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Effect of time step size on stress relaxation

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ABSTRACT

Many materials used in industry show time and temperature dependant stress strain relationship. While essentially most of the materials exhibit stress relaxation or in general viscoelastic material properties, some of them are assumed as linear elastic to be able to make their stress calculations simpler. On the contrary, there are some materials showing intense viscoelastic stress strain relationship even at lower temperatures and short time periods. Most of these materials are employed in construction industry as pavements on roads or bridges and needed a better understanding of their viscoelastic material properties and calculation methods for their design. For a better understanding and comparison between several material products in industry, their stress strain behavior shall be evaluated. Stress relaxation of materials, which shows time and temperature dependant properties, is investigated in this paper. For that reason first, relaxation test results existed in the literature are used to verify the numerical stress relaxation calculation of commercial FEM program, ANSYS. Second, the determination of Prony series parameters and the commands to be entered in ANSYS to perform stress relaxation are given. Finally, the amount of error in the numerical calculation depending on time step sizes at different temperatures is presented.

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1. Introduction

Viscoelasticity is an interesting topic to model time and temperature dependant material behavior. Therefore, several researchers use viscoelasticity to predict stress strain relationship of materials, which are of interest. (Delgadillo R, 2010; Findley WN et al., 1989). In this study, using viscoelasticity by means of Prony series in ANSYS is explained first. Then, stress relaxation test results of Monismith and Secor (1962) are used to indicate the validity of Prony series for viscoelastic analysis. Afterwards, amount of error occurred in ANSYS (2014) calculations depending on selected time step size is illustrated. Numerical ANSYS calculations and stress relaxation tests are presented at three different isothermal environmental temperatures to investigate the effect of time step size at different temperatures on the values calculated by ANSYS.

2. Properties of Tested Specimen

An 85-100 penetration asphalt cement is used in stress relaxation tests done by Monismith and Secor (1962). The properties of mix tested is briefly given below,

Test name	Result
• Penetration at 77°F, 100gm, 5 secs.	96
• Penetration at 39.2°F, 200gm, 60 secs.	24
• Penetration proportion	25
• Flashpoint, Pensky-Martens, °F	445
• Viscosity at 275°F, SSF	138
• Heptane-xylene Equivalent	20/25
• Soften point, Ring and Ball, °F	110
• Thin – film – oven – test, 325°F, 5h:	
◦ Percentage weight lose	0.51
◦ Percentage penetration back gained	53
◦ Ductility of rest	100+

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In addition to standardized tests, a lot of tests are done in this research paper of Monismith and Secor (1962), which provides detailed information regarding type of tests and methods employed. Stress relaxation tests are performed using universal test machine, but the hydraulic loading system of the test machine is not used. Here, only the relaxation curves obtained from these tests are employed to make a benchmark comparison with the numerically obtained stress relaxation curves using ANSYS.

3. Using Prony Series in ANSYS

In this section, the background of viscoelastic equations employed by ANSYS is explained (2014). A material is called viscoelastic, when its strain comprises both elastic (reversible) and viscos (irreversible) parts. Under loading elastic strains develop instantly, whereas developing viscos strains take time. Such materials behave at higher temperatures like a liquid, but at lower temperatures like a stiff body. The viscoelastic material behavior in ANSYS is defined using the theory of Thermorheological Simplicity (TRS). TRS means: The reaction of a material under loading at higher temperatures and a small time period is similar to the reaction of same material at lower temperatures and a larger time period. ANSYS employs two different methods to represent viscoelastic material behavior of bodies, namely, generalized Maxwell elements (for small strains and small displacements) and Prony series (for small strains and large displacements). In this study, Prony series method is used to represent stress relaxation, since it is more robust and reliable than generalized Maxwell elements method. The equations of viscoelasticity by means of Prony series are given below,

$$s_{ij}(t) = \int_0^t 2 \cdot G(t-\tau). \frac{de_{ij}}{d_\tau} d\tau \, , \tag{1}$$

