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Challenge Journal of STRUCTURAL MECHANICS

Research Article

Valorization of pinus and cedrus acicular leaves as bio-based fibers for reinforced cementitious composites: Extraction, microstructural characterization and mechanical performance

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ABSTRACT

In modern construction applications, various cement-based composite material systems are being developed using forest products and forest waste, such as bio-based fibers. These composites serve as sustainable building materials, including wood-cement composites, fiber-reinforced panels, lightweight building elements, and precast building components, especially in non-load-bearing architectural applications. In this study, the physical and mechanical properties of a composite produced in a similar manner, using acicular leaves of *Pinus nigra* and *Cedrus libani* as natural fibers, were examined. In this context, the fibers underwent a multi-stage pre-treatment, including hot water extraction, alkaline treatment, and $\text{Ca}(\text{OH})_2$ mineralization, followed by characterization; the treatments were verified by SEM, EDS, and FTIR analyses. Short *Cedrus* fibers are randomly distributed in the matrix to form an isotropic structure, while long *Pinus* fibers are oriented perpendicular to the load direction to obtain an orthotropic structure. Unit weight, ultrasonic pulse velocity, dynamic modulus of elasticity, and compressive, flexural, and splitting tensile strengths were determined. When compared, short, isotropic cedar fibers showed higher compressive strength, while long, oriented pine fibers exhibited superior bending and splitting tensile performance.

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1. Introduction

Research in building materials aims not only to meet high-strength requirements but also to satisfy multidimensional criteria such as lightness, material efficiency, workability, and architectural harmony. Extensive academic studies have examined fiber-reinforced cementitious composites, primarily because of their benefits in crack control, increased ductility, and modifications to the internal structure. Incorporating fibers into cementitious matrices alters mechanical behavior through fiber-matrix interaction, enabling the development of composite systems.

The primary purpose of reinforcing cementitious composites with fibers is to mitigate premature cracking and limited ductility arising from the brittle behavior and low tensile strength of conventional cementitious composites. Fibers added to the matrix delay the transformation of microcracks into macrocracks via the crack-bridging mechanism, thereby significantly increasing tensile and flexural strengths, toughness, and service life (Anas et al. 2022; Ünal and Canbaz 2025; Urtekin and Çelik 2025). Furthermore, fiber reinforcement substantially improves not only the mechanical strength of composites but also its post-damage deformation capacity and load-bearing behavior, making it an

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indispensable engineering approach in high-performance, long-lasting cementitious composites designs (Pham 2025). Natural fibers, while offering a sustainable reinforcement alternative with advantages such as renewability, low density, biodegradability, and low environmental impact, significantly increase mechanical strength, toughness, and deformation capacity by crack bridging and microcrack formation in the matrix. Literature studies indicate that the crack-bridging mechanism in natural fiber reinforced cementitious composites (NFRCC) is governed by fiber geometry, volume fraction, and fiber-matrix interfacial quality, and that this mechanism improves critical performance indicators, including post-peak bearing capacity, energy absorption, and residual strength in flexural and splitting tensile behavior (Lilargem Rocha et al. 2022). Additionally, natural fibers offer environmental advantages, as they are produced with a lower carbon footprint and lower energy consumption than synthetic fibers (Olanrewaju et al. 2025). However, achieving homogeneous distribution of natural fibers in matrix and improving the fiber-matrix interface pose significant design challenges for workability and durability; current studies aim to overcome these limitations through fiber surface modification, chemical pretreatments, and hybrid fiber systems. In the literature, natural fibers of vegetable (lignocellulosic) origin are mainly used. The classification of these fibers as bast fibers (hemp, jute, flax), leaf fibers (sisal, abaka), fruit fibers (coir), wood/cellulose origin fibers (wood fiber, kraft cellulose, micro/nano-cellulose), and agricultural residue fibers (rice husk, straw, bagasse) is widely adopted to explain how differences in fiber geometry, absorbency, surface roughness, and mechanical properties affect workability in fresh state and tensile behavior, toughness, and compressive strength in hardened state (Hamada et al. 2023). Vegetable fibers, from chemical and structural perspectives, have a lignocellulosic matrix consisting of cellulose, hemicellulose, and lignin. Cellulose content increases the fiber's tensile capacity and rigidity. In contrast, hemicellulose/pectin fractions enhance the fiber's hydrophilic behavior and interact with the high alkalinity of the cementitious medium, a critical mechanism in research on NFRCC (Camargo et al. 2020). Durability and fiber-matrix interface stability are the main topics of discussion in NFRCC, and since the high alkalinity of Portland cement can lead to degradation and mineralization of lignocellulose fibers over time, reducing the alkalinity of the cement matrix with pozzolanic admixtures and modifying the fibers by chemical or physical pretreatments stand out as the two main strategies in the literature to improve long-term performance (Camargo et al. 2020; Merta et al. 2017). A prominent trend in natural fiber research is the development of hybrid fiber systems by enhancing fiber surface roughness and fiber-matrix adhesion through pretreatments such as alkali treatment. It is emphasized that controlled removal of the hemicellulose/pectin fraction can increase interface strength. Nevertheless, excessive processing can reduce the fiber's tensile capacity, so optimization is a fundamental design parameter (Hamada et al. 2023; Camargo et al. 2020). Pretreatment can also improve fiber-matrix bond strength (Tarhan et al. 2025). Current literature reveals that the focus in NFRCC is no longer only

on strength increase, but also on microstructure-interface engineering, crack monitoring, durability and workability optimization, and that this holistic approach significantly improves the flexural and tensile behavior of this holistic approach, even if the compressive strength does not always increase, and changes the damage mode from brittle to controlled damage; It shows that the combination of fiber chemistry, geometry, pretreatment and matrix design has become a standard view in the field (Lilargem Rocha et al. 2022; Merta et al. 2017). In the NFRCC literature, regionally abundant biomass fibers such as bamboo, coconut, and palm fiber stand out; Studies with bamboo fibers reveal that fibers can improve mechanical strength and durability by limiting brittle cracking, but processability, water absorption behavior, and fiber-matrix interface design are critical parameters that determine performance (Dhakal et al. 2024; El Bourki et al. 2023). It has been reported that coconut/coir fibers significantly improve the tensile and flexural behavior of NFRCC. The optimal fiber ratio should be determined by considering workability and porosity. In the study of Abasi et al., it was stated that the 28-day flexural strength increased by 21%, 25% and 15.24%, the tensile strength increased by 29.66%, 38.66% and 22.81%, respectively, with the addition of 0.4–0.8–1.0% fiber by weight of cement, and the best performance was obtained with 0.8% fiber content (Kavitha and Kala 2017). In studies examining higher fiber volume ratios, an increase in tensile strength under compressive and splitting was observed when an appropriate mixture design was employed. The findings summarized by Lin et al. indicate that approximately 25% increases in compressive strength and 20.4% increases in splitting tensile strength can be achieved with 1.5% coconut fiber. These gains are closely related to matrix compaction, fiber-matrix adhesion, and the fiber-bridging mechanism (Lin et al. 2025). Studies on palm fibers reveal that the ductility and tensile ductility of NFRCC can increase significantly with increased fiber content. Nevertheless, adverse effects, such as loss of workability and reduced compressive strength, may occur if the optimal fiber ratio is exceeded (Ahmad et al. 2023; Althoey et al. 2022; Uzun 2024). Examples of fiber systems of natural origin and waste show that fibers behave not only as a reinforcing element but also as an active component that changes the fracture mode through friction and mechanical coupling on the crack surface, with Kilani et al. reporting an increase in tensile strength of about 52.5% in a system containing banana origin components, which quantitatively confirms this effect (Kilani et al. 2022).

The most critical design parameter for forest-derived fibers is fiber-matrix interface compatibility. In the study reported by Lin et al., fiber-matrix adhesion improved significantly after 2 hours of pretreatment of waste wood fiber with 2.5 M NaOH, and the flexural strength of the mortar increased by 34.7% relative to the control mixture (Althoey et al. 2022). In NFRCC containing sawdust, compressive strength generally decreases as the substitution rate increases because of higher porosity, but limited positive effects can be observed at low or optimum levels. In one study, an approximately 3% increase in compressive strength was reported with a 10% substitution of sawdust (El-Nadoury 2021). Studies on wood-cementitious composites reveal that density decreases as wood con-

tent increases (by up to ~11% over 28 days). In comparison, compressive strength can increase from early to later ages (~15%), indicating a balance between the "mitigation effect" and age-related strength gain (Dias et al. 2022). Similarly, in the literature on the use of lignocellulosic phases, such as wastepaper and cellulose fibers, in cementitious systems, it is commonly reported that these fibers provide crack control and increase toughness. However, at high dosages, they may adversely affect compressive strength due to increased porosity and water demand (Fernando et al. 2023).

In pine acicular leaf NFRCC, improvements in tensile behavior were reported to depend on fiber content. In one study, tensile strength increased by 23.5% at 0.5% fiber content and by 5.7% at 1.0% fiber content. It was emphasized that the bridging effect may be more effective at low dosages, whereas losses due to workability and porosity may predominate at high dosages (Khalid and Ali 2024). Studies examining how pine acicular-leaf fibers affect fracture mode and ductility, particularly in high-performance matrices such as self-compacting and ultra-high-performance concretes, indicate that this fiber type remains a limited area of research (Cogurcu 2022). Similarly, studies evaluating pine acicular-leaf fibers at the scale of cementitious mortar, considering viscosity, density, and mechanical properties together, are included in the literature (Zain-Ul-Abdin and Khitab 2020). Studies showing that *Cedrus libani* acicular-leaf is used in gypsum-based board products rather than concrete reveal that it can be used in mineral binder composites (Şahin and Çam 2022). The widespread investigation of cedar-derived sawdust and similar wastes in cement boards and wood-cement board systems presents the compatibility and mechanical behavior of cedar-derived lignocellulosic phases with cement (El Hamri et al. 2024). In summary, the literature findings show that the mechanical contribution of natural fibers is mainly realized through the crack control mechanism; It reveals that the strength and toughness are significantly improved, especially in flexural and splitting tensile behavior, thanks to the fact that the fibers reduce the stress accumulation at the crack end by bridging the microcracks and delaying the formation of macrocracks.

According to data from the General Directorate of Forestry for 2024, approximately 11 million hectares of *Pinus* and 580 thousand hectares of *Cedrus* are planted in Turkey. The acicular leaves of *Pinus* are long, whereas those of *Cedrus* are short. *Cedrus libani* A.Rich., a subspecies of *Cedrus*, has short acicular leaves ≈15–35 mm long, a rhomboidal cross-section, and a high specific surface area. It has a dense epicuticular wax layer typical of the Pinaceae family and a lignocellulosic chemical composition comprising cellulose, hemicellulose, lignin, resins, and hydrophobic extractives. In addition, the gradual deterioration of the wax layer due to natural aging increases surface roughness (Güney et al. 2016). *Pinus* species with acicular leaves, belonging to the subspecies *Pinus nigra*, are ≈75–175 mm long and 0.8–2.1 mm wide, and stand out for their fibrous morphology and high length/diameter ratio. These acicular leaves, characterized by anatomically dense sclerenchyma tissue, well-developed vascular bundles, and an epicuticular wax layer that may vary with environmental conditions, ex-

hibit important chemical properties that enhance the relative resistance of fibers to water, thanks to lignin and resinous extractive components, along with high cellulose and hemicellulose content (Öztürk Çoban and Seki 2025). *Cedrus libani* A.Rich., thanks to its short length, high specific surface area, and increasing surface roughness with age, has the potential to reduce density, control microcracking, and increase durability by forming an isotropic network of fibers in the cemented matrix; *Pinus nigra* acicular leaves, on the other hand, are a natural fiber that has the potential to enhance mechanical performance under loading conditions where tensile and flexural effects are dominant, thanks to their long, routable fiber geometry, high length/diameter ratios, and sufficient mechanical stiffness.

Today, various cement-based composite material systems are being developed using forest products and forest waste. These composites are emerging as alternatives, especially in non-load-bearing architectural applications. Forest products and forest waste are used as fibers or aggregates in composite systems, or in precast concrete composites containing fibers and in cement-based pressed board products. Mineral aggregates are rarely used in the design of these composites; instead, forest products and forest waste serve as reinforcement materials, such as fibers and chips, and the matrix consists of cement paste. Studies in the literature reveal that the mechanical properties of cement-based composites that do not contain aggregates and where wood waste is evaluated have been extensively investigated (Canbaz et al. 2021; Garcez et al. 2016; Mahzabin et al. 2013). These studies provide a clearer understanding of fiber-matrix interface interactions and charge transfer mechanisms. Similarly, in this study, only cement, fiber, and water were used in the production of the specimens, and no aggregate was used. No control specimens consisting of cement and water were produced.

In this study, dried, fallen acicular leaves of *Pinus nigra* Arnold and *Cedrus libani* A.Rich. were used as natural fibers, and the physical and mechanical properties of the resulting specimens were examined. SEM, EDS, and FTIR analyses were examined as qualitative and semi-quantitative evidence to verify the surface modification and mineralization of natural fibers before their incorporation into the cemented matrix. *Cedrus libani* A.Rich. short acicular leaves were incorporated into the mixture to form an isotropic fiber arrangement by being distributed randomly in all directions. By contrast, *Pinus nigra* Arnold's long acicular leaves were oriented perpendicular to the load direction and in one direction during specimen production, resulting in an orthotropic fiber structure. Thus, the effects of fiber length and fiber orientation on mechanical behavior were comparable. Aggregates and admixtures were omitted from the mixture to observe the physical and mechanical effects of the fibers fully. At the end of the 7 and 28 day curing periods, unit weight, ultrasonic pulse velocity, compressive and flexural strengths, and splitting tensile strength were measured for the produced specimens. Unit weight and ultrasonic pulse velocity measurements were used to reveal the effect of fiber addition and fiber orientation on matrix density and internal structure continuity; compressive, flexural, and splitting tensile tests were con-

ducted to evaluate how natural fiber-cement matrix interaction is reflected in mechanical performance. The role of fiber orientation in limiting crack progression and in load transfer mechanisms was examined in detail, particularly in flexural and tensile behavior.

2. Materials and Methods

2.1. Materials

In specimen production, CEM I 52.5 R cement from the Çimsa cement factory was used, and the cement properties are presented in Table 1. *Pinus nigra* Arnold and *Cedrus libani* A.Rich were used as natural fibers. The dried, fallen acicular leaves of the trees were used. The photograph and the technical specifications of the acicular leaves are presented in Table 2. In the experiments, tap water was used, and the laboratory chemical analysis report for the drinking water treatment plant is presented in Table 3.

2.2. Fiber pretreatment

Dry acicular leaves of *Pinus nigra* Arnold and *Cedrus libani* A.Rich, collected from the forest ecosystem, under-

went extensive multistage pretreatment before being incorporated into cement mixtures. Initially, the material was separated to remove biological contaminants, including soil, sand, ash, and fungal residues. The cleaned material was thoroughly rinsed with deionized water to remove surface contaminants, then dried in an oven at 60 °C for 24 hours. Subsequently, water-soluble organic compounds, including tannins and phenolics that may interfere with cement hydration, were extracted with hot water (80 °C) for 3 hours. This process was repeated twice with fresh deionized water. After extraction, the residues were filtered and dried for chemical treatment. Alkaline pretreatment was performed by soaking the materials in a 2% NaOH aqueous solution for four hours under continuous stirring, thereby removing hemicellulose components and partially increasing surface roughness. The processed residues were rinsed with deionized water until the pH approached neutrality, then dried in the oven. Mineralization was achieved by soaking the materials in a saturated $\text{Ca}(\text{OH})_2$ solution at 25 °C for 24 h, with a solid-to-liquid ratio of 1:10 (g mL^{-1}). The process was repeated twice with a fresh $\text{Ca}(\text{OH})_2$ solution to ensure uniform surface coverage. Finally, the mineralized residues were carefully rinsed to remove excess surface scale and then dried at 60 °C before being prepared for use in cementitious composite mixtures.

Table 1. Cement properties.

Chemical Content (%)				Physical Properties		
CaO	64.70	MgO	1.35	Density (g/cm^3)	3.06	
SiO ₂	21.20	Na ₂ O	0.38	Blain (m^2/g)	4300	
Al ₂ O ₃	4.60	SO ₃	3.50	Setting time (min)	Initial 95	Final 125
Fe ₂ O ₃	0.28	K ₂ O	0.20	Compressive strength (MPa)	7 th day 50	28 th day 60
LOI	3.5	Cl	0.01			

Table 2. *Pinus nigra* Arnold and *Cedrus libani* A.Rich. Acicular leaves characteristics.

