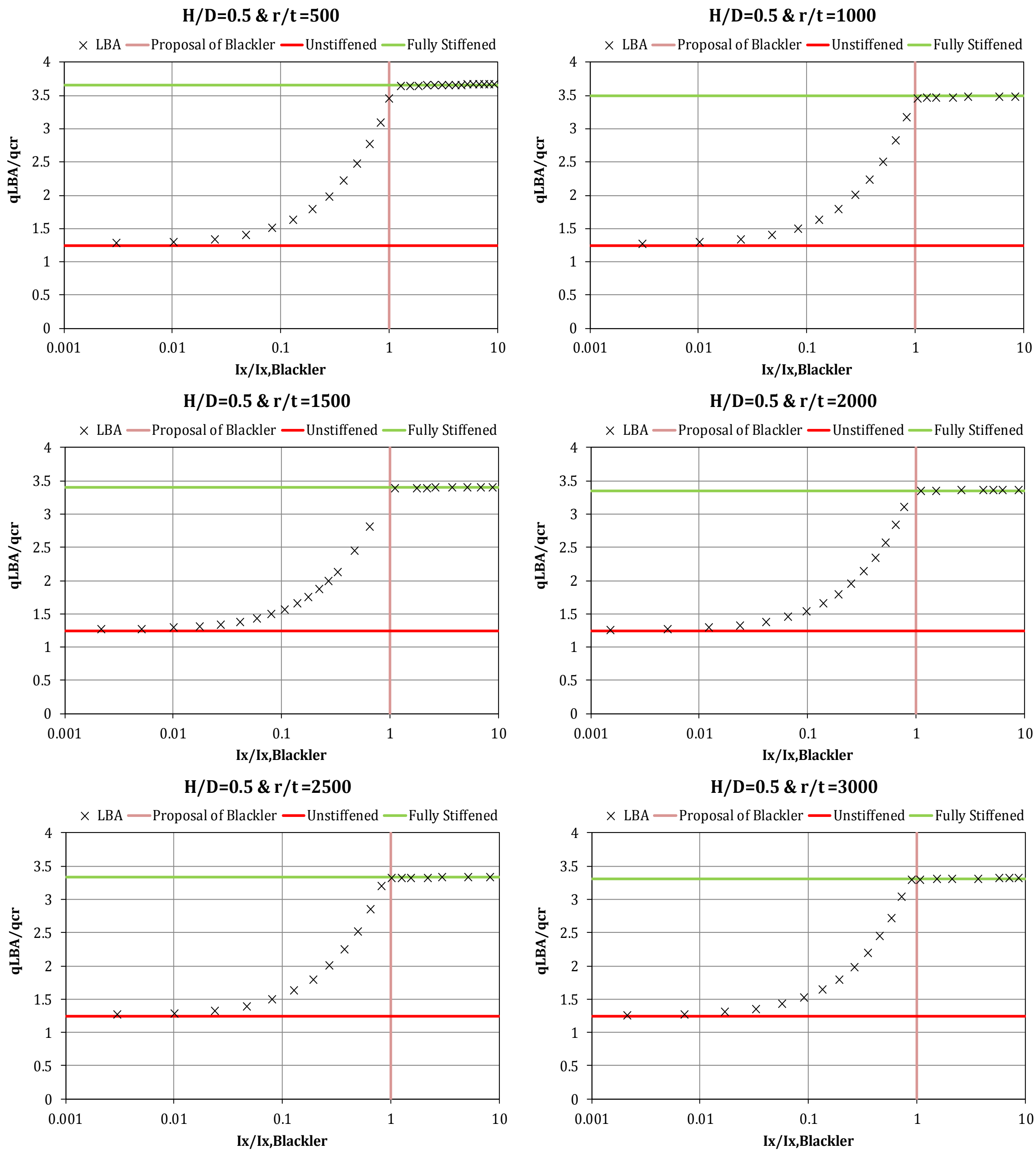


Fig. 7. Evaluation of SMA requirement proposed by Blackler for H/D=0.5.

