Non-linear behaviour modelling of the reinforced concrete structures by multi-layer beam elements

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ABSTRACT

A two-dimensional multi-layered finite elements modeling of reinforced concrete structures at non-linear behaviour under monotonic and cyclical loading is presented. The non-linearity material is characterized by several phenomena such as: the physical non-linearity of the concrete and steels materials, the behaviour of cracked concrete and the interaction effect between materials represented by the post-cracking field. These parameters are taken into consideration in this paper to examine the response of the reinforced concrete structures at the non-linear behaviour. Two examples of application are presented. The numerical results obtained, are in a very good agreement with available experimental data and other numerical models of the literature.

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1. Introduction

The non-linear behaviour modelling of reinforced concrete structures is an important objective for the civil engineering researchers. The response of a structure under a loading results from a strong interaction between the materials effects (local non-linearity), the structures effects (geometry, distribution of forces and stiffness, links) and the environment effect (soil-structure interaction). The local non-linearities are related particularly to the formation, the opening and reclosure of cracks, on one hand, to the link and to the behaviour of the reinforcements (plasticity of steels) on the other hand. A good description of these phenomena has to be done in order to represent the variations of the structural stiffness and to have access to the behavior until to the collapse (Khebizi and Guenfoud, 2015).

In this paper we have presented a numerical method for modelling planar reinforced concrete structure (2D) under static and cyclical loading. This method uses multi-layered beams elements of which the stiffness matrix is computed using a beam discretization according to the height in superimposed successive layers (Fig. 1). The summation of these layers allows the calculation of stiffness in a correct manner and takes into account the behaviour variations (Khebizi and Guenfoud, 2015). The Bernoulli hypothesis (section remaining plane and perpendicular to the neutral axis of the beam) confers for different layers a uniaxial behaviour. Hence, this allows as to treat the local behaviours through uniaxial laws for the concrete and steel, laws that are assigned to each layer. The calculation of inelastic efforts is carried through to an iteration method based on the initial secant stiffness.

A particular treatment is reserved for the layers including simultaneously concrete and steel (Khebizi and Guenfoud, 2015). The behaviour of the mixed layers (Fig. 1.) is homogenized by a mixing law permitting to calculate the stress layer in proportion to each material:

$$\sigma_{layer} = \left(1 - C_{a/b}\right)\sigma_{concrete} + C_{a/b}\,\sigma_{steel}\,, \qquad (1)$$

where σ_{layer} denote axial stresses in the layer, $\sigma_{concrete}$ and σ_{steel} axial stresses in the concrete and the steel respectively in the layer and $C_{a/b}$ is the ratio surface of steel within the reinforced layer. The steel-concrete adherence is supposed to be perfect (identical strain of the two materials at their frontier: $\varepsilon_{concrete} = \varepsilon_{steel}$).

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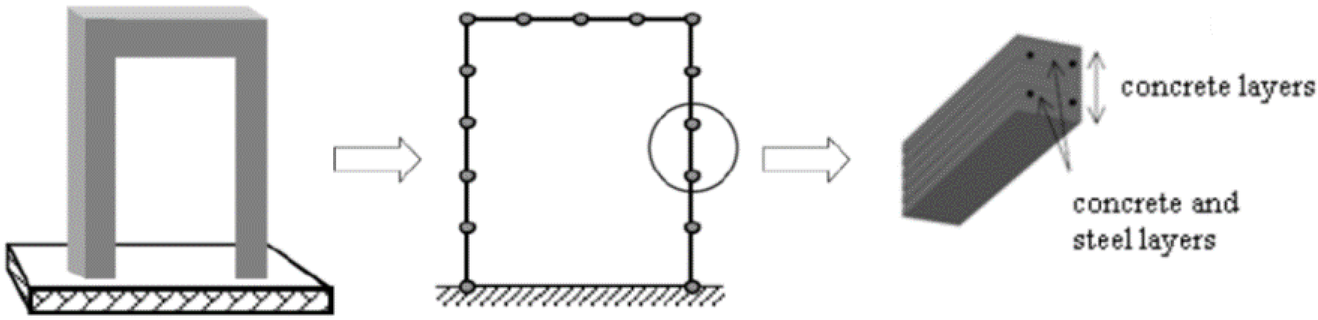

Fig. 1. Discretisation principal of reinforced concrete structures with multi-layered beam.