	<i>Pinus nigra</i> Arnold Leaves type : Acicular (Needle) Length : 75-175 mm Width : 0.8-2.1 mm Unit weight : 0.14 g/cm^3 Density* : 0.35-0.59 g/cm^3 Water absorption: 35.4 % (untreated) 26.9 % (treated)		<i>Cedrus libani</i> A.Rich. Leaves type : Acicular (Needle) Length : 15-35mm Width : 0.8-2.1 mm Unit weight : 0.13 g/cm^3 Density** : 0.48-0.51 g/cm^3 Water absorption: 50.1 % (untreated) 41.5 % (treated)
	* Ross (2010)		**Efe (2021)

Table 3. Properties of tap water.

Chemical Analysis Result (mg/L)						
SO ₄	NO ₃	Cl	NH ₄	Mn	Al	Fe
70	7.5	43	<0.05	<0.0007	<0.012	<0.0075
pH	Conductivity ($\mu\text{S/cm}$)		Hardness (Fr°)		Turbidity (NTU)	
7.28	664		32		0.10	

2.3. Preparation of specimens

In this study, dry *Pinus nigra* Arnold and *Cedrus libani* A.Rich were used as natural fibers derived from acicular leaves. The physical and mechanical properties of cementitious composites at different proportions were investigated. The cementitious composites mixture is given in Table 4, and no admixtures or aggregates other than cement, water, and natural fiber were used to isolate the fiber's effect. The natural fibers of *Pinus nigra* and *Cedrus libani* origin used in this study are not homogeneous materials produced by standard processes, but lignocellulosic structures collected as forest waste and then subjected to multi-stage processes such as pre-cleaning, screening, washing, extraction, characterization, and mineralization. As a result of these processes, the physical properties of the fibers vary, and significant differences are observed between fibers. Therefore, determining the volumetric ratios of the fibers precisely and repeatability is difficult. In this study, fiber amounts were expressed by weight to ensure the controllability and reproducibility of the experimental design, and the fiber/cement ratio was defined on this basis. Although volumetric ratios are more appropriate when standard, homogeneous, and industrially produced fibers are used, mass ratios were used in this study due to the natural and heterogeneous fiber structure.

The water/cement ratio and the amount of natural fiber were kept constant, while the cement content was increased. In this study, cement contents of 600–900 g were used to create a gradual binder content series with a constant fiber content (140 g) and a constant water/cement ratio (0.40). Thus, the fiber/cement ratio was reduced from 0.23 to 0.16, and the effect of this change on the physical and mechanical behavior of composites was examined.

Cedrus libani A.Rich. acicular leaves mixture C is symbolized by *Pinus nigra* Arnold acicular leaves mixture P. Standard specimens of 4x4x16 cm were produced. The curing of the specimens was carried out in accordance with the TS EN 196-1 standard. In this context, all specimens subjected to compressive strength, flexural strength, and splitting tensile strength tests were cured for 7 and 28 days at a temperature of 20 ± 2 °C. After curing, unit weights and ultrasonic pulse velocities were measured. Flexural, compressive, and splitting tensile tests were then performed, and the dynamic elastic modulus was calculated. A three-point flexural test was performed, and the compressive strengths of cube specimens were determined. Photographs of C and P type specimens, along with flexural, compressive, and splitting tensile test setups, are shown in Fig. 1.

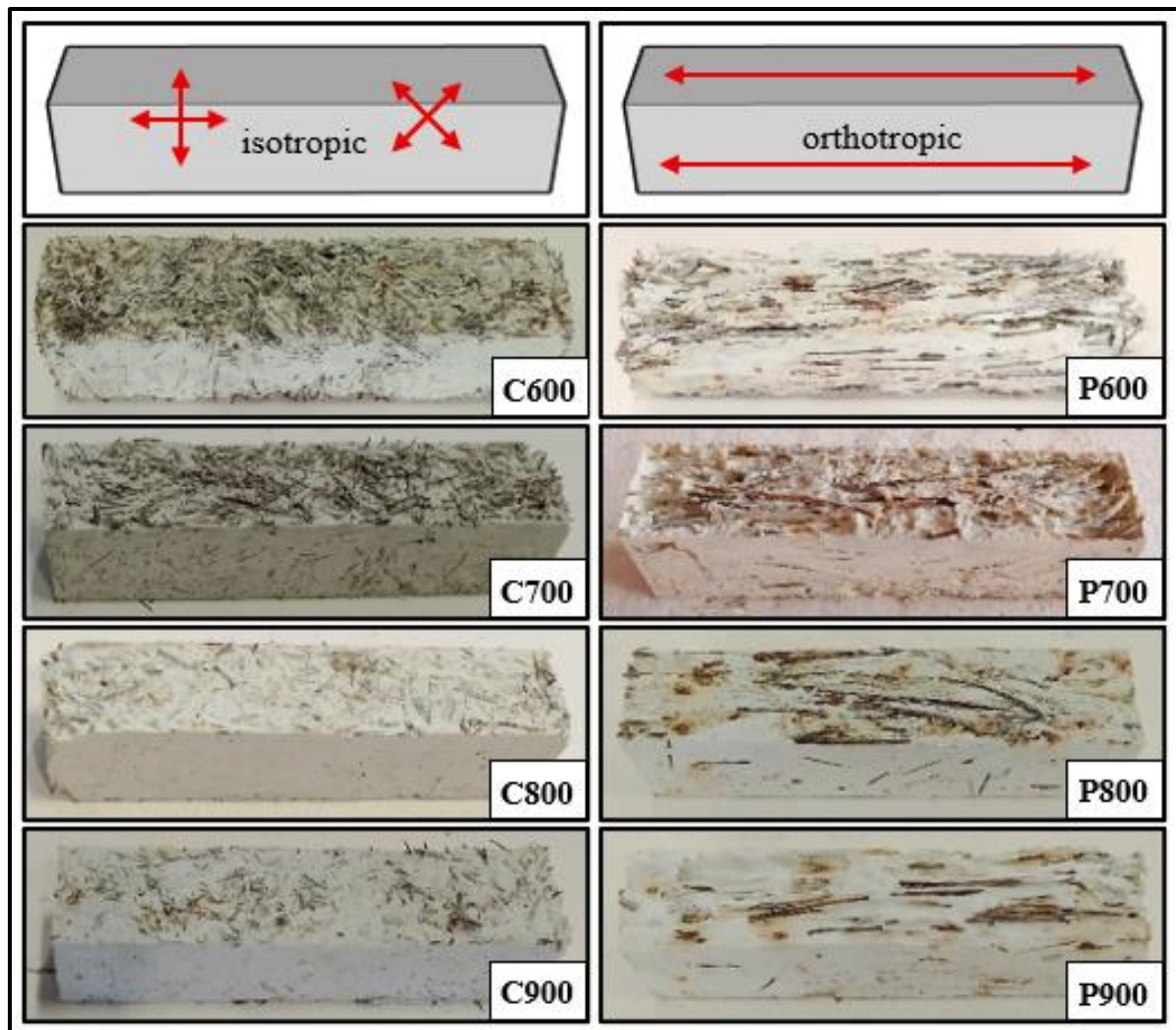


Fig. 1. Type C and P specimens.

The specimens were prepared in accordance with the mixing and production principles defined in the TS EN 196-1 standard for blends containing C-type fibers. The primary purpose of the production process is to ensure that the fibers are uniformly distributed within the cement matrix and to prevent the formation of agglomerations. This allows the fibers to be oriented in different directions within the composite structure, contributing to more uniform and isotropic mechanical properties. In specimens with P-type fibers, the fibers were mixed with the matrix in a specific direction and placed in the formwork. Manual orientation of long fibers perpendicular to the load direction was carried out under controlled, idealized laboratory conditions to examine the effect of fiber orientation on mechanical behavior in isolation. This method does not represent a production technique applicable at construction sites. Furthermore, once the

mixture was placed in the mold, pressure was applied, similar to wood–cement-based pressed board manufacturing techniques, resulting in a denser, more compact, and tidier internal structure.

The geometric properties of the natural fibers used in this study limit the determination of fresh mixture consistency by standard methods. C-type fibers from *Cedrus libani* have lengths of approximately 15–35 mm in nature and in the experimental study, while P-type fibers from *Pinus nigra* are longer, ranging from 75–160 mm. In particular, the high length/diameter ratio of P-type fibers makes one-way application a suitable measure of mixture workability. Therefore, standard fresh consistency tests, such as slump or the spreading table, do not yield feasible or representative results for this material system. Accordingly, fresh mixture properties were not evaluated using standard consistency tests in the study.

Table 4. Mix proportions.

Fiber type	Code	Cement (g)	Fiber (g)	Fiber/Cement	Water (g)	W/C
C type <i>Cedrus libani</i> A.Rich acicular leaves	C600	600	140	0.23	240	0.40
	C700	700	140	0.20	280	0.40
	C800	800	140	0.18	320	0.40
	C900	900	140	0.16	360	0.40
P type <i>Pinus nigra</i> Arnold acicular leaves	P600	600	140	0.23	240	0.40
	P700	700	140	0.20	280	0.40
	P800	800	140	0.18	320	0.40
	P900	900	140	0.16	360	0.40

2.4. Flexural strength test

Prismatic specimens measuring 40×40×160 mm, produced in accordance with the TS EN 196-1 (2016) standard, were subjected to a three-point flexural test at the end of 7 and 28 day curing periods. During the experiments, loading was performed using the span and loading speed specified in the standard. In specimens produced with natural fibers of varying compositions and geometries, the mechanical behavior was evaluated as a function of fiber distribution and orientation within the matrix. In this context, the flexural behavior of isotropic specimens, in which short fibers are randomly distributed in multiple directions, and orthotropic specimens, in which long fibers are aligned in one direction perpendicular to the load direction, was compared. In the experimental study, a total of 24 specimens were tested, with each of the two specimen groups, type C and type P, consisting of three replicates, to examine the flexural strength of the specimens.

2.5. Compressive strength test

A compressive test was conducted on cubic specimens (40×40×40 mm) prepared from C-type and P-type materials at various cement ratios, in accordance with TS EN 196-1 (2016). The experiments were performed on a universal testing press, and loading was continued

until the specimens failed under the specified standard conditions. During the compressive test, the effects of different cement ratios and specimen types on mechanical behavior were evaluated. In this experimental study, measurements were performed on a total of 24 specimens, with three replicates for each of the C and P type specimen groups, to evaluate compressive strength.

2.6. Splitting tensile strength test

The splitting tensile strength of the specimens was determined using the Brazilian test method. In this experiment, a compressive load was applied to prismatic specimens, inducing tensile stresses across the specimen's cross-section. As the applied load increased, the specimen fractured into two parts under tensile stress. To determine the splitting tensile strength of type C and type P specimens, experiments were conducted on a total of 24 specimens, with three replicates per group.

3. Results and Discussion

3.1. Characterization of C and P type acicular leaves

The microstructural characteristics of untreated and chemically treated C type and P type acicular leaves were investigated by scanning electron microscopy (SEM) to

clarify how fiber length and surface modification influence surface morphology prior to incorporation into cementitious composites.

SEM observations provide qualitative evidence regarding removal by extraction, surface roughening, and mineral deposition, all of which are known to affect fiber-matrix compatibility. Scanning electron microscopy (SEM) images of untreated C type leaves (Fig. 2) reveal a relatively smooth and compact surface characterized by longitudinal grooves and an intact epicuticular wax layer. At higher magnifications ($\times 500$ – $\times 1000$), subtle surface asperities and closed voids become apparent. These morphologies are representative of untreated lignocellulosic fibers, which are rich in waxy extractives

and hemi cellulosic components, potentially impeding interfacial interactions with cementitious matrices (De Aguiar et al. 2023; Melichar et al. 2024). Following the mineralization of $\text{Ca}(\text{OH})_2$, the surface of the fiber shows significant roughness due to the emergence of granular and platelet-like deposits. At higher magnifications ($\times 1000$ – $\times 4000$), these characteristics reveal the precipitation of calcium-based minerals on the fiber surface. The combined effects of alkaline pretreatment and mineralization diminish the impact of hydrophobic extractives and encourage a more varied surface topology. This change in surface properties indicates that surface modification following alkali pretreatment and mineralization was successful (Zhu et al. 2023; Maier et al. 2024).

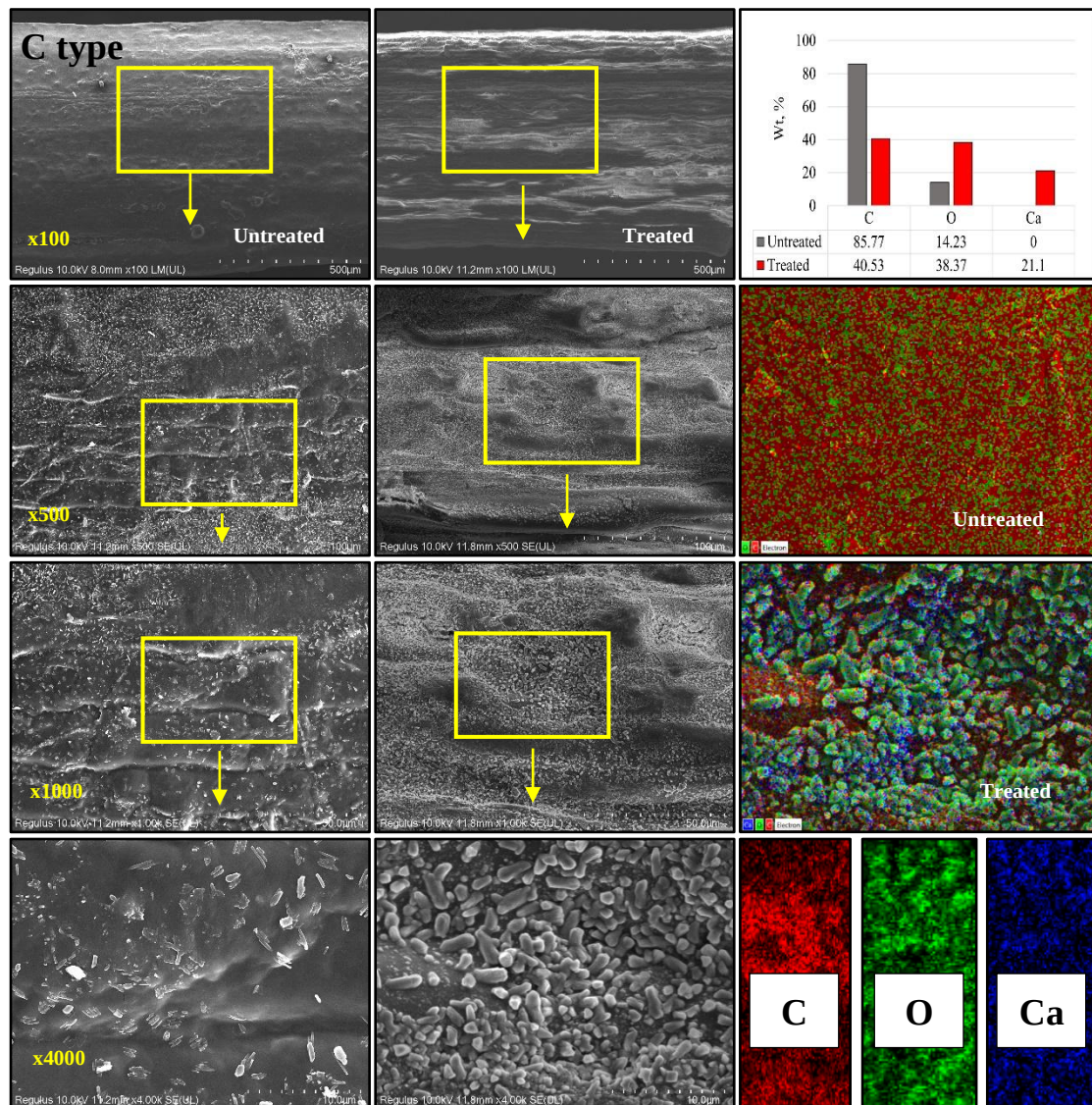


Fig. 2. SEM and EDS analysis of C type acicular leaves.