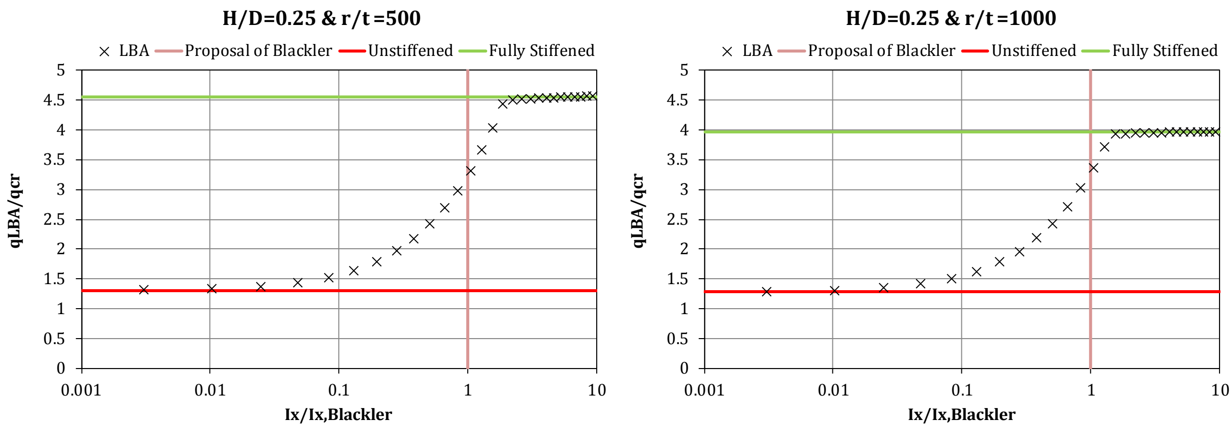


Fig. 8. (continued)