2. Formulation of Multi-Layered Beam Element

The elements used are beams with tow nodes, the Bernoulli hypothesis confers on the various layers a uniaxial behaviour. The relation giving the element equilibrium is obtained by the virtual work principle, expressed in terms of generalized coordinates.

$$\delta U^T F = \int_\Omega \delta\varepsilon^T \sigma dV = \int_\Omega \delta(BU)^T \sigma dV = \int_\Omega \delta U^T B^T \sigma dV \,, \quad (2)$$

where B depends on the derived shape functions.

If can be introduced a behaviour law with damage and inelastic,

$$\varepsilon = \frac{\sigma}{E(1-D)} + \varepsilon_{an}(D) \Rightarrow \sigma = E(1-D)(\varepsilon - \varepsilon_{an})\,. \quad (3)$$

The virtual work principle takes the following form:

$$\delta U^T F = \int_\Omega \delta U^T B^T E(1-D)(\varepsilon - \varepsilon_{an}) dV$$
$$\Rightarrow F = \int_\Omega B^T E(1-D)(\varepsilon - \varepsilon_{an}) dV\,. \quad (4)$$

Eq. (4) can be rewritten in the following form:

$$F = \left[\int_\Omega B^T E(1-D) B dV\right] U - \int_\Omega B^T E(1-D)\varepsilon_{an} dV\,. \quad (5)$$

By putting:

$$\begin{cases} K = \int_\Omega B^T E(1-D) B dV \\ F = -\int_\Omega B^T E(1-D)\varepsilon_{an} dV \end{cases}, \quad (6)$$

we end up with the final system to solve:

$$F = KU + F_{an}\,. \quad (7)$$

K is the element stiffness matrix:

$$K = \int_0^l B^T k_s B dx\,. \quad (8)$$

The section stiffness matrix is expressed as follows:

$$k_s = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}. \quad (9)$$

$$k_{11} = \int_s E ds \quad k_{12} = k_{21} = \int_s E y ds \quad k_{22} = \int_s E y^2 ds\,. \quad (10)$$

The discretization of the cross-section in superimposed layers according to the Bernoulli hypothesis allows to be obtaining the following stiffnesses (Belmouden, 2004; Khebizi and Guenfoud, 2015; Khebzi, 2015):

$$k_{11} = \sum_{k=11}^{nlayers} E_k A_k\,,$$

$$k_{12} = k_{21} = \sum_{k=11}^{nlayers} E_k y_k A_k\,,$$

$$k_{22} = \sum_{k=11}^{nlayers} E_k y_k^2 A_k\,. \quad (11)$$

E_k, A_k and y_k are respectively the Young's modulus, the layer area and the centre position layer to the reference axis.

3. Damage Model for the Concrete (Unilateral Model)

The unilateral model (Laborderie, 2003; Kotronis, 2000; Davenne et al., 2003) is an isotropic model where two scalar damage variables, are used to describe the consequences of the evolution of the mechanical characteristics of material, the irreversible strains and the unilateral effect when the sign of the stresses changes. Considering the partition of the strain tensor as the sum of an elastic part and an inelastic part, calculated as follows:

$$\varepsilon = \varepsilon_e + \varepsilon_{an}\,, \quad (12)$$

$$\begin{cases} \varepsilon_e = \frac{\sigma^+}{E_0(1-D_1)} + \frac{\sigma^-}{E_0(1-D_2)} + \frac{V}{E_0}(\sigma - (Tr\sigma)I) \\ \varepsilon_e = \frac{\beta_1 D_1}{E_0(1-D_1)}\frac{\partial f(\sigma)}{\partial\sigma} + \frac{\beta_1 D_2}{E_0(1-D_2)} I \end{cases}, \quad (13)$$

where E_0 is the initial Young's modulus and ν the Poisson's ratio. ‹•›+ denotes the positive part of a tensor, D_1 and D_2 are scalar damage variable in tension and scalar damage variable in compression respectively (their evolution between 0 – i.e, healthy material- to 1 - i.e, broken material- is related to the local elastic energy). β_1 and β_2 are material parameters to be identified in order to describe the evolution of the inelastic strains can be described, $f(\sigma)$ is the crack closure function which cancels the inelastic strains of the tension during the recovery of stiffness and σ_f the crack closure stress:

$$\begin{cases} Tr(\sigma)\ \epsilon\ [0,+\infty[\rightarrow \frac{\partial f(\sigma)}{\partial \sigma} = 1 \\ Tr(\sigma)\ \epsilon\ [-\sigma_f, 0[\rightarrow \frac{\partial f(\sigma)}{\partial \sigma} = (1 + \frac{Tr(\sigma)}{\sigma_f}) = 1\ . \\ Tr(\sigma)\ \epsilon\ [-\infty, -\sigma_f[\rightarrow f(\sigma) = 0.1 \end{cases} \quad (14)$$

The evolution laws for the damage are finally written as:

$$D_i = 1 - \frac{1}{1+(A_i(Y_i - Y_{0i}))^{B_i}}\ , \quad (15)$$

where Y_i is the variable associated to damage (energy refund ratio, tension or compression). A_i and B_i are material constants. Y_{0i} is the damage threshold (tension or compression).