Fig. 2 shows the EDS analysis results and elemental mapping images of C-type fibers before and after $\text{Ca}(\text{OH})_2$ mineralization, illustrating the evolution of surface chemistry and the spatial distribution of C, O, and Ca. C-type fibers contain 85.77% wt% C and 14.23% wt% O. This result is characteristic of lignocellulosic biomass. It also shows a relatively homogeneous spatial

distribution of carbon and oxygen throughout the analyzed area, suggesting a chemically homogeneous surface before processing (Xia et al. 2023; Gharieb et al. 2024). After treatment with $\text{Ca}(\text{OH})_2$, a significant compositional change occurred in type C sheets. EDS results show a significant addition of calcium (21.10 wt%), a decrease in carbon content (40.53 wt%), and an increase in

oxygen content (38.37 wt.). This change clearly demonstrates that calcium-based phases effectively precipitate onto the fiber surface. Elemental mapping confirms that calcium is not homogeneously distributed, but rather concentrated in localized regions, forming mineral-rich areas that partially cover the organic substrate. This type of heterogeneous Ca enrichment has been commonly associated with surface deposition mechanisms where calcium hydroxide reacts with exposed hydroxyl groups in cellulose chains, promoting interfacial bonding rather than forming a continuous mineral shell (Feng et al. 2025).

In Fig. 3 untreated P type leaves exhibit a distinct longitudinal morphology characterized by a high length-to-diameter ratio. Scanning Electron Microscope (SEM) images captured at $\times 100$ and $\times 500$ magnifications reveal aligned grooves on a relatively smooth surface, whereas higher magnifications reveal only minor surface irregularities. This observed morphology indicates limited po-

tential for mechanical interlocking in the untreated condition, consistent with previous findings on pine-derived fibers (El Hamri et al. 2024; Albadrani 2023). Following the mineralization with $\text{Ca}(\text{OH})_2$, as shown in Fig. 3, the fiber surface experiences a notable change in morphology, characterized by the development of plate-like and clustered mineral formations. Under higher magnification, these formations appear as a thick yet uneven layer that partially fills surface grooves and microvoids (De Michele et al. 2025; Ajibola et al. 2024). Comparative SEM analysis reveals that chemical treatment and mineralization alter the surface morphology of both C type and P type fibers; however, the implications of these changes vary based on fiber geometry and loading mode. C-type fibers generally exhibit more homogeneous surface modifications, while P-type fibers display more localized mineralized regions (Kirker et al. 2024; El-Feky et al. 2024).

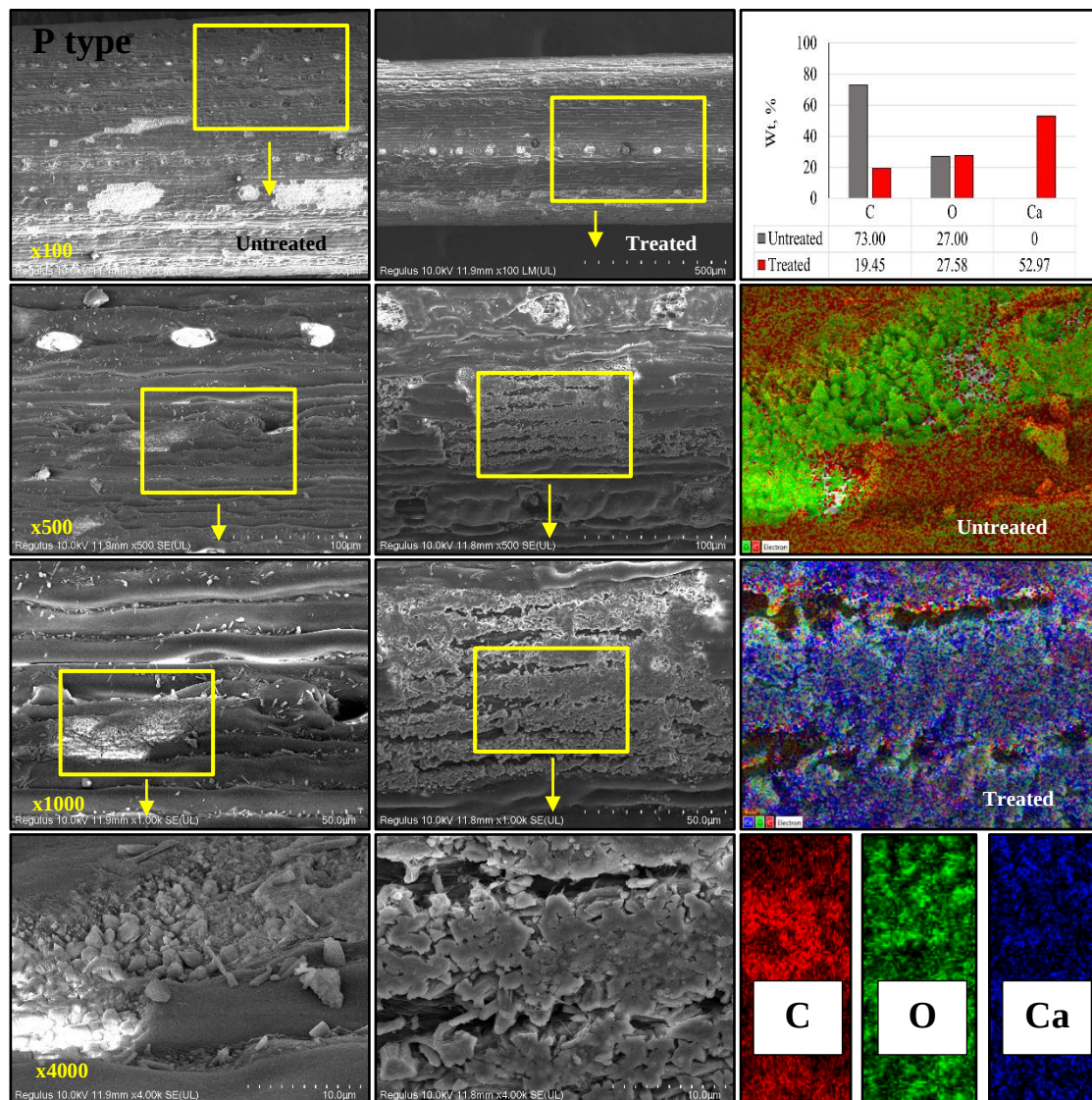


Fig. 3. SEM and EDS analysis of P type acicular leaves.

Fig. 3 shows the EDS results of the P-type fibers. The EDS results show a composition dominated by carbon (73.00 wt.) and oxygen (27.00 wt.) As a result of $\text{Ca}(\text{OH})_2$

mineralization, the fibers have a calcium-dominant surface composition; Ca is present at 52.97 wt., along with oxygen (27.58 wt.) and a significantly reduced carbon

fraction (19.45 wt). Elemental mapping reveals a dense and continuous distribution of calcium-rich phases throughout the analyzed area, indicating the formation of a robust inorganic layer that partially masks the underlying organic structure (Xu et al. 2025; Li et al. 2025; Zhang et al. 2025).

Fig. 4 illustrates the FTIR spectra of untreated and $\text{Ca}(\text{OH})_2$ -treated C type and P fibers, highlighting the chemical modifications induced by alkaline treatment and subsequent mineralization. In both untreated C type and P type fibers, broad absorption bands observed in

the 3600–3200 cm^{-1} region are attributed to O–H stretching vibrations associated with hydroxyl groups in cellulose, hemicellulose, and adsorbed moisture. These features are characteristic of lignocellulosic biomass and have been widely reported for untreated wood-based fibers (El Hamri et al. 2024; Maier et al. 2024). Following $\text{Ca}(\text{OH})_2$ treatment, a noticeable reduction in band breadth and intensity is observed, suggesting partial removal of hemicellulosic components and a decrease in free hydroxyl groups due to chemical interaction with calcium species.

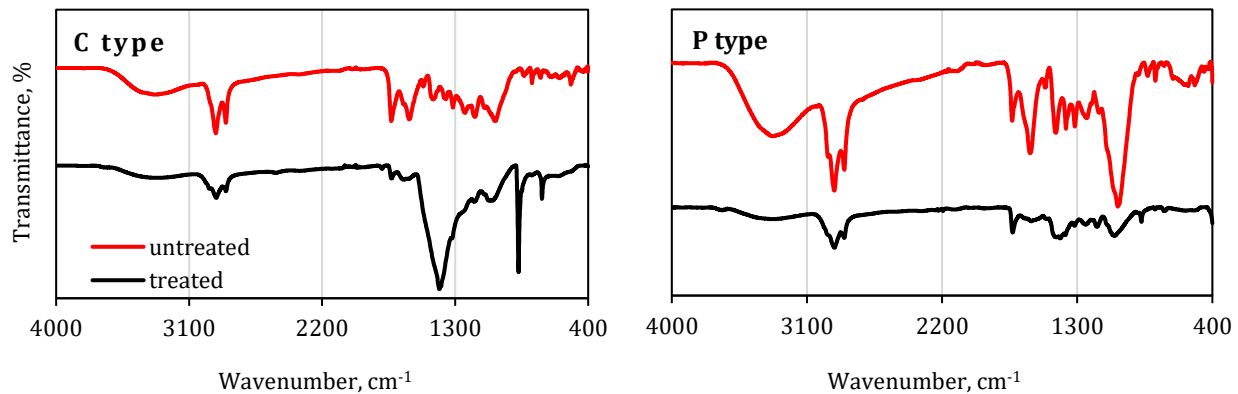


Fig. 4. FTIR analysis of C and P type acicular leaves.

The absorption bands located at approximately 2916 and 2849 cm^{-1} correspond to asymmetric and symmetric C–H stretching vibrations of aliphatic $-\text{CH}_2$ and $-\text{CH}_3$ groups originating from cellulose and lignin structures. In treated fibers, these bands exhibit reduced intensity, indicating partial degradation or extraction of amorphous organic constituents during alkaline treatment. Similar trends have been reported for alkali-treated lignocellulosic fibers intended for cementitious composites (Melichar et al. 2024; Zhu et al. 2023). In the carbonyl region, the band around 1720–1730 cm^{-1} , which is associated with C=O stretching of acetyl and ester groups in hemicellulose, is clearly visible in untreated fibers but becomes significantly weakened or nearly disappears after $\text{Ca}(\text{OH})_2$ treatment. This result indicates that the acetyl and ester groups associated with hemicellulose are reduced after $\text{Ca}(\text{OH})_2$ treatment (Kirker et al. 2024; Bufalino et al. 2023). The fingerprint region (1200–1000 cm^{-1}) exhibits notable bands at approximately 1170 and 1040 cm^{-1} , which are associated with C–O–C and C–O stretching vibrations of polysaccharide structures. These bands are observed after treatment with $\text{Ca}(\text{OH})_2$, suggesting that the fundamental cellulose framework remains unchanged (Zhao et al. 2025; Lin et al. 2026). Distinct differences between untreated and treated fibers are most evident in the emergence and intensification of bands around ~1400–1420 cm^{-1} , ~871 cm^{-1} , and ~712 cm^{-1} in $\text{Ca}(\text{OH})_2$ -treated specimens. These bands are characteristic of carbonate (CO_3^{2-}) vibrational modes and indicate the formation of calcium carbonate phases through carbonation of $\text{Ca}(\text{OH})_2$ (Zhao et al. 2025; Lin et al. 2026).

3.2. Compressive strength test

The cement-based composites developed in this study are similar to wood-cement composites and wood-based cement binder products in their applications. These materials are widely used in the construction industry, especially in the production of precast composite elements containing wood fibers or particles and in wood-cement-based pressed boards. Because such composite systems are generally not designed as load-bearing structural elements, high compressive strength is not considered a basic performance criterion. In fact, the literature frequently reports that the compressive strength of similar composite materials is relatively low (Canbaz et al. 2021; Garcez et al. 2016; Mahzabin et al. 2013). The main reason for this is the limited bonding capacity of wood-based phases with the cement matrix, which results in higher void ratios within the composite structure (Canbaz et al. 2021).

Fig. 5 shows the compressive strength results obtained at the end of 7 and 28 day curing periods for cementitious composites reinforced with P and C type natural fibers at different cement ratios; these values are key mechanical indicators reflecting changes in matrix development, fiber-matrix adhesion, and internal structural continuity. Across all specimen series, the curing process produced significant increases in compressive strength, associated with the formation of hydration products and a more compact matrix structure. In C type specimens, compressive strength ranged from 5.41 to 6.30 MPa during the 7 day curing period and increased to 7.15–9.09 MPa at the end of the 28 day curing period. With increasing cement content, compressive strength

rose by approximately 16.4% at 7 days and by approximately 27.2% at 28 days. This trend can be explained by the isotropic fiber arrangement formed by the random, multidirectional distribution of type C fibers, which allows the matrix to develop more homogeneously and transfer load more evenly during curing. In P type specimens, compressive strength was lower than in C type specimens. During the 7 day curing period, it ranged from 2.93 to 5.70 MPa, and at the end of the 28 day curing period, it increased to 4.02–7.98 MPa. When C and P type specimens are compared at the same cement ratios, C type specimens exhibit higher compressive strength at both curing times. During the 28 day curing period, the compressive strength of the C900 specimens was approximately 14.0% higher than that of the P900 specimens. This difference can be attributed to the isotropic arrangement of short fibers, which provides a more balanced stress distribution under compressive loads and increases the load-bearing capacity of the fibrous and porous structure, while effective stress redistribution is

also recognized as an important contribution of fiber-reinforced composite systems (Alshabrawi and Uysal 2025). The C type isotropic short fiber arrangement provided higher absolute compressive strength values than the P type orthotropic long fiber arrangement. These findings make it clear that fiber length and orientation are key design parameters that govern the compressive behavior of cementitious composites. In general, the 7- and 28-day compressive strength values for both C-type and P-type specimens are considerably lower than the compressive strength of concrete used in conventionally produced load-bearing systems. The main reason for this is the design approach and target application area of the developed composite system. The targeted application area is non-structural elements such as partition boards, lightweight panels, and rehabilitation layers. Mean results were calculated for each variable's experimental outcome, and the standard deviation (SD) and coefficient of variation (CV) values are shown in Table 5.

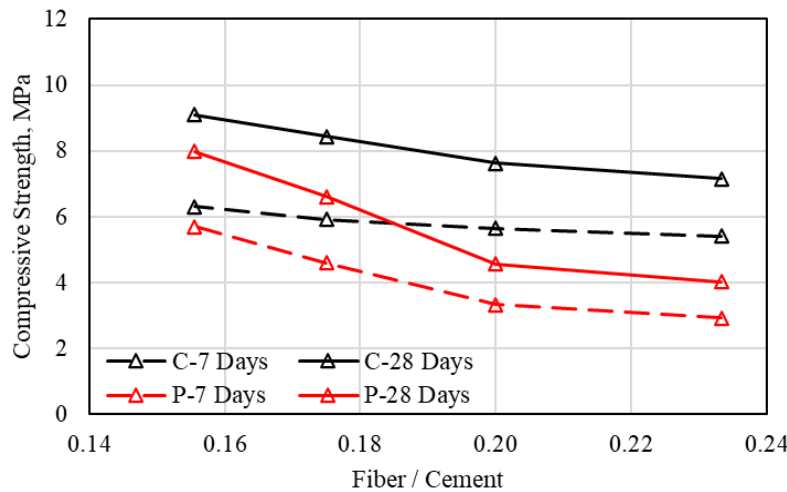


Fig. 5. Compressive strength at different fiber/cement ratios.

3.3. Flexural strength test

In Fig. 6, the flexural strength values obtained at the end of 7 and 28 day curing periods for different fiber/cement ratios in reinforced cementitious composites with P and C type natural fibers are presented. Flexural strength is a mechanical parameter directly related to the crack bridging capacity of the fibers, fiber orientation, and fiber-matrix interaction in fiber-cement composites. In C type specimens, flexural strength ranged from 0.91 to 2.32 MPa during the 7 day curing period and increased to 1.27–3.45 MPa at the end of the 28 day curing period. In P type specimens, flexural strength was significantly higher than in C type specimens. During the 7 day curing period, flexural strength ranged from 4.83 to 6.38 MPa, and at the end of the 28 day curing period, it ranged from 6.59 to 9.33 MPa. When C and P type specimens are compared at the same curing time and fiber/cement ratios, the flexural strength of P type specimens is much higher. For example, in the mixture with the highest cement content, the flexural strength of P type specimens was approxi-

mately 170% higher than that of C type specimens during the 28 day curing period. Similarly, even with a 7 day curing period, the flexural strength of P type specimens was approximately 2.5-5 times that of C type specimens. This higher-than-expected flexural strength is directly related to the fact that the long fibers in P type specimens are oriented in one direction perpendicular to the flexural plane. The tensile stresses that occur during the flexural test are concentrated on the lower surface of the specimen; long fibers placed in this direction cut off the direction of crack advancement, creating an effective crack bridging mechanism. The orientation of the fibers perpendicular to the load direction enabled them to experience the tensile stresses directly. It was found that isotropic fiber orientation made a minor contribution to flexural behavior compared to orthotropic orientation; the effect of fiber orientation on flexural behavior is much more pronounced than on compressive behavior. Mean results were calculated for each variable's experimental outcome, and the standard deviation (SD) and coefficient of variation (CV) are shown in Table 5.