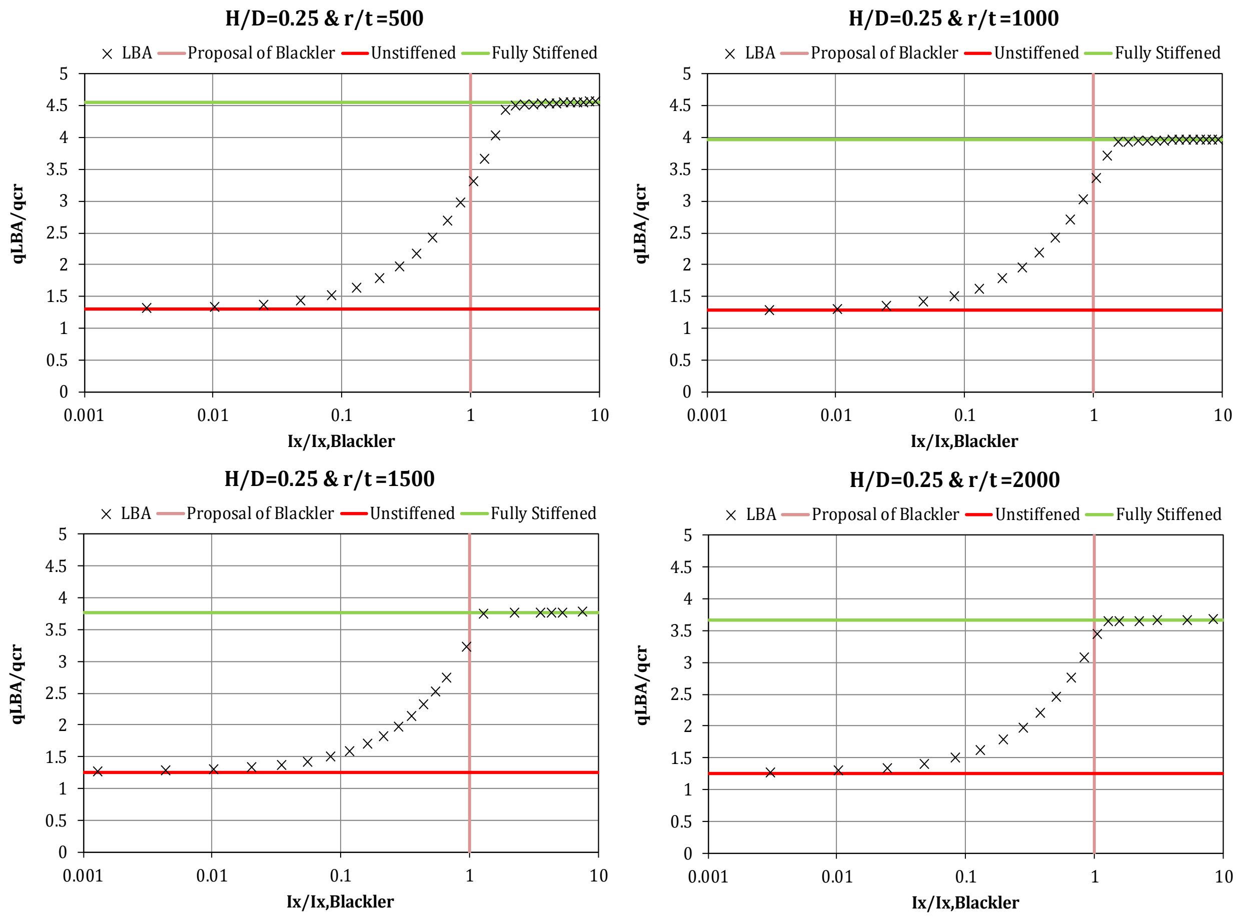
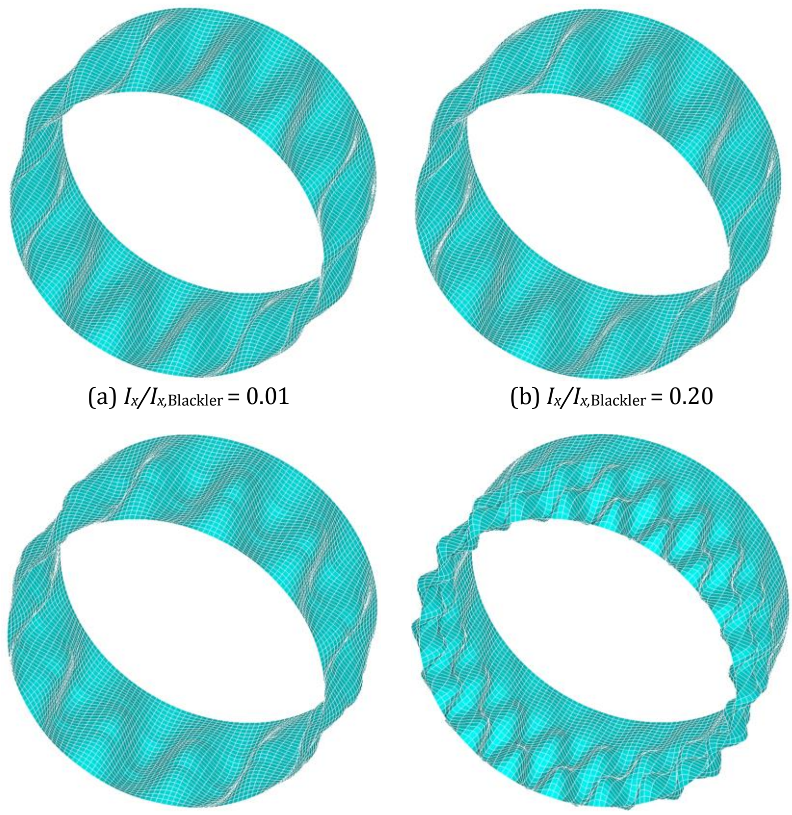


Fig. 8. Evaluation of SMA requirement proposed by Blackler for H/D=0.25.