The stress-strain relation-ship and the crack closure function in the uniaxial model (Fig. 2) can be written as follows:

$$\varepsilon = \frac{\sigma^+}{E_0(1-D_1)} + \frac{\sigma^-}{E_0(1-D_2)} + \frac{\beta_1 D_1}{E_0(1-D_1)} F(\sigma) + \frac{\beta_2 D_2}{E_0(1-D_2)}\ , \quad (16)$$

$$\begin{cases} F(\sigma) = 1\ if\ \sigma \geq 0 \\ F(\sigma) = 1 - \frac{\sigma}{\sigma_f}\ if\ -\sigma_f \leq \sigma \leq 0\ . \\ F(\sigma) = 0\ if\ \sigma < -\sigma_f \end{cases} \quad (17)$$

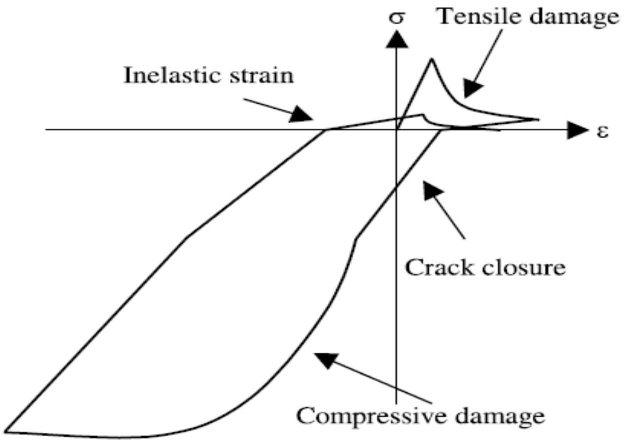

Fig. 2. Uniaxial response of the unilateral model.

4. The Behaviour of Steel

In order to describe the non-linear behaviour of reinforcement, one chooses the classical plasticity model which take into account the non-linear kinematic hardening is used.

The reinforcement has a privileged orientation and the uniaxial law is sufficient to reproduce its behaviour (Kotronis, 2000; Khebizi et al., 2014). The reinforcement can be considered as concentrate or diffuse in the concrete elements. In the first case, elements bars with non-linear behaviour, whose position and section coincide with the position and section of real reinforcement, are used. In the second case the behaviour of the mixed layers (Fig. 1) is homogenized by a law of mixtures to calculate the stress layer in proportion to each material (The adherence steel-concrete is supposed perfect; i.e, identical strain on the two materials at their frontier). Thus, in each layer (Mazars, 2001):

$$\begin{cases} \varepsilon_{concrete} = \varepsilon_{steel} \\ E = (1 - C_{a/b}) \times E_{an-concrete} + C_{a/b} \times E_{an-steel} \\ \varepsilon_{an} = (1 - C_{a/b}) \times \varepsilon_{an-concrete} + C_{a/b} \times \varepsilon_{an-steel} \\ C_{a/b} = \frac{A}{B} \end{cases}\ , \quad (18)$$

where E is the homogenized Young's modulus (steel + concrete), $C_{a/b}$ is the ratio surface of reinforcement, A is the relative steel air within the reinforced layer and B is the relative concrete air within the reinforced layer.

$\varepsilon_{an\text{-}concrete}$ is the inelastic concrete stain, $\varepsilon_{an\text{-}steel}$ is the inelastic steel stain; and ε_{an} is the inelastic strain homogenized of the reinforced layer (steel + concrete).

5. Applications

5.1. Column buckling

The purpose of this example is to perform a modelling of a reinforced concrete column with rectangular section subjected to an axial loading with an eccentricity e=1.50cm (Fig. 3(a)). The same column was studied experimentally by Fouré (Fouré, 1978) and numerically discretization by Franz (Franz, 1994) with multi-fiber elements (Willam-Warnke behaviour law).