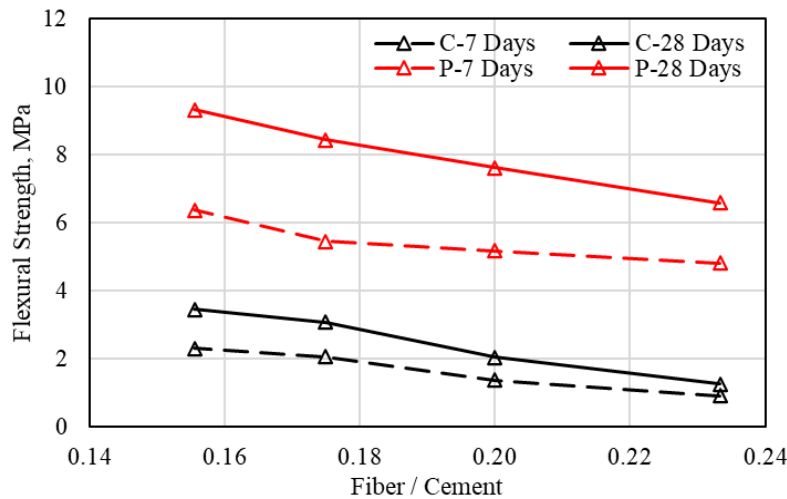


Fig. 6. Flexural strength at different fiber/cement ratios.

3.4. Splitting tensile strength test

In Fig. 7, the tensile strength values of composites reinforced with P and C type natural fibers, obtained at the end of 7 and 28 day curing periods for different fiber/cement ratios, are presented. Tensile strength at fracture is a mechanical parameter that is highly sensitive to fiber-matrix interactions and directly reflects resistance to crack initiation and propagation in cementitious composites. For C type specimens, tensile strength at splitting was measured at 1.10–1.55 MPa during the 7 day curing period, and these values increased to 1.37–2.12 MPa at the end of the 28 day curing period. With a decrease in the fiber/cement ratio (higher cement content), the strength increased by approximately 40.7% after 7 days and by approximately 55.5% after 28 days. This increase can be attributed to the matrix becoming more continuous as curing progresses, with cracks becoming multifaceted and demarcated due to the isotropic distribution of short C type fibers. In P type speci-

mens, tensile strength at splitting was higher in both absolute terms and in terms of increase rates. During the 7 day curing period, the strength of P type specimens ranged from 1.53 to 2.97 MPa, and at the end of the 28 day curing period, it reached 2.24–2.97 MPa. With a decrease in the fiber/cement ratio, the strength increased by approximately 34.8% after 7 days of curing and by approximately 32.7% after 28 days of curing. When C type and P type specimens are compared at the same curing time and fiber/cement ratios, the tensile strength of P type specimens at splitting is significantly higher. For example, in the mixture with the highest cement content, the tensile strength of the P type specimen at splitting during the 28 day curing period was measured to be approximately 40% higher than that of the C type specimen. A similar trend is observed during the 7 day curing period. This higher-than-expected tensile strength at splitting is directly related to the fact that the long fibers in P type specimens are oriented in one direction and perpendicular to the flexural and tensile planes.

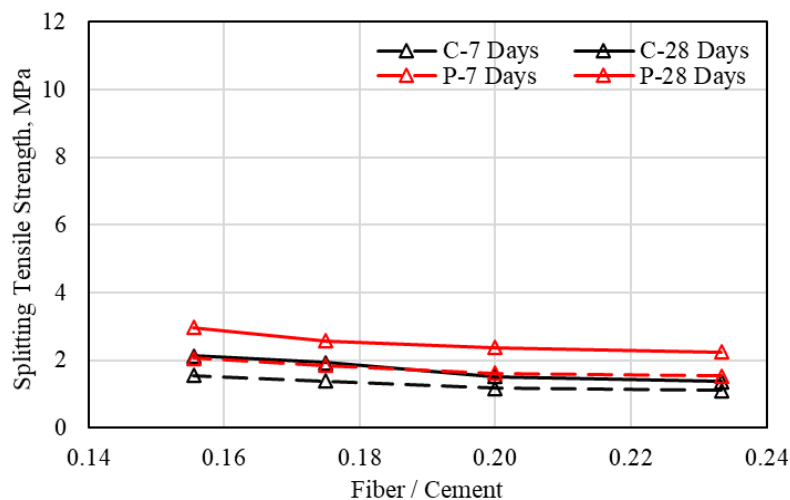


Fig. 7. Splitting tensile strength at different fiber/cement ratios.

In the split tensile test, tensile stresses along the specimen are concentrated in a central plane; long fibers placed perpendicular to this plane impede crack propa-

gation, creating an effective crack-bridging and tensile load carrying mechanism, consistent with the tensile-strength contribution of fiber reinforcement reported by

Youssef et al. (2026). The length and orientation of the fibers increase tensile strength to levels higher than expected. The increase in tensile strength at splitting in C type specimens was more balanced but more limited compared to orthotropically oriented P type specimens. When tensile strength results from the split are evaluated alongside the compressive and flexural strength results, it is evident that the role of fiber orientation in mechanical behavior varies with the type of loading. While the C type isotropic short fiber arrangement provided more balanced and high strengths in compressive behavior, the P type orthotropic long fiber arrangement showed significant superiority in loading situations where tensile stresses, such as flexure and splitting tensile, were dominant. Mean results were calculated for each variable's experimental outcome, and the standard deviation (SD) and coefficient of variation (CV) are shown in Table 5.

3.5. Unit weight

Fig. 8 presents the unit weight values for composites reinforced with P and C type natural fibers at different

cement ratios. The results clearly show that fiber type, fiber length, and fiber orientation have a pronounced effect on composite density. Across all specimen series, unit weight increased with higher cement content. This trend can be attributed to increased cement content, greater matrix volume, and more efficient filling of fiber-induced voids. In particular, in mixtures containing 800 g and 900 g of cement, a denser matrix structure develops, and the internal void ratio decreases markedly. In type C specimens, the unit weight increased from 1.394 kg/dm³ to 1.596 kg/dm³ with increasing cement content, and in type P specimens, it increased from 1.273 kg/dm³ to 1.675 kg/dm³. Mean results were calculated for each variable's experimental outcome, and the standard deviation (SD) and coefficient of variation (CV) values are shown in Table 5. In addition, the decrease in unit weight may not be solely due to the low density of the fibers. It is also associated with increased porosity and an increase in the void ratio in the matrix with the addition of fiber. In particular, the fibers partially disrupt the matrix continuity and reduce the compactness of the mixture in the composite structure.

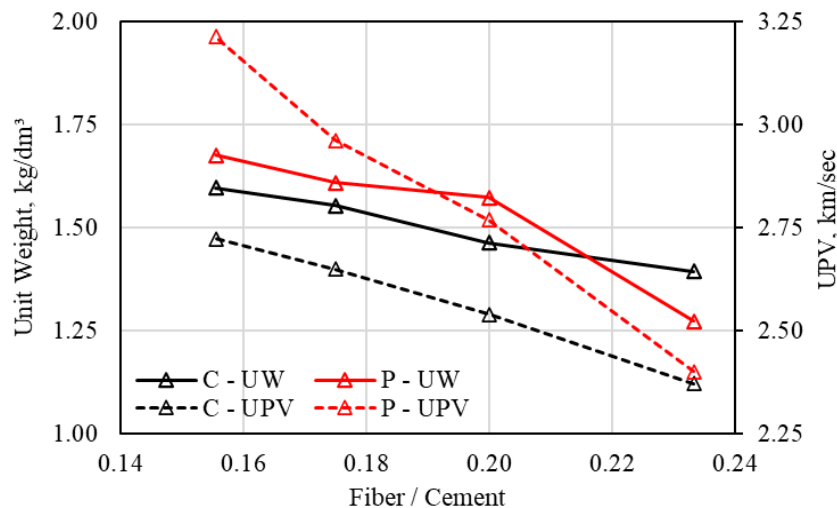


Fig. 8. Unit weight and UPV at different fiber/cement ratios.

3.6. Ultrasonic pulse velocity test

In Fig. 8, the measured ultrasonic pulse velocity (UPV) values of composites reinforced with natural fibers of types P and C at different cement ratios are presented. UPV results provide information on the continuity of the internal structure, the distribution of gaps, and fiber-matrix interactions in composites. Similar sensitivity of UPV to internal structural continuity was reported by Ünal et al. (2025). Across all specimen series, a significant upward trend in UPV values was observed with increasing cement content. This can be attributed to a denser matrix, a lower void ratio, and more continuous ultrasonic wave propagation through the material. In type C specimens, UPV was measured at 2.371 km/s in the C600 specimen and 2.723 km/s in the C900 specimen, representing an increase of approximately 14.9%. The isotropic fiber arrangement, formed by the random, multi-

directional distribution of short type C fibers, provides a more homogeneous structure within the matrix, allowing more uniform propagation of ultrasonic waves. This behavior is consistent with the increase in homogeneous density observed in the unit-weight results. In P type specimens, the increase in UPV was more pronounced. UPV, measured at 2.400 km/s in the P600 specimen, increased to 3.214 km/s in the P900 specimen, representing approximately a 34.0% increase. The orthotropic fiber arrangement, created by orienting long P type fibers perpendicular to the load direction and in a single direction, produced discontinuities around the fibers at low cement ratios; however, with increasing cement content, these discontinuities were largely eliminated, resulting in a significant improvement in ultrasonic wave transmission. Mean results were calculated for each variable's experimental outcome, and the standard deviation (SD) and coefficient of variation (CV) values are shown in Table 5.

In addition, water absorption tests were conducted to determine the water absorption behavior of the specimens at the end of the 28-day curing period. According to the results, the water absorption rates of C600–C900 type specimens were 17.5%, 17.20%, 16.70%, and 16.10%, respectively. In addition, the water absorption values of the fibers are given in Table 2, and the fibers in the mixture are also likely to absorb some of the mixing water. On the other hand, these rates were measured as 19.10%, 18.60%, 18.30%, and 17.90% in P600–P900 type specimens. The findings show that the void ratio decreases as the matrix gains a denser, more continuous structure with increasing cement content, and water absorption capacity decreases accordingly. They also show that an increase in fiber content leads to higher water absorption values, indicating increased pore volume and connectivity within the composite structure. This increase in porosity is thought to negatively impact the propagation of ultrasonic waves, resulting in lower UPV values.

3.7. Dynamic modulus of elasticity

Fig. 9 shows the dynamic modulus of elasticity for composites reinforced with P- and C-type natural fibers, calculated at different cement contents. The dynamic modulus of elasticity is an important parameter that

reflects the rigidity and continuity of the internal structure of composites. When evaluated alongside the unit weight and UPV results, it enables a holistic interpretation of material behavior. Across all specimen series, a significant upward trend in the dynamic modulus of elasticity was observed with increasing cement content. This may be attributed to the matrix developing a denser, more continuous structure, increased fiber-matrix contact area, and a reduced void ratio. Increases in unit weight and UPV values support the results for the dynamic modulus of elasticity. In type C specimens, the dynamic modulus of elasticity ranged from 7.99 to 12.06 GPa, and the modulus increased by approximately 50.9% with higher cement content. The isotropic fiber arrangement, formed by the random, multidirectional distribution of C-type short fibers, yields a more homogeneous internal structure, thereby increasing rigidity. In P-type specimens, the dynamic modulus of elasticity increased from 7.48 GPa to 17.65 GPa, a rise of approximately 136.1%. This is because the UPV value used in the calculation was measured in the orthotropic plane. For this reason, the rigidity appears relatively high in this direction. Mean results were calculated for each variable's experimental outcome, and the standard deviation (SD) and coefficient of variation (CV) values are shown in Table 5.

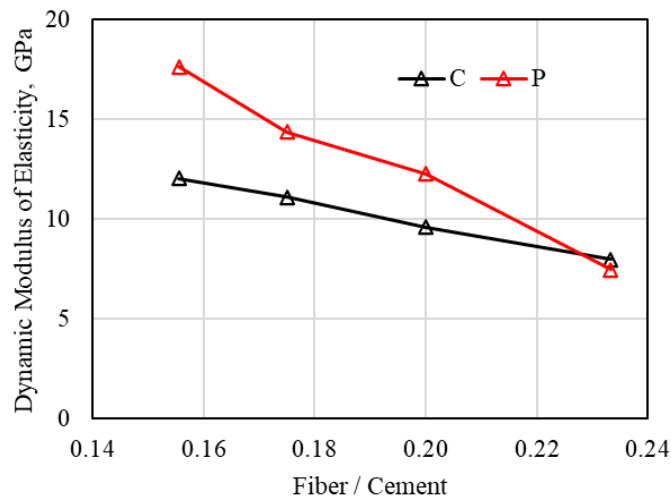


Fig. 9. Dynamic modulus of elasticity at different fiber/cement ratio.

3.8. Fracture analysis

In Fig. 10, the observed fracture forms after flexural, compressive, and splitting tensile tests on P and C type natural fiber-reinforced cementitious composites produced at different cement ratios were evaluated, and the damage patterns qualitatively confirm the mechanical performance results.

Spread crushing behavior is dominant in both C type and P type specimens under compressive loading, and the compressive behavior is mainly controlled by matrix continuity. On the other hand, under tensile stress conditions such as flexural and splitting tensile loading, fiber length and orientation become decisive; the long, orthotropically oriented P-type fibers delay fracture through crack bridging, resulting in more ductile and

controlled damage development. Type C specimens with an isotropic short-fiber arrangement exhibit more homogeneous crack control but limited crack control and complete fracture, with a more classical separation mechanism.

4. Conclusions

In this study, Cedrus libani A. Rich. and Pinus nigra Arnold were evaluated for their effects on the physical and mechanical behavior of cementitious composites as a function of fiber length and orientation. The results clearly demonstrated that fiber geometry and orientation govern distinct mechanical responses under different loading conditions. It was determined that isotropic

composites produced with short, randomly distributed *Cedrus libani* acicular leaves exhibited a more homogeneous internal structure, which positively affected compressive strength, unit weight, and ultrasonic pulse velocity. The isotropic fiber arrangement supported matrix continuity, providing a more balanced stress distribution under compressive loads and promoting stiffness development. On the other hand, in orthotropic composites in which long *Pinus nigra* acicular leaves are oriented perpendicular to the load direction, tensile strength in flexural and splitting tests was observed to increase significantly. This increase has been attributed to the fact that long fibers provide an effective crack-

bridging mechanism under loading conditions dominated by tensile stresses. The results for ultrasonic pulse velocity and dynamic modulus of elasticity indicated differences in internal structural continuity and rigidity with respect to fiber orientation. It has been determined that ultrasonic wave transmission increases with stronger fiber-matrix interaction, particularly at high cement ratios, and that a more continuous structure form. Fracture observations indicate that matrix-controlled failure behavior predominates under compressive loads; in tensile loading in flexural and splitting tests, fiber orientation significantly affects the fracture mode and limits crack propagation.

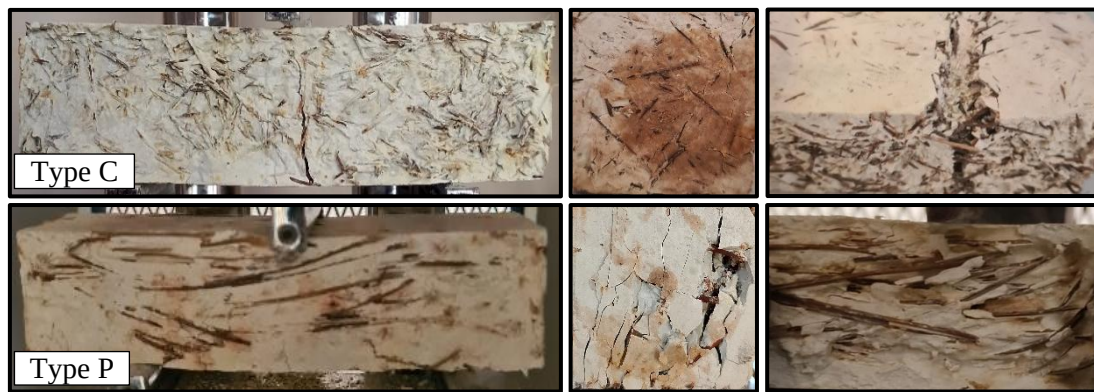


Fig. 10. Fracture analysis of type C and P specimen (flexural, compressive, and splitting tensile tests).

Table 5. Statistical analysis of data.