(a) $I_x/I_{x,\text{Blackler}}$ = 0.01
(b) $I_x/I_{x,\text{Blackler}}$ = 0.20
(c) $I_x/I_{x,\text{Blackler}}$ = 1.00
(d) $I_x/I_{x,\text{Blackler}}$ = 1.88

Fig. 9. Buckling mode shapes considering different SMA ratios ($I_x/I_{x,\text{Blackler}}$).

As shown in Fig. 9, the SSRs participate in the buckling of CS when $I_x/I_{x,\text{Blackler}}$=0.01, $I_x/I_{x,\text{Blackler}}$=0.2. Furthermore, for the proposed SMA by Blackler (1986) case ($I_x/I_{x,\text{Blackler}}$=1.0), SSRs are still involved in the buckling mode and critical buckle height is long. To obtain sufficient stiffness, the critical ring size should be at least 1.88 times proposal of Blackler (1986) for the case considering H/D=0.5 with r/t=500.

3. Imperfection Sensitivity of CSTs with two SSRs – Geometrically Nonlinear Analysis Including Imperfections (GNIA)

CSTs generally exhibit severe imperfection sensitivity since they have small wall thickness. Thus, changes in the geometry of CSTs during loading and small deviations in geometry should be taken into account in the design stage. Sonat et al. (2015) reported that geometrical imperfections decrease the elastic buckling capacity considerably under linear bifurcation value. There are many reasons of imperfections (deviations from ideal circular shape) such as constructional defects owing to welded seams between two shell strakes, out-of straightness or ovalization of the CS surface due to fabrication, handling and transporting stages (de Paor et al. 2012; Yan et al. 2021; Godoy 2016). Generally, there are two types of imperfections used in the shell analysis, i.e depressions due to welds and eigen-mode affine imperfection shapes. The most detrimental shape of imperfection (also called eigen-mode affine imperfection shapes) was considered in this phase of the study. This pattern is based on the lowest eigenvalue and, the results are conservatively estimated in terms of strength (Godoy 2016; Sun et al. 2018). Greiner and Derler (1995) reported that short or broad CS structures (low H/D ratio) are the most sensitive to eigen-mode affine imperfections when compared to slender shell structures. Thus, GNIA was carried out on a specific case (H/D=0.25 with r/t=500) to investigate the influence of geometrical imperfections on CSTs subjected to external pressure, 0.1t, 0.25t, 0.5t and 1.0t are chosen as imperfection amplitudes in the numerical model. The radial displacement values (u_r) at mid-height along the stagnation meridian were recorded and normalized by the tank shell thickness (t). Ratio of the buckling pressure loads ($q_{\text{GNIA}}/q_{\text{LBA}}$) are plotted against normalized values (u_r/t) as shown in Fig. 10.

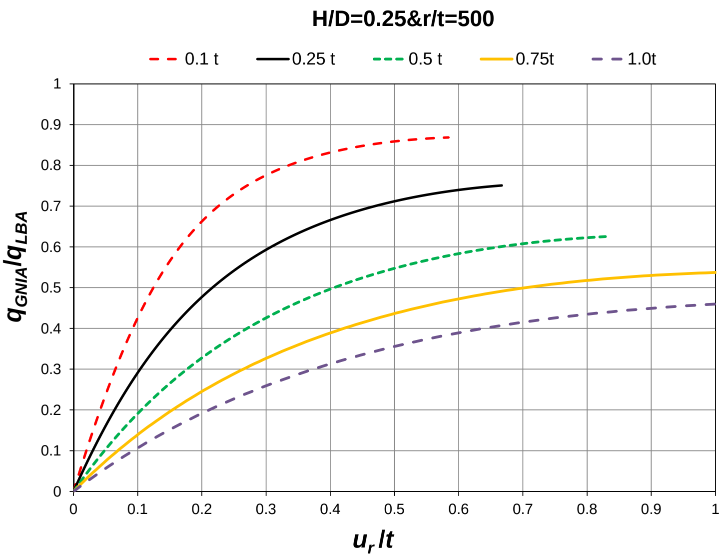


Fig. 10. Relationship between normalized buckling load values and radial displacement for H/D=0.25 with r/t=500 case.