In this paper, the column is modeled by 11 multi-layered elements with 2 nodes and 2 integration points. The section of each element is discretized by 6 superimposed layers, of which 4 in concrete alone and 2 in concrete and steel (Fig. 3(b)). The eccentric axial load is modelled by a centered axial load F and a bending moment M=F×e. The weight of the column is neglected. The concrete behaviour obeyed the Laborderie damage model (unilateral law behaviour). The characteristics considered for the concrete are shown on Table 1. The steel behaviour is supposed elastoplastic with kinematic hardening. The steel's characteristics used are: Young's modulus of 200,000 MPa and elastic limit of 400 MPa.

Fig. 4 shows the load variation according to the horizontal displacement of the top of the column. This figure gives a comparison between the results obtained by the present modelling, the experimental results of Fouré (1978) and those obtained by Franz (1994). As it can be seen from Fig. 4, there is a good agreement between these models.

Table 1. Concrete characteristics for Laborderie model (Khebizi and Guenfoud, 2015).

Parameter	Value
Young's modulus	30000e6 Pa
Density	2500 kg/m^3
Damage threshold in tension	220 Pa
Damage threshold in compression	9000 Pa
Damage parameter in tension	9e-3 Pa^{-1}
Damage parameter in compression	5.30e-6 Pa^{-1}
Parameter for tension	1.20
Parameter for compression	1.40
Permanent strain activation in tension	1.00e6 Pa
Permanent strain activation in compression	-40e6 Pa
Crack closure stress	1.30e6 Pa

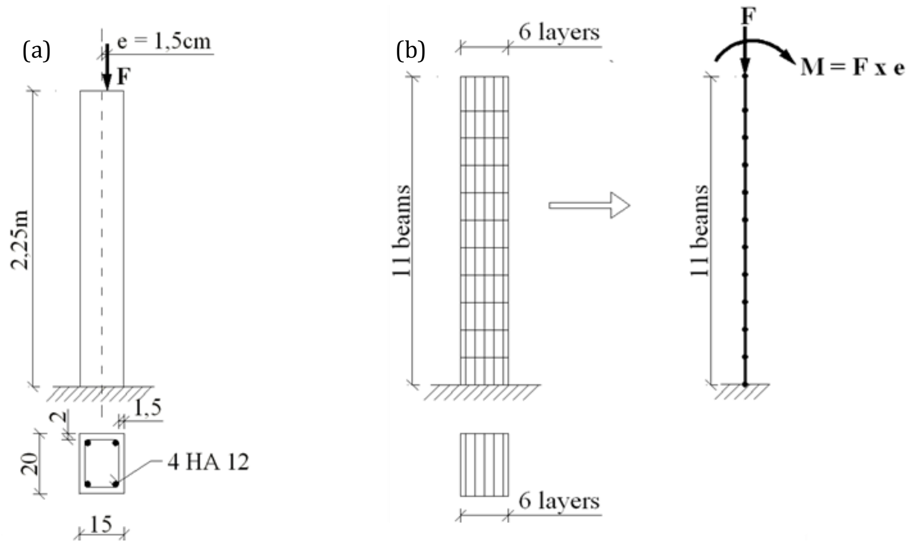

Fig. 3. Fouré Column: (a) Geometry and loading system; (b) Numerical model (2D).