Code	Cure	Compressive Strength (MPa)			Flexural Strength (MPa)			Splitting Tensile Strength (MPa)		
		Mean	SD	CV (%)	Mean	SD	CV (%)	Mean	SD	CV (%)
C600	7 days	5.41	0.22	4.1	0.91	0.07	7.7	1.10	0.07	6.4
C700		5.64	0.32	5.7	1.38	0.11	8.0	1.17	0.05	4.3
C800		5.93	0.30	5.1	2.06	0.14	6.8	1.39	0.11	7.9
C900		6.30	0.44	7.0	2.32	0.10	4.3	1.55	0.10	6.5
C600	28 days	7.15	0.37	5.2	1.27	0.08	6.3	1.37	0.06	4.4
C700		7.63	0.53	7.0	2.05	0.11	5.4	1.51	0.07	4.6
C800		8.44	0.62	7.3	3.09	0.16	5.2	1.93	0.11	5.7
C900		9.09	0.45	5.0	3.45	0.20	5.8	2.13	0.17	8.0
P600	7 days	2.93	0.22	7.5	4.83	0.29	6.0	1.53	0.10	6.5
P700		3.33	0.16	4.8	5.18	0.32	6.2	1.62	0.09	5.6
P800		4.59	0.23	5.0	5.46	0.22	4.0	1.86	0.09	4.8
P900		5.70	0.30	5.3	6.38	0.43	6.7	2.07	0.15	7.2
P600	28 days	4.02	0.23	5.7	6.58	0.27	4.1	2.24	0.11	4.9
P700		4.57	0.33	7.2	7.63	0.31	4.1	2.37	0.17	7.2
P800		6.62	0.33	5.0	8.44	0.62	7.3	2.57	0.20	7.8
P900		7.97	0.59	7.4	9.33	0.51	5.5	2.97	0.14	4.7
Code		Unit Weight (kg/dm ³)			UPV (km/s)			$E_{dynamic}$ (GPa)		
		Mean	SD	CV (%)	Mean	SD	CV (%)	Mean	SD	CV (%)
C600	28 days	1.39	0.10	7.2	2.37	0.10	4.2	7.99	0.61	7.6
C700		1.46	0.11	7.5	2.54	0.20	7.9	9.61	0.46	4.8
C800		1.55	0.12	7.7	2.65	0.21	7.9	11.12	0.74	6.7
C900		1.60	0.11	6.9	2.72	0.12	4.4	12.06	0.73	6.1
P600	28 days	1.27	0.08	6.3	2.40	0.14	5.8	7.48	0.56	7.5
P700		1.57	0.09	5.7	2.77	0.22	7.9	12.29	0.54	4.4
P800		1.61	0.09	5.6	2.96	0.19	6.4	14.38	0.96	6.7
P900		1.68	0.12	7.1	3.22	0.22	6.8	17.65	0.88	5.0

SEM analyses showed that $\text{Ca}(\text{OH})_2$ treatment produced partial surface mineralization and increased surface roughness, while EDS and elemental mapping confirmed progressive calcium enrichment and a corresponding decrease in surface carbon content. FTIR results further indicated the removal of hemicellulosic components and the formation of carbonate-related bands without altering the cellulose backbone. These findings confirm the surface modification and mineralization of the fibers.

In general, the short, isotropic fiber arrangement offers advantages in terms of compressive behavior and internal structural continuity. In contrast, the long, oriented fiber arrangement makes a greater contribution to mechanical behavior when tensile effects predominate. These findings reveal that fiber length and fiber orientation are key parameters that should be evaluated together in the design of natural fiber cementitious composites, depending on the targeted mechanical performance. Cedrus libani A. Rich. and Pinus nigra Arnold species as natural fibers, showing that it is a suitable approach to improve the mechanical behavior of cementitious composites. Based on the twenty-eight-day cure results, the inclusion of these fibers in the matrix limited crack initiation and propagation, thereby making charge transfer more controlled. The isotropic distribution of short Cedrus libani acicular leaves has a positive effect on compressive strength and internal structural homogeneity by supporting matrix continuity; the directed use of long Pinus nigra acicular leaves provided significant increases in tensile strength in flexural and splitting, where tensile stresses are dominant. It has been shown that as fiber-matrix interaction strengthens, fracture behavior tends toward a more controlled mode of damage, and fibers play an active role in improving mechanical performance.

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Conflict of Interest

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Data Availability

The datasets generated and/or analyzed during the current study are not publicly available but are available from the corresponding author upon reasonable request.

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No AI-based tools were used in the preparation of this manuscript.

Author Contributions

All authors made substantial contributions to the conception and design of the study, acquisition of data, analysis and interpretation of data; drafted or critically revised the manuscript for important intellectual content; and approved the final version to be published.

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Challenge Journal of STRUCTURAL MECHANICS

Research Article

Experimental investigation of material and joint influence on timber I-beam performance

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ABSTRACT

This study experimentally investigates the influence of web material and joint configuration on the flexural performance of timber I-beam configurations. A total of 27 specimens representing nine beam types were tested, including oriented strand board (OSB), laminated timber, and plywood-web beams produced as continuous, web-joined, and flange-joined members. Three-point bending tests were conducted over a clear support span of 1000 mm, and the load–displacement response, maximum load capacity, stiffness, energy absorption, strength-to-weight ratio, and failure modes were evaluated. The results showed that continuous specimens generally achieved higher load-bearing capacities than their joined counterparts, indicating the importance of uninterrupted load transfer. Among the tested configurations, the continuous plywood-web beam exhibited the highest mean maximum load of 28.569 kN and the highest strength-to-weight ratio of 6.93 kN/kg. OSB-web beams showed lower absolute load capacity but a competitive strength-to-weight ratio of 5.45 kN/kg, suggesting potential for lightweight applications. Web-joined specimens generally performed better than flange-joined specimens because flange continuity provided more effective resistance to bending-induced tensile and compressive stresses. Failure observations indicated that joint location influenced damage initiation, web-flange separation, adhesive debonding, and post-peak behavior. Since the specimens represented practical beam configurations incorporating locally available, unused, or previously used wood-based components, the findings should be interpreted at the configuration level rather than as isolated material-property comparisons. Overall, the study provides practical insight into material selection and joint detailing for sustainable timber I-beam applications.

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1. Introduction

Timber structures, particularly timber I-beams, have gained significant attention in the construction industry due to their sustainability and favorable mechanical properties, while efficient use of alternative and waste-derived resources has similarly become an important strategy in sustainable construction-material development (Mohamad et al. 2025; Wee Chit et al. 2025). As the demand for efficient and sustainable building materials

grows, researchers have been exploring various ways to enhance the performance of timber beams. This study aims to contribute to this field by investigating the flexural behavior of different types of timber I-beams, including those made with oriented strand board (OSB), laminated timber, and plywood, with and without joints in the web and flange regions.

Recent research in timber beam performance has made significant strides in various aspects of design, reinforcement, and application. Several studies have fo-

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cused on reinforcing timber beams using various materials and methods. Studies (Ling et al. 2020; Shekarchi et al. 2020; Chen et al. 2021; Yeboah and Gkantou 2021; Zhang et al. 2022; Wdowiak-Postulak 2023) investigated the use of fiber-reinforced polymers (FRP), steel bars, basalt, glass, and even bamboo for strengthening timber beams. These studies generally found that reinforcement significantly improved the load-bearing capacity, stiffness, and flexural performance of timber beams. More broadly, the beneficial effect of waste-derived composite laminates on beam flexural performance has also been experimentally demonstrated (Çelik et al. 2024). The study by Yeboah and Gkantou (2021) showed that near-surface mounted (NSM) basalt and glass FRP bars enhanced the moment resistance of timber beams under bending.

Research has also explored the potential of composite and hybrid beam designs. Previous studies (Ghanbari-Ghazijahani et al. 2020; Müller et al. 2021; Jurkiewicz et al. 2022; Chen et al. 2022a; Zhao et al. 2023; Zhang et al. 2023; Wen and Xiao 2023) examined various combinations of timber with other materials such as concrete, steel, and bamboo. These studies demonstrated that composite designs often result in improved mechanical properties compared to traditional timber beams. Jurkiewicz et al. (2022) concluded that hybrid steel-timber beams offered advantages in terms of stiffness, strength, and ductility compared to conventional all-wood or all-steel beams.

Several studies focused on developing and analyzing new connection methods for timber structures. Numerous studies (Sotayo et al. 2020; Müller et al. 2021; Karimi-Nobandegani and Valipour 2022; Zhao et al. 2023) investigated various connection techniques, including micro-notches for timber-concrete composites and self-tapping screws for cross-laminated timber panels. These studies aimed to simplify assembly processes and improve the overall performance of composite structures.

A significant portion of the research was dedicated to understanding and modeling the structural behavior of timber beams. Several studies employed analytical and numerical methods to predict the performance of timber beams under different loading conditions (Karimi-Nobandegani and Valipour 2021; Dönmez et al. 2022; Hammad et al. 2022; Goodwin et al. 2022; Patalas et al. 2022; Patalas et al. 2023). For example, Dönmez et al. (2022) investigated the performance of timber formwork beams under static, fatigue, and sudden dynamic impact loads, providing valuable insights into their behavior in construction applications. The study found that the stiffness of timber formwork beams decreases with an increase in load repetition number, and their energy dissipation capacity decreases with an increase in the number of impact loads.

Several studies compared the performance of different engineered wood products. Researchers (Li et al. 2022; Sciomenta et al. 2022; Wen and Xiao 2023) examined the mechanical properties of various timber products such as glued laminated timber, cross-laminated timber, and laminated bamboo-timber composites. These comparisons help in selecting the most appropriate materials for specific structural applications.

Application-oriented studies have investigated several specific topics in timber construction, including

curved laminated bamboo-timber composites, timber beams with openings, cross-laminated timber walls, and timber formwork beams under different loading conditions (Karimi-Nobandegani and Valipour 2021; Dönmez et al. 2022; Li et al. 2022; Chen et al. 2022b). These studies provide useful insights into the broader application of timber-based structural systems. Recent studies have also continued to investigate the flexural behavior and structural performance of I-shaped timber and wood-based members, including I-joists with alternative web configurations, OSB-based I-beams, and I-shaped laminated bamboo composite beams (Li et al. 2024; Jiloul et al. 2024; Denisiewicz et al. 2025).

Although previous studies have provided important insights into reinforced, hybrid, and composite timber beam systems, limited comparative experimental evidence is available on the flexural performance of timber I-beams produced with different web materials and joint configurations. In particular, the influence of joint location in critical regions, such as the web and flanges, remains insufficiently clarified, although such joints may be unavoidable in practical large-span applications due to manufacturing and transportation constraints. Therefore, this study addresses this gap by experimentally investigating OSB, laminated timber, and plywood I-beams produced as continuous, web-joined, and flange-joined members under identical testing conditions.

This study focuses on a unique set of timber I-beams, designed to address practical considerations and explore the potential of various materials, including those typically considered as waste or byproducts. The beam configurations used in this study are illustrated in Fig. 1, showcasing different types of I-beams made from OSB, laminated timber, and plywood, with variations in joint placement. The joint placement and the fabrication process of the beams are explained in the following sections.

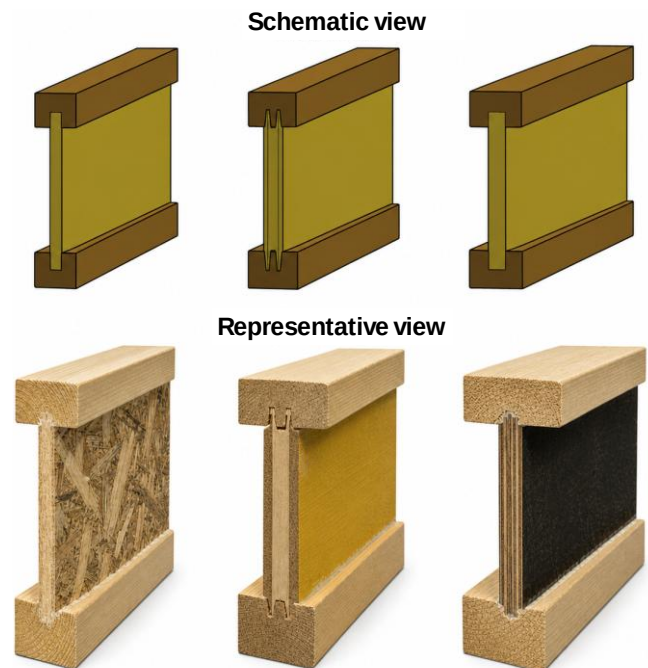


Fig. 1. Schematic and representative views: (a) Oriented strand board; (b) Laminated beams; (c) Plywood beams.

The contribution of this study lies in its comparative evaluation of material type and joint location in timber I-beams under identical experimental conditions. By testing continuous, web-joined, and flange-joined specimens made with OSB, laminated timber, and plywood webs, the study provides practical insight into how joint placement affects load-bearing capacity, stiffness, energy absorption, and failure behavior. In addition, the use of locally available materials, including unused timber pieces and previously used plywood, highlights the potential of resource-efficient and sustainable material utilization in engineered timber beam production, consistent with broader construction-material research linking waste reuse with resource conservation and circular-economy goals (Çelik et al. 2024; Mahmud et al. 2025; Tanash et al. 2025; Ünal and Canbaz 2025, 2026). Therefore, the findings are expected to support more informed material selection and joint-detailing decisions in practical timber I-beam applications.

2. Materials and Methods

2.1. Specimen preparation

In this study, a total of 27 timber I-beam specimens were tested. Unlike laboratory-produced samples, these specimens were sourced randomly from local manufacturers in the market to reflect real-world conditions and assess the performance of commercially available timber I-beams. These specimens represented 9 different beam types, with 3 replicates for each type. The beams were 1.2 m in length and were categorized based on their material composition and joint configuration.

The I-beam specimens were fabricated using a combination of web and flange materials. The web material varied according to the specimen group and consisted of OSB, laminated timber, or plywood. In all configurations, the web acted as the central vertical component of the I-section, while the upper and lower flanges provided the main resistance to bending stresses. All specimens featured solid timber flanges made from Ardanuç spruce (*Picea orientalis*), a species native to the Black Sea region of Türkiye. This material was selected because of its structural suitability, local availability, and practical use in timber products.

The OSB and plywood specimens had a similar general I-beam layout, consisting of a flat web panel bonded to solid timber flanges at the top and bottom. In these specimens, the web-flange connection was formed through continuous adhesive-bonded contact surfaces along the beam length. In contrast, the laminated timber specimens had a different web configuration, in which unused or waste Ardanuç spruce pieces were arranged in an interlocking pattern within the web region. This configuration was intended to improve material utilization while maintaining the overall I-beam geometry. Thus, although all specimens shared the same basic I-section concept, they differed in terms of web material, web configuration, and joint location. The specimens were coded as shown in Table 1, where the first letter indicates the beam material (O: OSB, L: Laminated, P: Plywood), and the following letters indicate the joint

type (WJ: Web Joined, FJ: Flange Joined) or continuous beams without joints.

Table 1. Specimen coding.

Type	OSB	Laminated	Plywood
Web Joined	OWJ	LWJ	PWJ
Flange Joined	OFJ	LFJ	PFJ
Continuous	O	L	P

Note: OSB = oriented strand board; WJ = web-joined; FJ = flange-joined; O = OSB-web beam; L = laminated-web beam; P = plywood-web beam.

OSB beam consists of an oriented strand board web with solid timber flanges at the top and bottom. The OSB web provides good shear resistance, while the solid timber flanges offer strength in tension and compression. The OSB web was connected to the upper and lower solid timber flanges through continuous flat adhesive-bonded interfaces along the beam length. Polyurethane adhesive was applied to the full web-flange contact surfaces, and no toothed or interlocking profile was used at this interface.

The laminated-web beam differs from the OSB- and plywood-web beams in terms of its web configuration. Its web consists of laminated timber elements, while the flanges are solid Ardanuç spruce members, as described above. The toothed connection structure of the laminated beam is a significant feature that distinguishes this type from the others. This design may provide a stronger connection and more effective load transfer between the web and flanges. The laminated beam web design shows an innovative approach to timber engineering. It features a continuous outer siding for structural integrity, while the interior utilizes unused or waste Ardanuç spruce in an interlocking pattern as seen in Fig. 2. This design maximizes material efficiency by incorporating otherwise discarded wood, enhancing sustainability. The interlocking structure likely improves the beam's shear strength and stability. This approach balances structural performance with eco-friendly practices, demonstrating a creative solution for efficient resource use in timber construction.

Similar in overall shape to the OSB beam, this design uses a plywood web with solid timber flanges. Plywood, made from thin layers of wood veneer, offers good strength and dimensional stability. The plywood web was connected to the solid timber flanges through continuous flat adhesive-bonded web-flange interfaces, similar to the OSB-web specimens.