As shown in Fig. 10, as the imperfection amplitude is rised, the buckling strengths determined from GNIA (q_{GNIA}) deviates from q_{LBA}. According to the analysis results, the $q_{cr,\text{GNIA}}/q_{cr,\text{LBA}}$ ratios extend to 0.87, 0.75, 0.63 and 0.46 for imperfection amplitudes of 0.1t, 0.25t , 0.5t, 0.75t and 1.0t, respectively.

4. Conclusions

This paper has examined buckling capacity of CSTs with two identical SSRs under external pressure. Pursuant to this aim, a numerical study was carried out on CSTs with different shell geometries and SSR dimensions by making use of LBA. The analysis results demonstrated that SMA requirement proposed by Blackler (1986) generally provides sufficient strength when the tank structures have high H/D *ratios* and r/t *ratios*. Yet, for low H/D ratios and low r/t ratios, proposal of Blackler underestimates critical buckling pressure of tanks with two identical SSRs. In other words, Blackler's (1986) recommendation generally provides sufficient stiffness for all cases of H/D ratio of 0.75 and 1.0. Proposal of Blackler (1986) slightly underestimates required ring sizes for the r/t of 500, 1000 and 1500 when H/D is equal to 0.5 and for all cases when H/D is equal to 0.25. What is more, GNIA was also executed to examine the influence of geometrical imperfections on buckling strength of the tanks. The first eigenmode (eigenmode-affine pattern) was utilized for imperfection patterns such as 0.1t, 0.25t, 0.5t, 0.75t and

1.0*t*. According to the analysis results, the buckling pressure values obtained from GNIA diverge considerably from LBA results, with a rising difference as the amplitude of imperfection is increased. This means that high imperfection amplitudes lead to a great amount of strength reduction. This affirms that load bearing capacity of the thin-walled CSTs are very sensitive to geometrical imperfections.

Acknowledgements

None declared.

Funding

The authors received no financial support for the research, authorship, and/or publication of this manuscript.

Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this manuscript.

References

Ansourian P (1992). On the buckling analysis and design of silos and tanks. *Journal of Constructional Steel Research*, 23, 273-294.

ANSYS (2010). Version 12.1 On-line User's Manual.

API 650 (2013). Welded tanks for oil storage (12th ed.), American Petroleum Institute, Washington, DC, USA.

Batdorf SB (1947). A simplified method of elastic stability analysis for thin cylindrical shells. NACA TN No.1341.

Biezeno CB, Koch JJ (1939). The buckling of a cylindrical tank of variable thickness under external pressure. *5th International Congress for Applied Mechanics*, ASME, 34-39.

Blackler MJ (1986). Stability of Silos and Tanks under Internal and External Pressure. *PhD thesis*, School of Civil and Mining Engineering, University of Sydney, Australia.

Cao QS, Zhao Y, Zhang R (2018). Wind induced buckling of large circular steel silos with various slenderness. *Thin-Walled Structures*, 130, 101-113.

Çelik Aİ (2022). Impact analysis of cylindrical steel water storage tanks under the seismic action. *International Journal of Advances in Engineering and Pure Sciences*, 34(1), 1-13.

Çelik Aİ, Köse MM (2020). Dynamic buckling analysis of cylindrical steel water storage tanks subjected to Kobe earthquake loading. *Steel Construction*, 13, 128-138.

Çelik Aİ, Köse MM, Akgül T, Apay AC (2018). Directional-deformation analysis of cylindrical steel water tanks subjected to El-Centro Earthquake loading. *Sigma Journal of Engineering and Natural Sciences*, 36(4), 1033-1046.