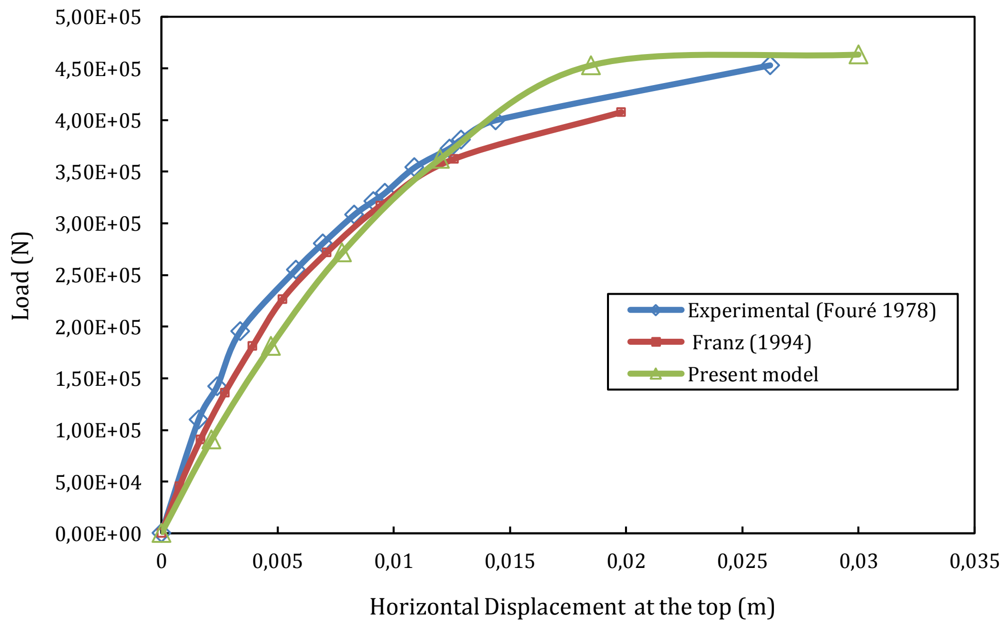

Fig. 4. Load-displacement graph of the top column.

5.2. Cyclic response modelling of a reinforced concrete beam

This example is used to validate the cyclic bending behaviour of a reinforced concrete beam (Fig. 5(a)). The loading is composed of an amplitude cycle of 1mm followed by an amplitude cycle of 2mm (Fig. 5(b)).

The model used in this paper is a structure of 20 beams elements with 2 nodes and 2 integrations points. The section of each element is discretized by 10 superimposed layers, including 8 out of concrete alone and 2 simultaneously including concrete and steel (Fig. 6). The same concrete and steel behaviour as the previous example is used in this case.

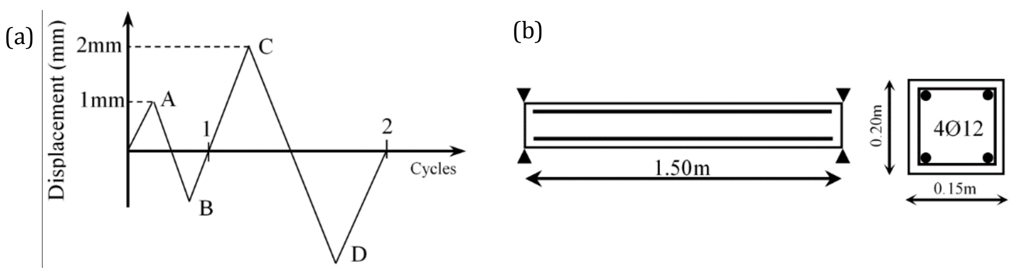

Fig. 5. Reinforced concrete beam: (a) Geometry; (b) Numerical model (2D).

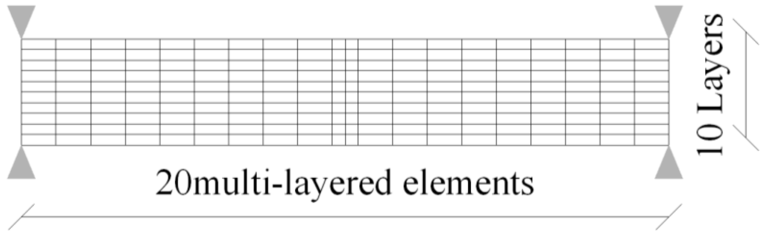

Fig. 6. Beam discretisation in multi-layered elements.

The cyclic response of the beam shown on the Fig. 7, is compared with the test results. As it can be seen from this figure, a very good coherence between the two results. This figure shows also presents a comparison of the load-displacement response obtained by the present simulation (modeling by multi-layered elements with a Laborderie law) and that obtained by Matallah (2009). The two numerical models gave similar results in first cyclic loading. However, for the second cyclic loading, a light difference is observed.