All three beam types share the I-beam geometry, which is efficient in resisting bending and shear forces. The different web materials (OSB, laminated timber, and plywood) each bring unique properties to the beam's overall performance. The solid timber flanges in all designs provide the primary resistance to bending stresses. All specimens were tested over a support span of 1000 mm. The nominal cross-sectional dimensions of the specimens, including the overall beam depth, flange dimensions, and web thickness, are provided in Fig. 3.

These beams represent different approaches to engineered wood products, each balancing factors such as

material availability, manufacturing processes, and desired structural properties. The specific choice between

these types would depend on the intended application, load requirements, and economic considerations.

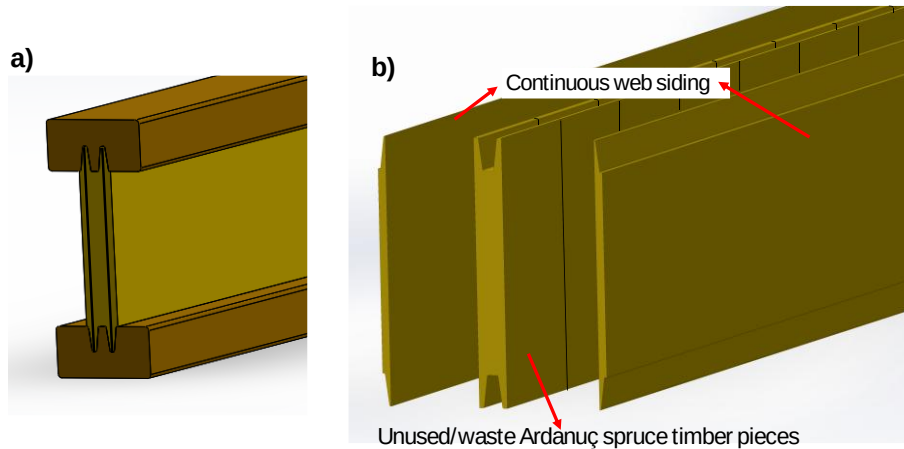


Fig. 2. (a) Perspective view of the laminated timber I-beam; (b) Detailed view of the laminated web configuration.

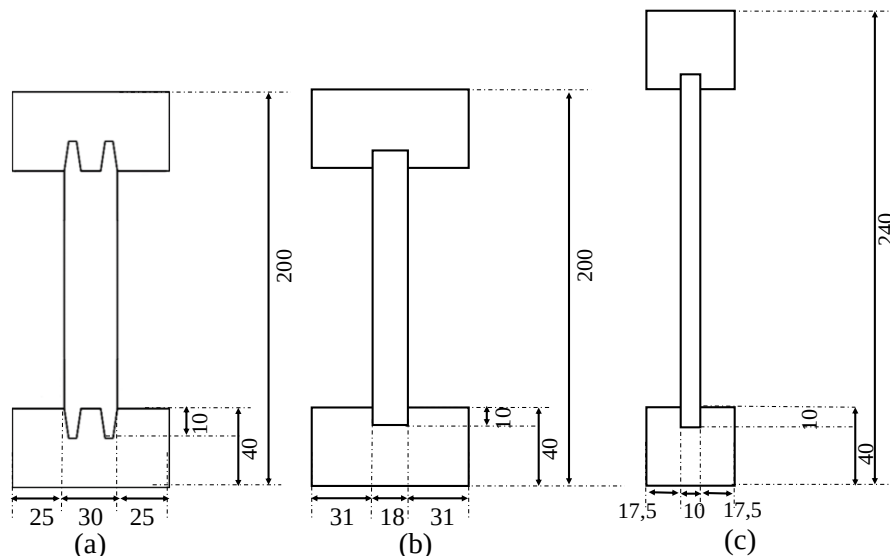


Fig. 3. Dimensions of the specimens (mm): (a) Laminated beam; (b) Plywood beam; (c) OSB beam.

2.1.1. Materials

OSB is an engineered wood product made of wood strands or flakes compressed and bonded with resin under high pressure and temperature. It is widely used in construction due to its strength, durability, and cost-effectiveness. While specific properties of the OSB used in this study were not measured, typical OSB panels have a density ranging from 600 to 680 kg/m³, and modulus of elasticity (MOE) between 3500 and 4800 N/mm². The exact properties can vary based on the manufacturing process and wood species used. The measured density and costs of the materials are provided in Table 2. In this study, the OSB used for the web had a density of 670.76 kg/m³ and cost \$444.61 \$/m³.

The laminated timber used in this study was made from Ardanuç spruce (*Picea orientalis*) as mentioned earlier. Laminated timber, also known as glued laminated timber or glulam, is an engineered wood product made by bonding together individual wood laminations

with durable, moisture-resistant adhesives. The laminated timber used for the web had a density of 431.20 kg/m³ and approximately cost 294.12 \$/m³.

The Ardanuç spruce used for the flanges and laminated beams exhibits notable physical and mechanical properties. The air-dry density of the wood is 0.448 g/cm³, indicating a moderate weight suitable for structural applications. The wood demonstrates varying shrinkage rates: 4.56% radially (β_g), 8.13% tangentially (β_k), 0.23% longitudinally (β_l), and a volumetric shrinkage (β_v) of 12.93% (Keskin et al. 2009). These shrinkage characteristics are important for understanding the dimensional stability of the material under changing moisture conditions.

In terms of mechanical properties, the Ardanuç spruce shows considerable strength. It has a bending strength (N/mm²) of 72.88, which is crucial for its performance in beam applications. The elasticity modulus (N/mm²) is 10070.09, indicating good stiffness and resistance to deformation under load. The compression

strength (N/mm^2) of 40.92 demonstrates the wood's ability to withstand compressive forces along the grain (Keskin et al. 2009). Additionally, the shear strength (N/mm^2) of 6.28 is important for its resistance to shearing forces (Keskin et al. 2009), particularly relevant in beam design where shear stresses can be significant. These properties make Ardanuç spruce a suitable choice for the flanges and laminated beams in this study, offering a balance of strength, stiffness, and dimensional stability. Although all flanges were produced from the same wood species, the flange materials used in the OSB-web beams and those used in the laminated- and plywood-

web beams were obtained from different commercial batches. Therefore, their measured density and cost values are reported separately in Table 2.

Plywood is a panel product made from thin layers of wood veneer, called plies, which are glued together with adjacent layers having their wood grain rotated up to 90 degrees from the adjacent layer. This cross-graining improves the panel's strength and dimensional stability. Although the specific properties of the plywood used in this study were not measured, the modulus of elasticity of plywood generally ranges from 6000 to 10000 N/mm^2 , depending on the specific type and grade of plywood.

Table 2. Material density and cost values.

Material Type	Density (kg/m^3)	Cost ($\$/\text{m}^3$)	Cost ($\$/\text{kg}$)
OSB web	670.76	444.61	0.66
Laminated timber web	431.20	294.12	0.68
Plywood web	663.48	915.03	1.38
Ardanuç spruce flange for laminated- and plywood-web beams	382.43	337.44	0.88
Ardanuç spruce flange for OSB-web beams	331.75	441.18	1.33

Note: OSB = oriented strand board.

It should be noted that the density values reported in Table 2 were directly measured for the materials used in this study, whereas some mechanical properties of OSB and plywood, such as typical modulus of elasticity ranges, were taken from literature or manufacturer-based reference ranges rather than being measured separately for each material batch. Therefore, these values are provided only to describe the general material characteristics and should not be interpreted as direct material test results. The experimental comparison in this study focuses on the flexural performance of the complete I-beam configurations under identical testing conditions, rather than on the isolated mechanical characterization of the individual web materials.

For all these materials, it's important to note that their performance in the I-beam configuration may differ from their individual properties due to the composite nature of the beams and the specific manufacturing processes used by the local producers. The experimental results from this study will provide valuable insights into how these materials perform in real-world, commercially produced I-beam configurations.

Polyurethane adhesive was used for bonding in the web-flange connections and at the joints. In addition to the adhesive, simple staples were used in both web and flange joints for added strength.

The cost analysis for 1 m I-beams showed that OSB beams cost \$3.62 (flange: \$2.82, web: \$0.80), laminated beams cost \$3.31 (flange: \$2.16, web: \$1.15), and plywood beams cost \$4.47 (flange: \$2.16, web: \$2.31).

It's important to note that the prices mentioned in this study are local market prices in Türkiye as of [09/2024] and may vary in different regions or countries. These costs are provided to give a general idea of the relative expenses associated with each material and beam type, but readers should be aware that prices can fluctuate based on local market conditions, availability, and economic factors.

2.1.2. Joint configuration

In this study, two distinct joint configurations were investigated: web-joined (WJ) and flange-joined (FJ) specimens. Continuous specimens without web or flange joints were also tested as reference specimens to evaluate the influence of joint location on flexural performance. The joint regions are shown in Fig. 4, where enlarged details are provided to emphasize the web and flange joint locations.

In the web-joined (WJ) configuration, the web panel was produced from two separate web segments, and the joint was located at the midspan of the beam. The upper and lower flanges remained continuous along the beam length. This configuration was selected to evaluate the influence of discontinuity in the web region while maintaining flange continuity, which is important for bending resistance.

In the flange-joined (FJ) configuration, the upper and lower flanges were each produced from two separate flange segments, and the joints were also located at the midspan of the beam. In this configuration, the web panel remained continuous, while the discontinuities were introduced in the flange members. Since the flanges carry the dominant tensile and compressive stresses under bending, this configuration was used to evaluate the effect of flange discontinuity on flexural performance and failure behavior.

For both jointed configurations, the mating surfaces were carefully aligned before bonding. Polyurethane adhesive was applied uniformly to the contact surfaces of the joint region and to the web-flange interfaces. Simple staples were additionally used in the jointed regions to assist mechanical fixing during assembly and to provide supplementary resistance against local separation. All joints were located at the beam midspan so that the effect of joint type could be evaluated under the same three-point bending configuration.

During specimen procurement, beams with visible manufacturing or material defects were not accepted from the manufacturer. Only visually sound specimens,

without apparent cracks, excessive warping, delamination, or major surface damage, were selected for testing.

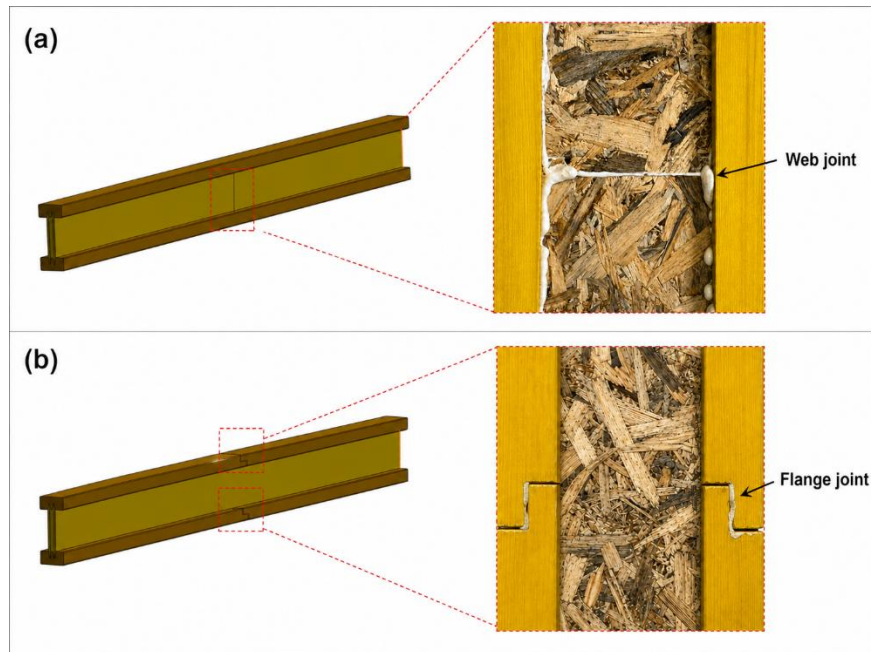


Fig. 4. Joint configurations of the tested timber I-beam specimens: (a) Web-jointed specimen with an enlarged detail of the web joint region; (b) Flange-jointed specimen with an enlarged detail of the flange joint region.

2.2. Experimental setup and procedure

The static bending tests were conducted using a three-point bending setup adapted from the general principles of ASTM D198 for structural-size wood and wood-based members. The experimental program was designed as a comparative flexural performance assessment rather than as a formal certification test for establishing standardized design values. Therefore, the same loading arrangement, boundary conditions, loading rate, and measurement procedure were used for all specimens to ensure a consistent comparison among the different material and joint configurations.

The tests were carried out using a BCO-FC fully automatic Sematron touch-controlled servo-hydraulic bending test machine (BESMAK, column-type frame). The machine is designed for three- and four-point bending tests and is equipped with a servo-hydraulic loading system,

adjustable bending supports, a hydraulic piston/loading head, and a computer-controlled Sematron Touch data acquisition system. The control unit supports load cell, LVDT, strain gauge, and pressure sensor inputs and has a data acquisition rate of up to 1 kHz. In the present study, the applied load was measured using the load cell of the testing machine, and the corresponding displacement was recorded from the crosshead displacement.

The specimens were placed on two roller supports and tested under simply supported boundary conditions. The clear span between the supports was 1000 mm, and the vertical load was applied at midspan through a single loading nose, as illustrated in Fig. 5. Prior to testing, each specimen was carefully measured, marked, and positioned symmetrically on the supports. The loading nose was aligned with the midspan to minimize eccentric loading and local stress concentrations.

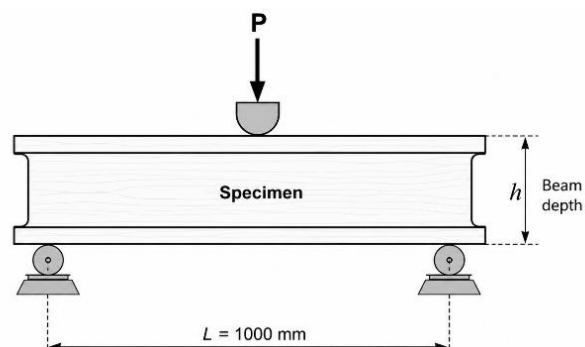


Fig. 5. Three-point bending test setup showing the midspan load, simply supported boundary conditions, clear support span ($L = 1000 \text{ mm}$), and beam depth (h).

The tests were performed under displacement control at a constant crosshead displacement rate of 1 mm/s, equivalent to 60 mm/min. Load and displacement data were continuously recorded throughout the test using the computer-based data acquisition system. The collected data were used to determine the load–displacement response, maximum load capacity, displacement at maximum load, maximum displacement, initial stiffness, post-peak behavior, failure mode, and energy absorption capacity of each specimen.

For each specimen, the cleaned load–displacement signal recorded by the testing machine was used without additional smoothing. The maximum load $P_{\max}$ was taken as the maximum value of the load array, and $\delta P_{\max}$ was defined as the displacement corresponding to $P_{\max}$. The maximum displacement $\delta_{\max}$ was defined as the largest displacement recorded before the end of the test. The initial stiffness K was calculated as the slope of an ordinary least-squares linear regression fitted to the loading branch between 10% and 40% of $P_{\max}$. Energy absorption was calculated by trapezoidal numerical integration of the load–displacement curve and is reported in

kN·mm. For each beam group, the mean, sample standard deviation, and coefficient of variation were calculated from the three replicate specimens.

3. Results and Discussion

This section presents the experimental results obtained from the flexural tests conducted on the nine different types of timber I-beams. The results are analyzed in terms of load-displacement behavior, maximum load capacity, stiffness, and energy absorption. Additionally, the failure modes observed during the tests are described and illustrated. Load-displacement graphs for all tested specimens are given in Fig. 6. This comprehensive figure presents the results of all test specimens together, allowing for the observation of general trends and comparisons. However, to conduct more detailed and specific analyses, the specimen results have been categorized and examined separately in subsequent sections. This approach enables a more in-depth evaluation of the unique performance characteristics and behaviors of each configuration.