Çelik Aİ, Köse MM, Akgül T, Apay AC (2019). Effects of the shell thickness on the directional deformation and buckling on the cylindrical steel water tanks under the Kobe Earthquake loading. *Sakarya University Journal of Science*, 23(2), 269-281.

de Paor C, Cronin K, Gleeson JP, Kelliher D (2012). Statistical characterisation and modelling of random geometric imperfections in cylindrical shells. *Thin-Walled Structures*, 58, 9-17.

Donnell LH (1956). Effect of imperfections on buckling of thin cylinders under external pressure. *Journal of Applied Mechanics*, ASME, 23(4), 569-575.

Donnell, LH (1958). Effect of imperfections on buckling of thin cylinders with fixed edges under external pressure. *3rd US National Congress of Applied Mechanics*, ASME, 305-311, New York.

Ebner H (1952). Theoretical and experimental investigations on buckling of cylindrical tanks subjected to reduced pressure. *Stahlbau*, 21, 153-159. (in German)

EN 1993-4-2 (2007). Eurocode 3: Design of Steel Structures, Part 4.2: Tanks, Eurocode 3 Part 4.2, CEN, Brussels.

EN 14015 (2004). Specification for the Design and Manufacture of Site Built, Vertical, Cylindrical, Flat-Bottomed, above Ground, Welded, Steel Tanks for the Storage of Liquids at Ambient Temperature and above. CEN, Brussels.

Esslinger M, Geier B (1976). On the buckling behavior of thin-walled isotropic circular cylinders subjected to external pressure. DVFLR, Institut fur Flugzeugbau, Braunschweig, IB 152-76/03.

Flores FG, Godoy LA (1998). Buckling of short tanks due to hurricanes. *Engineering Structures*, 20(8), 752-760.

Flügge W (1932). Die Stabilität der Kreiszylinderschale. *Ingenieur-Archiv*, 3(5), 463-506. (in German)

Godoy LA (2016). Buckling of vertical oil storage steel tanks: Review of static buckling studies. *Thin-Walled Structures*, 103, 1-21.

Greiner R (2001). Cylindrical Shells under Uniform External Pressure. In: *Buckling of Thin Metal Shells (Ed. Teng JG and Rotter JM)*, Spon Press, London, 154-174.

Greiner R, Derler P (1995). Effect of imperfections on wind-loaded cylindrical shells. *Thin-Walled Structures*, 23(1-4), 271-281.

Guggenburger W (1995). Buckling and postbuckling of imperfect cylindrical shells under external pressure. *Thin Walled Structures*, 23(1-4), 351-366.

Güray E, Yazici G (2015). Sloshing in medium sized tanks under earthquake load. *Gradevinar,* 67(7), 655-662.

Hamdan FH (2000). Seismic behaviour of cylindrical steel liquid storage tanks. *Journal of Constructional Steel Research*, 53(3), 307-333.

Haroun MA, Housner GW (1981). Seismic design of liquid storage tanks. *Journal of Technical Councils of ASCE*, 107(1), 191-207.

Hu W, Bohra H, Azzuni E, Guzey S (2019). The uplift effect of bottom plate of aboveground storage tanks subjected to wind loading. *Thin-Walled Structures*, 144, 106241.

Maheri MR, Abdollahi A (2013). The effects of long term uniform corrosion on the buckling of ground based steel tanks under seismic loading. *Thin-Walled Structures*, 62, 1-9.

Maraveas C, Balokas GA, Tsavdaridis KD (2015). Numerical evaluation on shell buckling of empty thin-walled steel tanks under wind load according to current American and European design codes. *Thin-Walled Structures*, 95, 152-160.

Mehretehran AM, Maleki S (2022). Axial buckling of imperfect cylindrical steel silos with isotropic walls under stored solids loads: FE analyses versus Eurocode provisions. *Engineering Failure Analysis*, 137, 106282.