$$\sigma_K(t) = \int_0^t 3 \cdot K(t-\tau) \cdot \frac{d\Delta}{d\tau} d\tau \, , \tag{2}$$

$$\sigma_{ij}(t) = s_{ij}(t) + s_{ij} \cdot \frac{\sigma_K(t)}{3} . \tag{3}$$

Replacing Eqs. (1, 2) in Eq. (3), the following equation is supplied,

$$\sigma_{ij}(t) = \int_0^t 2 \cdot G(t-\tau). \frac{de_{ij}}{d_\tau} d\tau + \delta_{ij} \cdot \int_0^t K(t-\tau) \cdot \frac{d\Delta}{d\tau} d\tau \, . \tag{4}$$

The definitions of symbols in Eqs. (1-4) are given below,

σ_{ij} : Cauchy stress
e_{ij} : Deviatoric part of strain
Δ : Hydrostatic part of strain
$G(t)$: Deviatoric relaxation module
$K(t)$: Hyrostatic relaxation module
t : Real time
τ : Elapsed time

For Prony series (Park and Kim, 2001; Ghoreishy, 2012), the expressions of relaxation moduli are given as

$$G(t-\tau) = G_\infty + \sum_{i=1}^{n_G} G_i \cdot e^{-(t-\tau)/\tau_i^G} \, , \tag{5}$$

$$K(t-\tau) = K_\infty + \sum_{i=1}^{n_K} K_i \cdot e^{-(t-\tau)/\tau_i^K} \, , \tag{6}$$

whereas,

$$G_0 - G_\infty = \sum_{i=1}^{n_G} G_i \, , \tag{7}$$

$$K_0 - K_\infty = \sum_{i=1}^{n_K} K_i \, . \tag{8}$$

Or with introducing relative moduli,

$$a_i^G = \frac{G_i}{G_0}, \qquad a_\infty^G = \frac{G_\infty}{G_0} \, , \tag{9a}$$

$$a_i^K = \frac{K_i}{G_0}, \qquad a_\infty^K = \frac{K_\infty}{K_0} \, , \tag{9b}$$

expressions of relaxation moduli result in following forms:

$$G(t-\tau) = G_0 \cdot \left[a_\infty^G + \sum_{i=1}^{n_G} a_i^G \cdot e^{-(t-\tau)/\tau_i^G} \right] , \tag{10}$$

$$K(t-\tau) = K_0 \cdot \left[a_\infty^K + \sum_{i=1}^{n_K} a_i^K \cdot e^{-(t-\tau)/\tau_i^K} \right] . \tag{11}$$

4. Verification of ANSYS Results

A three dimensional brick element is modeled in ANSYS to enable visual observation of shear deformation. Afterwards, this three dimensional geometry is reduced into a single freedom system using necessary restraints, where only shear deformations are permitted. Below, Figs. 1 and 2 serve for illustration of applied restraints and deformed shape of FE- model used in this study.

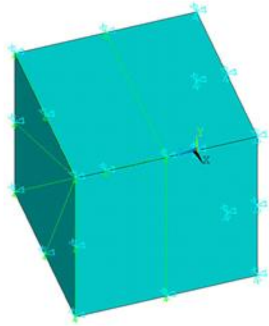

Fig. 1. The geometry and restraints of FE- model.

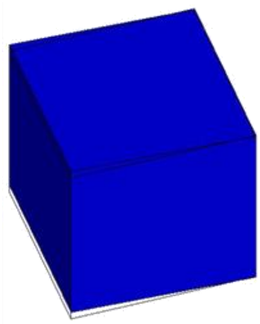

Fig. 2. The deformed geometry of FE- model.