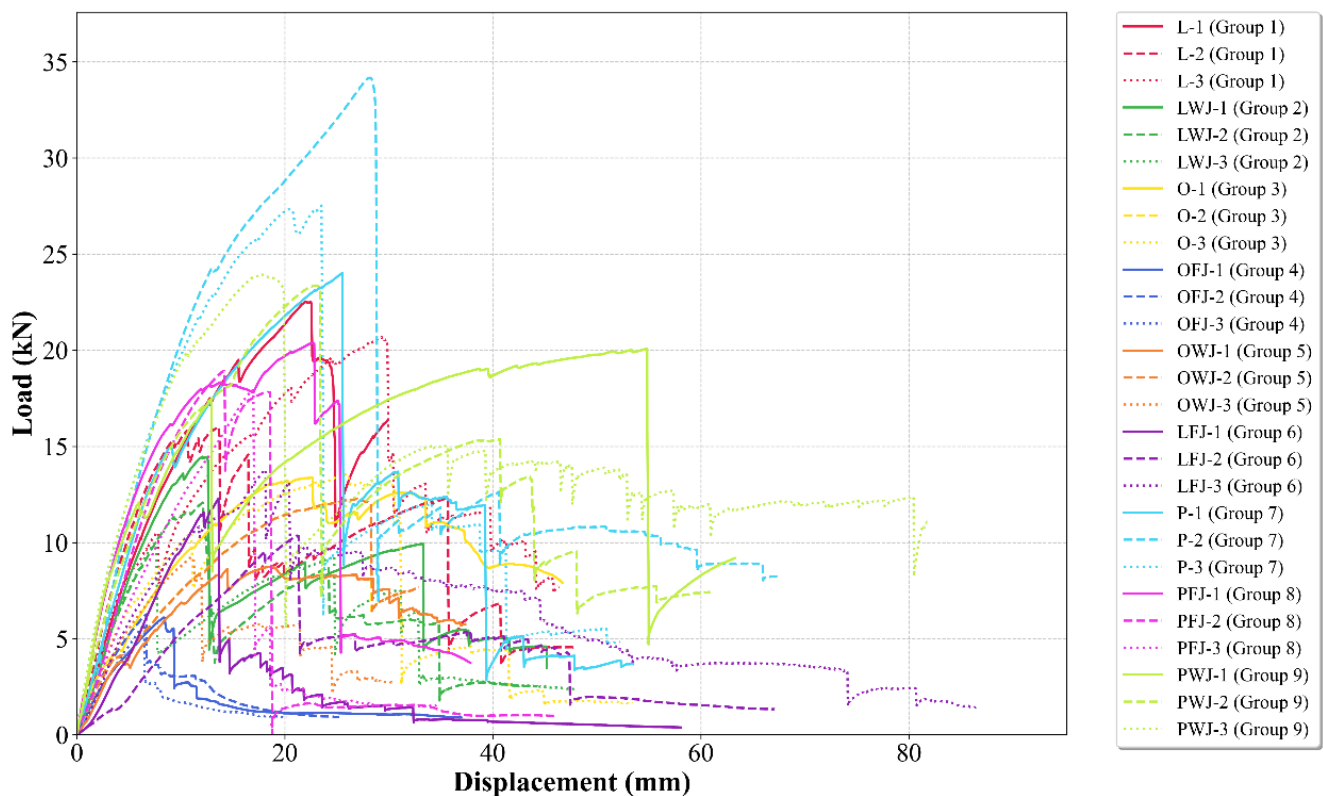


Fig. 6. Load displacement curves for all specimens.

The peak points of the curves represent the maximum load-bearing capacities of the individual specimens, while the post-peak branches reflect the progression of damage and the residual deformation capacity after peak load. Some specimens showed abrupt post-peak load reduction, indicating relatively brittle failure, whereas others maintained residual load over larger displacements, indicating progressive damage development and higher deformation capacity. Since Fig. 6 includes all individual specimens, the curves appear dense and overlapping;

therefore, the grouped mean curves in Fig. 7 and the numerical summary in Table 3 are used for clearer quantitative comparison among beam types and joint configurations.

The performance of each beam type is evaluated, with particular attention given to the effects of different materials (OSB, laminated timber, and plywood) and joint configurations (web-joined, flange-joined, and continuous). The data presented here forms the basis for the subsequent discussion on the comparative performance

of these engineered timber I-beams. To provide a clearer quantitative comparison of the experimental results, the main mechanical parameters obtained from all specimens are summarized in Table 3. The table includes the maximum load ($P_{\max}$), displacement at maximum load ($\delta P_{\max}$), maximum recorded displacement ($\delta_{\max}$), initial

stiffness (K), and energy absorption capacity of each specimen. For each beam group, the mean value, sample standard deviation (SD), and coefficient of variation (CoV) were calculated based on the three replicate specimens to evaluate the variability and repeatability of the test results.

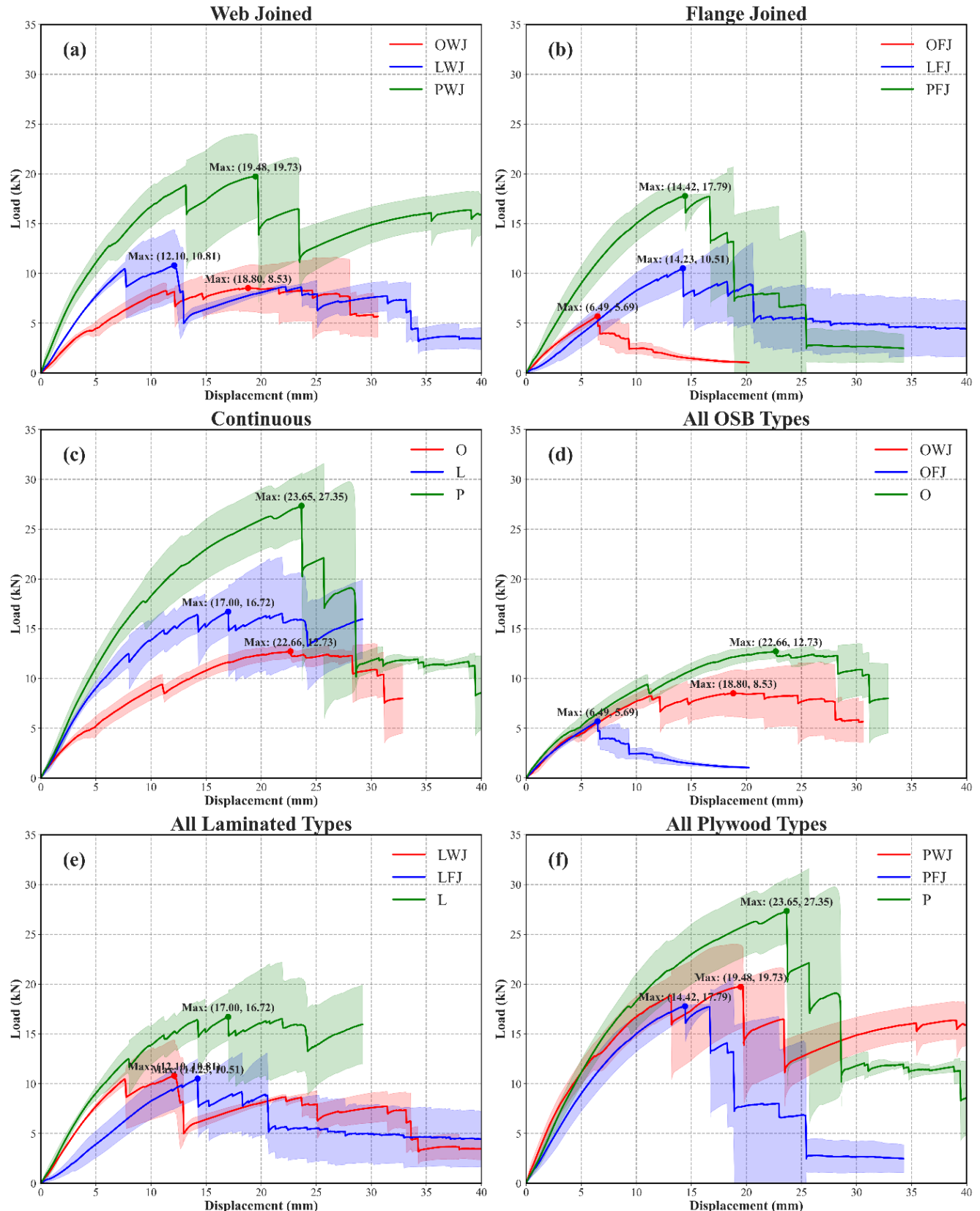


Fig. 7. Average load–displacement curves for specified groups of specimens.

Table 3. Comprehensive experimental results of timber I-beam specimens.

Beam group	Specimen ID	$P_{\max}$ (kN)	$\delta P_{\max}$ (mm)	$\delta_{\max}$ (mm)	K (kN/mm)	Energy absorption (kN·mm)
O	O-1	13.368	35.096	59.276	0.7735	460.62
O	O-2	12.282	41.144	46.952	0.6441	275.36
O	O-3	13.557	35.924	63.363	1.3687	380.57
O	Mean	13.069	37.388	56.53	0.9288	372.19
O	SD	0.688	3.279	8.543	0.3865	92.91
O	CoV (%)	5.265	8.77	15.112	41.6101	24.96
OWJ	OWJ-1	8.735	31.674	50.455	1.0304	249.31
OWJ	OWJ-2	9.125	28.144	48.452	1.1421	215.36
OWJ	OWJ-3	9.243	24.885	44.314	1.2935	155.48
OWJ	Mean	8.989	28.279	47.385	1.1619	202.39
OWJ	SD	0.359	4.801	4.342	0.186	66.35
OWJ	CoV (%)	3.998	16.975	9.164	16.012	32.78
OFJ	OFJ-1	6.1	74.989	103.569	1.0706	91.26
OFJ	OFJ-2	6.257	21.502	40.092	1.1812	65.2
OFJ	OFJ-3	5.742	20.383	34.114	0.9464	44.47
OFJ	Mean	6.033	38.958	59.258	1.0661	66.98
OFJ	SD	0.264	31.209	38.49	0.1175	23.44
OFJ	CoV (%)	4.373	80.109	64.954	11.0191	35.0
L	L-1	22.523	30.955	38.333	1.7685	437.32
L	L-2	15.956	23.123	57.231	2.3345	450.14
L	L-3	20.687	49.733	66.814	1.0502	590.97
L	Mean	19.722	34.604	54.126	1.7177	492.81
L	SD	3.388	13.675	14.492	0.6437	85.25
L	CoV (%)	17.18	39.519	26.775	37.4727	17.3
LWJ	LWJ-1	14.459	30.305	63.401	1.7083	346.31
LWJ	LWJ-2	12.078	31.386	63.595	1.7036	270.76
LWJ	LWJ-3	9.976	22.275	62.684	1.8338	263.77
LWJ	Mean	12.171	27.989	63.227	1.7485	293.61
LWJ	SD	2.243	4.978	0.48	0.0739	45.77
LWJ	CoV (%)	18.43	17.784	0.759	4.2237	15.59
LFJ	LFJ-1	12.283	13.632	58.082	0.9591	145.76
LFJ	LFJ-2	10.359	21.191	67.102	0.5905	273.4
LFJ	LFJ-3	13.794	18.084	86.534	1.3065	523.39
LFJ	Mean	12.145	17.636	70.573	0.9521	314.18
LFJ	SD	1.722	3.799	14.54	0.358	192.09
LFJ	CoV (%)	14.176	21.544	20.603	37.6067	61.14
P	P-1	24.018	25.544	53.483	1.8946	629.02
P	P-2	34.168	28.194	67.444	2.5441	1037.57
P	P-3	27.522	23.543	52.273	2.3603	697.96
P	Mean	28.569	25.76	57.733	2.2663	788.18
P	SD	5.155	2.333	8.431	0.3348	218.71
P	CoV (%)	18.045	9.057	14.604	14.7715	27.75
PWJ	PWJ-1	20.084	54.826	63.285	2.7511	926.04
PWJ	PWJ-2	23.363	22.966	60.897	1.8751	766.08
PWJ	PWJ-3	23.93	17.824	81.863	2.5898	1120.9
PWJ	Mean	22.459	31.872	68.682	2.4053	937.67
PWJ	SD	2.076	20.044	11.478	0.4662	177.7
PWJ	CoV (%)	9.246	62.89	16.711	19.3818	18.95
PFJ	PFJ-1	20.383	51.963	67.062	2.6281	444.76
PFJ	PFJ-2	18.915	14.174	46.173	1.9702	269.37
PFJ	PFJ-3	17.914	16.908	34.606	1.3423	209.48
PFJ	Mean	19.07	27.682	49.28	1.9802	307.87
PFJ	SD	1.242	21.073	16.45	0.6429	122.27
PFJ	CoV (%)	6.51	76.125	33.38	32.4682	39.72

Note: $P_{\max}$ = maximum recorded load; $\delta P_{\max}$ = displacement at $P_{\max}$; $\delta_{\max}$ = maximum recorded displacement before end of test; K = slope of OLS linear regression of P vs δ over the 10-40 % $P_{\max}$ window of the loading branch (fallback 20-50 % $P_{\max}$); Energy absorption = trapezoidal integration of P - δ over the cleaned load-displacement curve. Mean = arithmetic mean of valid replicates; SD = sample standard deviation (divisor $n - 1$); CoV (%) = SD / Mean x 100.

3.1. Load-displacement behavior

Fig. 7 illustrates the grouped load-displacement curves for various timber I-beam configurations. The curves represent the mean values calculated from three specimens for each beam type. The shaded areas around these mean curves represent the standard deviation of the measurements, providing a visual representation of the variability within each beam type.

This visualization method, combining mean curves with standard deviation bands, offers a comprehensive view of the structural performance and inherent variability in each beam configuration. It allows for a nuanced understanding of not only the average performance but also the consistency and reliability of different beam types under load.

The comparative analysis of different beam types shown in Fig. 7 reveals various structural performance characteristics and sustainability approaches. Continuous beams consistently demonstrated superior load-bearing capacities compared to their joined counterparts across all subplots Figs. 7(d-f). This highlights the significant impact of joints on overall structural integrity. The superior performance of continuous beams is attributed to the continuity in load distribution and the absence of stress concentrations.

The effect of material properties on load-bearing capacity is evident. As seen in Fig. 7(f), plywood (P) beams exhibited the highest peak loads, followed by laminated (L) and OSB beams. This ranking is explained by the high strength and stability provided by the cross-layered structure of plywood. Notably, the webs of the plywood beams were produced from plywood panels previously used as formwork. This demonstrates that waste materials can be transformed into high-performance structural elements.

The performance of laminated beams (L) is particularly noteworthy from a sustainability perspective. The web portion of these beams is made from timber pieces typically considered waste or unused. This innovative design ensures efficient resource utilization while providing satisfactory structural performance.

The effect of joint types on performance is clearly visible in Figs. 7(a) and (b). Generally, web-joined (WJ) beams exhibited higher maximum load capacities compared to flange-joined (FJ) beams. This indicates that the

position of the joint in the beam cross-section has a significant impact on load-bearing capacity. The superior performance of web-joined specimens is attributed to two key factors. First, the continuity of the flanges provides more effective resistance to bending moments. Second, and crucially, the continuous flanges in web-joined beams prevent premature failure by maintaining the integrity of the flange-web connection under shear stresses. In web-joined specimens, failure does not occur until either the continuous flange fails or the adhesive bond between the flange and web fails under shear stress. This mechanism allows web-joined beams to sustain higher loads and exhibit more ductile behavior compared to flange-joined beams, where the discontinuity in the flanges can lead to earlier failure initiation.

Performance variability, represented by the shaded areas in the graphs, differs among beam types. For example, in Fig. 7(a), PWJ beams show a wider range of variability compared to OWJ and LWJ beams. This variability is more pronounced in beams made from recycled or waste materials, as the properties of these materials can show greater variation.

Maximum load points are marked for each beam type in the graphs. These points allow us to compare the load-bearing capacities of different beam types and the displacement values at which they reach this capacity.

An important consideration in structural design is the strength-to-weight ratio of materials. Analysis of 1-meter beam sections reveals significant differences among the tested materials. OSB beams weigh 2.40 kg, substantially less than laminated beams at 4.13 kg and plywood beams at 4.12 kg. When considering the maximum load capacity in relation to weight, plywood beams demonstrate the highest strength-to-weight ratio of 6.93 kN/kg, followed by OSB beams at 5.45 kN/kg, while laminated beams show a ratio of 4.78 kN/kg. This analysis reveals that plywood beams offer both the highest absolute load-bearing capacity and the best strength-to-weight ratio. Interestingly, OSB beams, despite their lower absolute strength, provide a competitive strength-to-weight ratio that significantly outperforms laminated beams. Such characteristics make OSB an attractive option for applications where balancing weight and strength is crucial, such as in renovations of existing buildings or in designs where foundation loads need to be minimized.

Table 4. Strength-to-weight comparison of continuous timber I-beam specimens.

Type	Weight (kg)	Max load (kN)	Strength-to-weight ratio (kN/kg)
OSB Beam	2.40	13.069	5.45
Laminated Beam	4.13	19.722	4.78
Plywood Beam	4.12	28.569	6.93

Overall, this comprehensive analysis details the effects of different beam types, materials, and joining methods on structural performance. It is particularly important in demonstrating that waste or used materials can be transformed into high-performance structural el-

ements. This approach makes a significant contribution to sustainable construction practices and circular economy principles. At the same time, understanding the impact of joint locations on beam performance is critical, especially in large-span applications. This information

will serve as a valuable guide in selecting the most appropriate beam configuration for specific applications and in sustainable structural design.