Rotter JM, Kerr E, Lam HT, Holst JM (2015). Secondary rings for large tanks under external pressure. *Eighth International Conference on Advances in Steel Structures Lisbon*, Portugal.

RV Southwell BA (1913a). On the collapse of tubes by external pressure. *Philosophical Magazine Series 6*, 25(149), 687-698.

RV Southwell BA (1913b). On the collapse of tubes by external pressure–II. *Philosophical Magazine Series 6*, 26(153), 502-511

RV Southwell MA (1915). On the collapse of tubes by external pressure–III. *Philosophical Magazine Series 6*, 29(169), 67-77.

Schmidt H, Binder B, Lange H (1998). Postbuckling strength design of open thin walled cylindrical tanks under wind load. *Thin-Walled Structures*, 31, 203–220.

Shokrzadeh AR, Sohrabi MR (2016a). Buckling of ground based steel tanks subjected to wind and vacuum pressures considering uniform internal and external corrosion. *Thin-Walled Structures*, 108, 333-350.

Shokrzadeh AR, Sohrabi MR (2016b). Strengthening effects of spiral stairway on the buckling behavior of metal tanks under wind and vacuum pressures. *Thin-Walled Structures*, 106, 437-447.

Sonat C, Topkaya C, Rotter JM (2015). Buckling of cylindrical metal shells on discretely supported ring beams. *Thin-Walled Structures*, 93, 22-35.

Sosa, EM, Godoy LA (2010). Challenges in the computation of lower-bound buckling loads for tanks under wind pressures. *Thin-Walled Structures* 48 (12), 935-945.

Spritzer JM, Guzey S (2017a). Nonlinear numerical evaluation of large open-top aboveground steel welded liquid storage tanks excited by seismic loads. *Thin-Walled Structures*, 119, 662-676.

Spritzer JM, Guzey S (2017b). Review of API 650 Annex E: Design of large steel welded aboveground storage tanks excited by seismic loads. *Thin-Walled Structures*, 112, 41-65.

Sun T, Azzuni E, Guzey S (2018). Stability of open-topped storage tanks with top stiffener and one intermediate stiffener subject to wind loading. *ASME Journal of Pressure Vessel Technology*, 140(1), 011204.

von Mises R (1914). Der kritische Ausendruck zylindrischer Rohre. *Zeitschrift des Vereines Deutscher Ingenieure*, 58, 750-755.

Yan XL, Li GQ, Wang YB (2021). Behavior and design of high-strength steel members under bending moment. In: *Behavior and Design of High Strength Constructional Steel (Ed. Li GQ and Wang YB)*, Woodhead Publishing, United Kingdom.

Zdravkov L (2022). Influence of the roof structure on the forces in the top angle under wind loading. *Annual of the University of Architecture, Civil Engineering and Geodesy*, 55(2), 245-254.

Zeybek Ö (2021). Design of cylindrical steel liquid tanks with stepped walls using One-foot method. *Challenge Journal of Structural Mechanics*, 7(4), 162-169.

Zeybek Ö (2022). The stability of anchored cylindrical steel tanks with a secondary stiffening ring. *International Journal of Pressure Vessels and Piping*, 198, 104661.

Zeybek Ö, Topkaya C (2022). Stiffness requirements for wind girders in open-top cylindrical steel tanks. *Thin-Walled Structures*, 176, 109353.

Zeybek Ö, Topkaya C, Rotter JM (2019). Stress resultants for wind girders in open-top cylindrical steel tanks. *Engineering Structures*, 196, 109347.

Zeybek Ö, Topkaya C, Rotter JM (2021). Stability of open-top cylindrical steel tanks with primary stiffening ring under wind loading, *ce/papers*, 4, 1781-1788.

Zhao Y, Lin Y (2014). Buckling of cylindrical open-topped steel tanks under wind load. *Thin Walled Structures*, 79, 83-94.