4.1. Stress relaxation at 4.444°C

First, using Eq. (12), the values of shear relaxation module is calculated at specific times from the test output provided by Monismith and Secor (1962).

$$s_{11}(t) = G(t) \cdot 2e_{11} \, . \qquad (12)$$

Second, to calculate the constants of Prony series, the values of shear relaxation module, $G(t)$, is written in a text file.

```
/temp,4.4444
0              32316.48878
0.205789129    26394.01207
0.57880812     20759.52444
0.790884086    19802.05426
0.977393582    18700.58954
1.997118533    15781.42751
2.991277014    14268.54982
3.985016372    13127.81385
4.977917485    12190.91432
5.980458415    11549.48410
7.992246234    10655.59592
9.962960052    9774.798151
11.93996071    9376.475593
```

Third, this text file has been read into ANSYS using Viskoelastic Material Curve Fiting (VMCF). Then, again using VMCF the constant of Prony series are determined as follows,

$PRXY = 0.4$ (This value is arbitrarily selected to be able to enter K_0 in ANSYS)

$EX = 2 \cdot G_0 \cdot (1 + PRXY) = 90486.1686$

$a_1^K = 0.3494$

$\tau_1^K = 4.2965$

$a_2^K = 0.38013$

$\tau_2^K = 0.36619$

Finally, commands required to be entered in ANSYS to perform the analysis are determined as follows,

```
MP, EX, 1, 90486.1686
MP, PRXY, 1, 0.4
TB, PRONY, 1, ,2, SHEAR
TBDATA, 1, 0.34941, 4.2965
TBDATA, 3, 0.38013, 0.36619
```

The results given in Fig. 3 show that Prony series can represent the test data good in general, nevertheless the initial stress values cannot be approximated like other time points.

4.2. Stress relaxation at 25°C

Because the steps of procedure explained in the previous subsection applies also here, solely the data used at 25°C is given here.

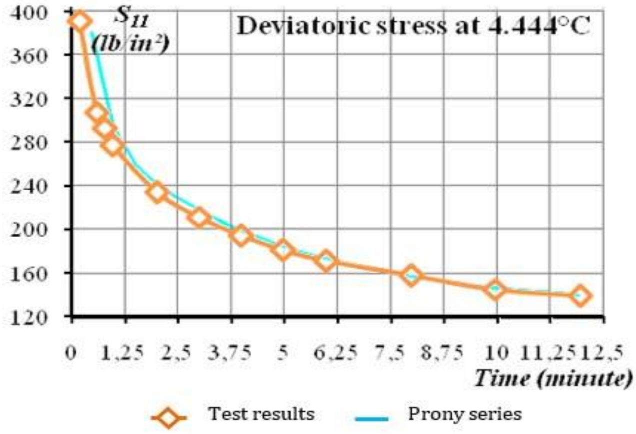

Fig. 3. Comparison of stress relaxation test results of Monismit and Secor (1962) with ANSYS results at 4.4444°C under constant strain e_{11}=0.0074.

The text file of $G(t)$:

```
/temp,25
0              12660.930968302
0.171748276     6800.598244922
0.359977447     6012.191506357
0.577698747     5609.332973279
0.795420046     5412.624705174
1.000130112     5252.897591472
2.025415275     4813.057903990
3.034219543     4608.481305161
4.056468751     4539.239994788
6.055427853     4372.431383435
8.068265603     4235.522428834
10.07763369     4189.099277561
12.09047144     4120.644800261
14.09290020     4014.422335484
16.03027280     3968.786017284
18.00798022     3961.704519632
19.98091686     3961.704519632
```

Constants of Prony series are as follows,

$PRXY = 0.4$ (This value is arbitrarily selected to be able to enter K_0 in ANSYS)

$EX = 2 \cdot G_0 \cdot (1 + PRXY) = 35450.606716$

$a_1^K = 0.13548$

$\tau_1^K = 0.56364$

$a_2^K = 0.45824$

$\tau_2^K = 0.066043$

$a_3^K = 0.093375$

$\tau_3^K = 5.4215$

The commands required to be entered in ANSYS to perform the analysis are as follows,

```
MP, EX, 1, 35450.606716
MP, PRXY, 1, 0.4
TB, PRONY, 1, , 3, SHEAR
TBDATA, 1, 0.13548, 0.56364
TBDATA, 3, 0.45824, 0.066043
TBDATA, 5, 0.093375, 5.4215
```

The results given in Fig. 4 show the same result obtained at 4.444°C, that Prony series can represent the test data good in general, nevertheless the initial stress values cannot be approximated like other time points.