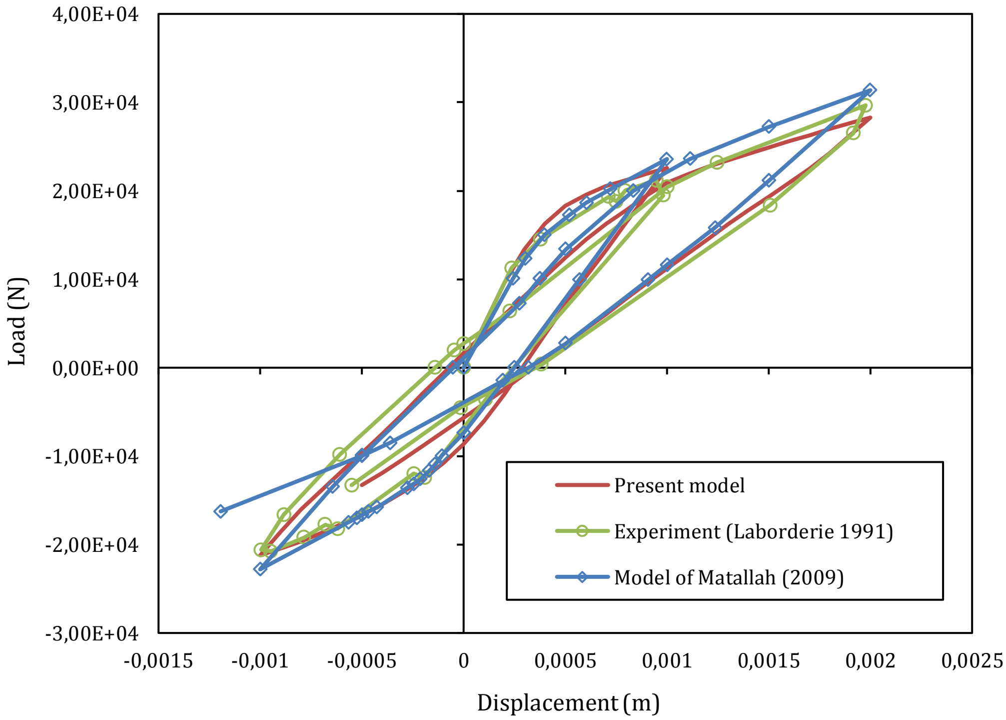

Fig. 7. Load-displacement response for different models.

Fig. 8 presents the damage chart of the beam for the first cyclic loading. In the loading state «A», the higher part of the beam is damaged (Fig. 8(a)). The loading state «B» corresponds to an opposed loading, the damage state initially product is always stored whereas a new damage state is created in the lower part of the beam (Fig. 8(b)). The damage chart of the beam during the second cyclic loading is shown in Fig. 9.

6. Conclusions

A simple modelling of the non-linear behaviour of the reinforced concrete structure is presented. It uses multi-layered beam elements which obeyed the Bernoulli hypothesis to confer to the various layers a uniaxial behaviour. It also allows the description of the structures damage state during a loading. Two examples of applications were presented. The first on is a column buckling test (Fouré column) and the second one was a beam subjected to a 3 points flexion with cyclic loading applied to the mid-span of the beam (cyclic bending). According to these examples it was noticed:

- A very good coherence between the present numerical results and the experimentation results.
- A good concordance between the results of present numerical models and those of other numerical models of references.
- The non-linear analysis reflects the real behavior of reinforced concrete structures.
- If the material is discharged after having undergone a damage state, it restores its stiffness, the crack previously open are closed again but the internal structure of material remains always damaged.

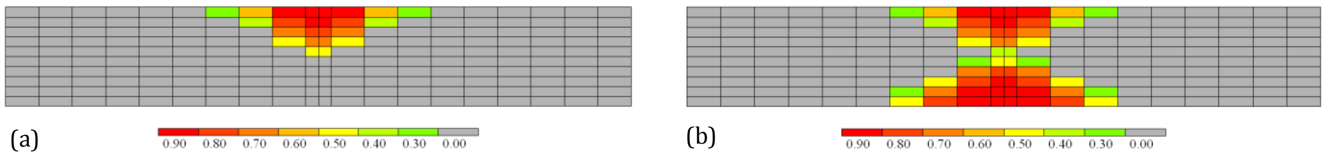

Fig. 8. Damage chart in tension «D1» for the first cyclic loading: (a) Loading State «A»; (b) Loading State «B».

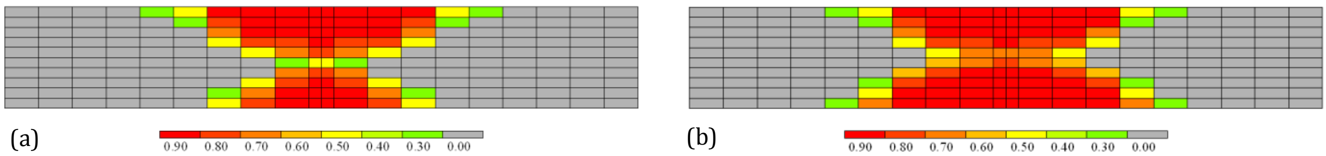

Fig. 9. Damage chart in tension «D1» for the second cyclic loading: (a) Loading State «C»; (b) Loading State «D».

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