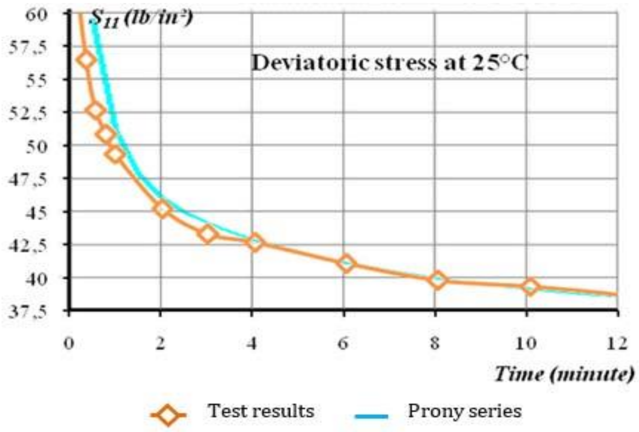

Fig. 4. Comparison of stress relaxation test results of Monismith and Secor (1962) with ANSYS results at 25°C under constant strain e_{11}=0.0047.

4.3. Stress relaxation at 60°C

Because the steps of procedure explained in the previous subsection applies also here, solely the data used at 60°C is given here.

The text file of $G(t)$:

```
/temp,60
0               3067.281051
0.071628428     2139.044636
0.182326906     1951.960548
0.337386173     1793.313241
0.536399247     1715.935261
0.758203185     1650.829999
0.924250903     1614.311184
1.356056367     1557.287954
1.942514117     1508.496424
2.939207407     1459.704894
3.935493717     1418.995396
4.976140815     1394.599631
5.950450221     1390.558614
6.946736532     1370.203866
7.87668515      1358.080817
```

Constants of Prony series are as follows,

$PRXY = 0.4$ (This value is arbitrarily selected to be able to enter K_0 in ANSYS)

$EX = 2 \cdot G_0 \cdot (1 + PRXY) = 8588.3871$

$a_1^K = 0.15448$

$\tau_1^K = 0.15675$

$a_2^K = 0.09066$

$\tau_2^K = 0.74528$

$a_3^K = 0.23555$

$\tau_3^K = 0.00090614$

$a_4^K = 0.087006$

$\tau_4^K = 3.8438$

The commands required to be entered in ANSYS to perform the analysis are as follows,

```
MP, EX, 1, 8588.3871
MP, PRXY, 1, 0.4
TB, PRONY, 1, , 4, SHEAR
TBDATA, 1, 0.15448, 0.15675
TBDATA, 3, 0.09066, 0.74528
TBDATA, 5, 0.23555, 0.00090614
TBDATA, 7, 0.087006, 3.8438
```

Fig. 5 supports the results obtained in Figs. 3 and 4, that the Prony series are in good agreement with the test data except the initial values.

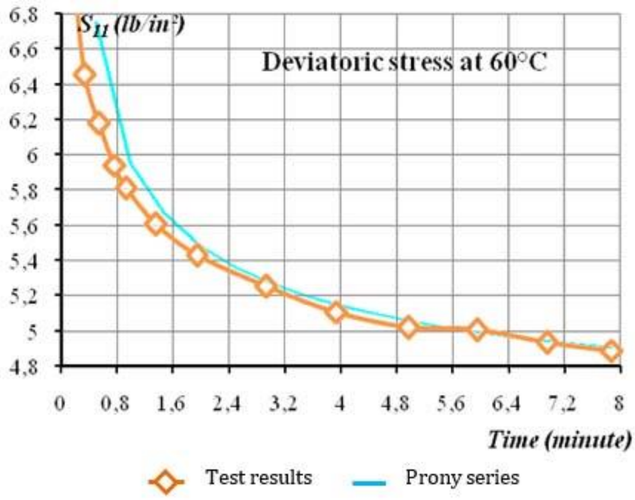

Fig. 5. Comparison of stress relaxation test results of Monismith and Secor (1962) with ANSYS results at 60°C under constant strain e_{11}=0.0018.

5. Results and Discussion

From the assessment of Figs. 3, 4 and 5; it is seen that Prony series approximate the relaxation test data well except the initial values. The reason of this error is that the relaxation test data is composed of scattered test data, whereas numerically calculated stress values using Prony series form a smooth curve. In addition to Figs. 3-5, Figs. 6-8 are given below to indicate the amount of error in calculated stress depending on selected time step size in minutes. It is concluded that the error in calculated stress values rise proportional to time step size. Briefly, the smaller time step size, the better for accuracy of stress results calculated using Prony series in ANSYS.

6. Conclusions

In this study it is verified that Prony series in ANSYS can approximate the stress strain behavior of viscoelastic bodies. In addition the effect of time step size on the amount of error occurred in calculated stress values is presented. Briefly, Prony series can approximate the general stress relaxation test result successfully except for the initial stress values due to the scattered stress values of test data versus smooth transition of Prony series curve. This result is supported by the curves corresponded at 4.444°C, 25°C and 60°C. Later conclusion of this study is, the smaller the selected time step size for numerical calculation using Prony series, the better accuracy is obtained.

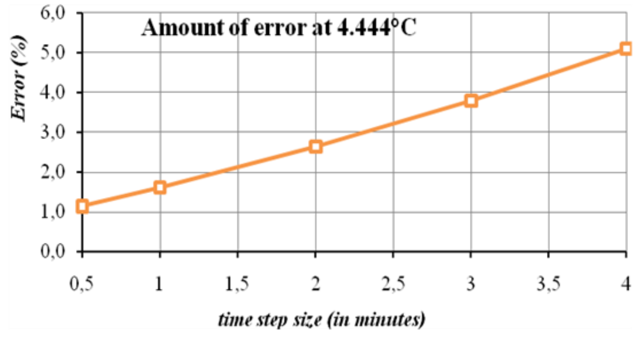

Fig. 6. The amount of error in max stress depending on time step size (in minutes) at 4.4444°C.

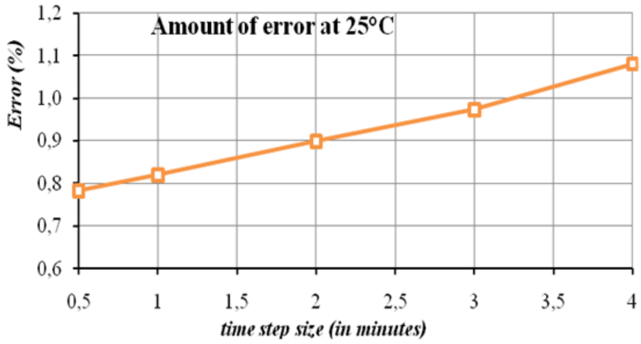

Fig. 7. The amount of error in max stress depending on time step size (in minutes) at 25°C.

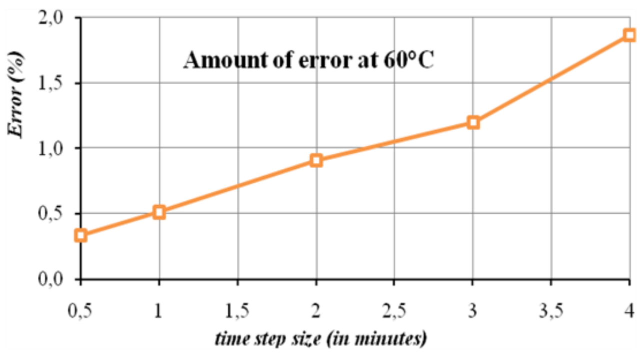

Fig. 8. The amount of error in max stress depending on time step size (in minutes) at 60°C.

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