3.2. Energy absorption

Fig. 8 presents the energy absorption capacities of different timber I-beam types. These results provide experimental evidence for comparing the flexural response of timber I-beams with different web materials and joint configurations.

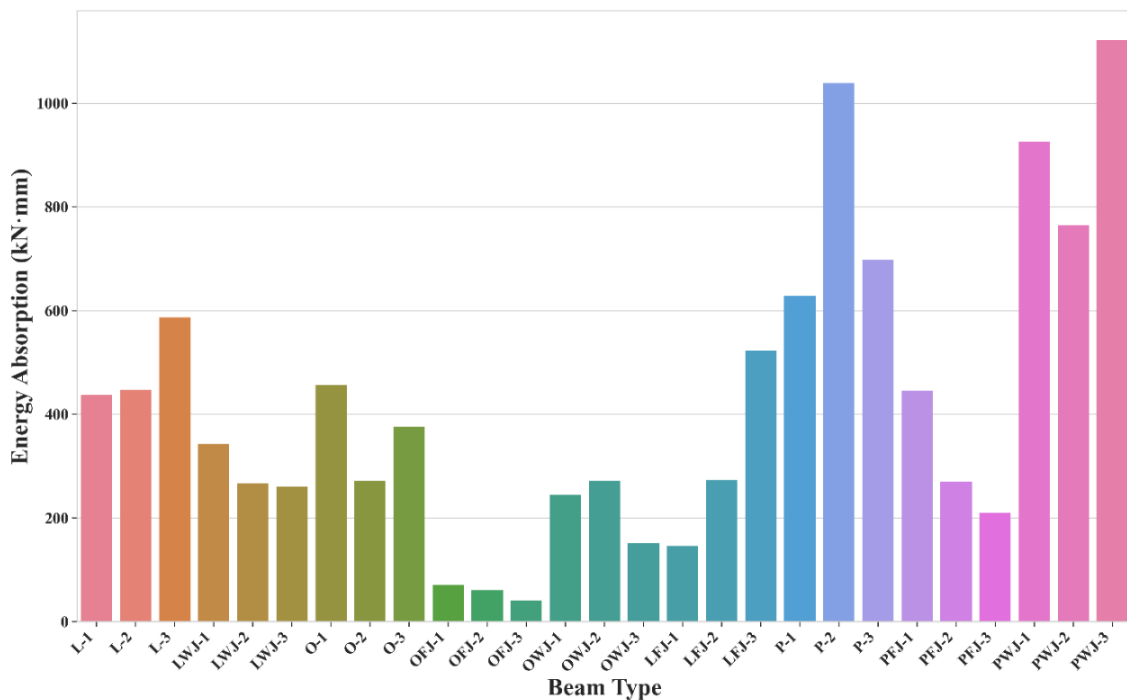


Fig. 8. The comparison of energy absorption capacity of the specimens.

It should be emphasized that the energy absorption values reported in this study were obtained from monotonic three-point bending tests and were calculated as the area under the load–displacement curves. Therefore, these values represent the absorbed energy under monotonic loading and should be interpreted as a comparative indicator of deformation capacity and post-peak response among the tested configurations. They should not be directly interpreted as cyclic energy dissipation capacity. Under cyclic loading, stiffness degradation, damage accumulation, adhesive debonding, and progressive joint deterioration may influence the hysteretic response and energy dissipation behavior. Therefore, future studies should include cyclic or repeated loading tests to evaluate the dissipative performance of these timber I-beam configurations more comprehensively.

3.3. Maximum load capacity

Fig. 9 presents the maximum load capacity of each beam type. Load capacity is vital for determining safety factors and load limits in structural design. P-series beams consistently demonstrated the highest maximum load capacities, suggesting superior performance in applications requiring high strength-to-weight ratios.

PWJ beams, particularly PWJ-3, demonstrated exceptional energy absorption capacity, suggesting potential applications in structures requiring high ductility. Energy absorption generally decreased in several joined configurations; however, some specimens, particularly in the PWJ group, exhibited higher energy absorption due to their larger deformation capacity. OSB beams showed the lowest energy absorption among the tested configurations, indicating limitations in their use for applications requiring high energy dissipation.

Joined beams, particularly flange-joined types, exhibited lower maximum load capacities compared to continuous beams, highlighting the need for careful consideration of joint placement in design. Within each material category (OSB, laminated, plywood), continuous beams outperformed joined beams in terms of maximum load capacity, emphasizing the structural advantages of uninterrupted beam sections.

3.4. Stiffness analysis

Fig. 10 presents the stiffness values for different timber I-beam types. Stiffness, calculated as the slope of the linear portion of the load–displacement curve, indicates the beam's resistance to deformation under load - a critical factor in serviceability limit state design.

P-series beams, especially the continuous ones, demonstrated the highest stiffness values, suggesting their suitability for applications where minimal deflection is crucial. OSB beams generally showed lower stiffness compared to laminated and plywood beams, indicating potential limitations in their use for long-span or heavily loaded structures. The effect of joints on stiffness was configuration-dependent; however, flange-joined specimens generally showed a less favorable stiffness response in the laminated and plywood groups.

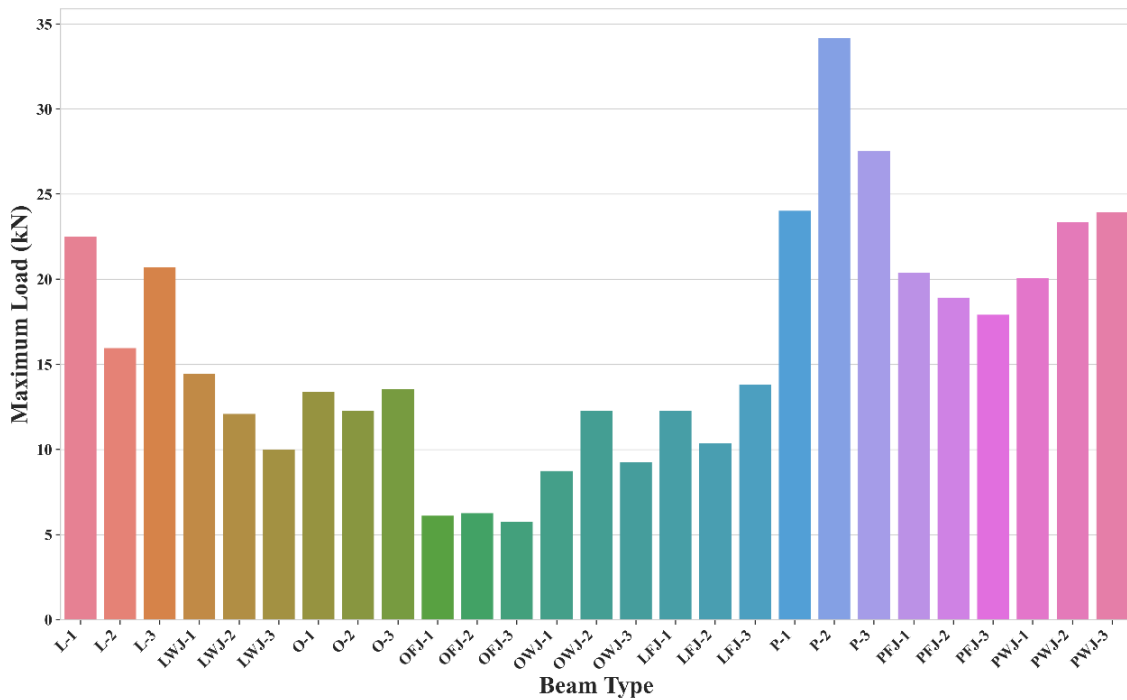


Fig. 9. Maximum load capacity of each specimen.

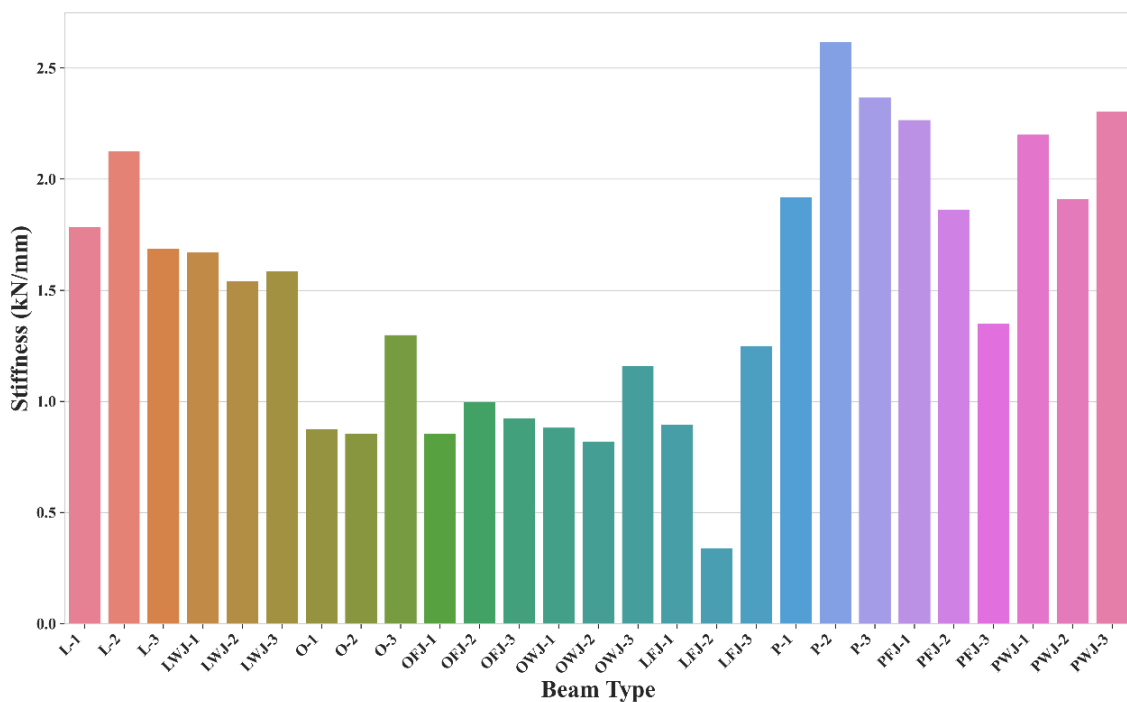


Fig. 10. Stiffness comparison of the specimens.

3.5. Failure modes

In the three-point bending configuration, the specimens were subjected to a combination of bending moment and shear force. While the flanges primarily resisted the tensile and compressive stresses generated by bending, the web and the web-flange adhesive interfaces were mainly responsible for shear transfer. Therefore, shear effects played an important role in the initiation and progression of several observed failure modes, particularly web cracking, web tearing, web-flange sep-

aration, adhesive bond failure, and damage around the jointed regions.

The observed failure modes provide valuable insights into the structural behavior and weak points of different beam configurations. These observations offer critical information for refining design methodologies and improving structural resilience.

Laminated timber beams exhibited distinct failure patterns depending on their joint configuration. In web-joined (LWJ) beams, failure typically initiated in the web, often near the stapled joint area. This initial web failure

was followed by progressive failure of the flanges, with the bottom flange usually failing before the top flange (Fig. 11(a)). On the other hand, flange-joined (LFJ) beams highlighted the critical nature of the flange joint, with failure often initiating as cracking or delamination at the bottom flange joint, followed by web separation and potential progression to the top flange (Fig. 11(b)).

Continuous laminated timber (L) beams demonstrated more varied failure modes. Some specimens experienced initial crushing or breaking of the top flange due to load-induced slippage, while others exhibited web failure followed by bottom flange separation from the web and upward crack propagation (Fig. 11(c)).

OSB beams also showed distinct failure characteristics based on their joint configuration. Web-joined (OWJ) beams exhibited more varied failure modes compared to their flange-joined (OFJ) counterparts. In OWJ beams, delamination between the web and flange was more pronounced, especially under high loads, potentially leading to ultimate failure (Fig. 11(d)). In contrast, OFJ beams consistently demonstrated a characteristic failure pattern, with cracks initiating at the bottom flange joint and propagating upwards through the web (Fig. 11(e)). This consistent behavior across OFJ specimens highlights the critical nature of the flange joint in OSB beams.

Continuous OSB (O) beams typically failed in a sequence starting with the top flange, followed by the bot-

tom flange, and finally the web. However, some variations were observed, such as initial top flange crushing followed by web cracking and bottom flange failure (Fig. 11(f)).

Plywood beams exhibited failure modes that were influenced by both joint configuration and the brittle nature of the material. Web-joined (PWJ) beams typically failed through web separation at the joint, followed by flange failure (Fig. 11(g)). The failure often initiated with the bottom flange sliding relative to the web, accompanied by progressive web tearing. Similarly, flange-joined (PFJ) beams exhibited bottom flange slippage occurring simultaneously with web failure initiation (Fig. 11(h)).

Continuous plywood (P) beams often showed a distinct failure sequence, with initial failure in the top flange, followed by brittle web failure and subsequent bottom flange separation. The brittle web failure was a notable characteristic of plywood beams, setting them apart from the other materials tested (Fig. 11(i)).

The diverse failure modes observed across different beam types and joint configurations underscore the complex interplay between material properties, joint design, and load distribution mechanisms. Understanding these failure mechanisms is crucial for optimizing beam design and improving overall structural performance. By carefully considering the strengths and weaknesses of each configuration, engineers can develop more resilient and efficient structural solutions.

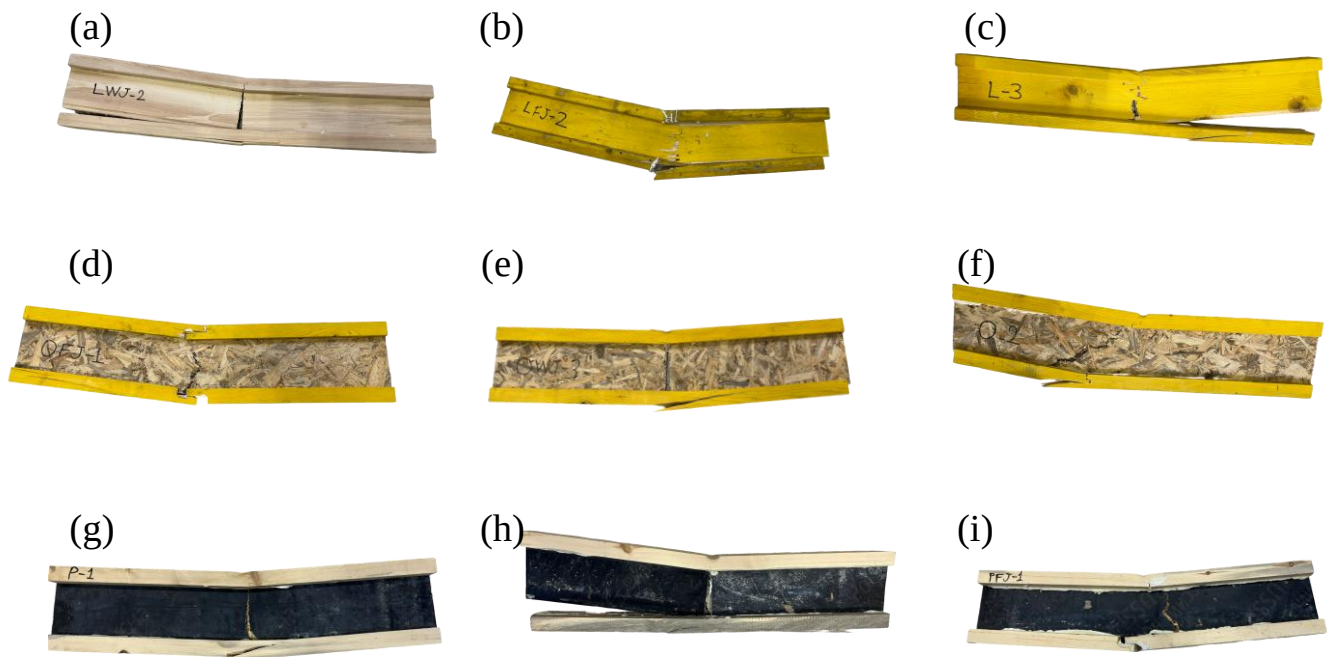


Fig. 11. Representative failure modes observed in the tested timber I-beam specimens:

(a) LWJ specimen; (b) LFJ specimen; (c) Continuous laminated specimen; (d) OWJ specimen; (e) OFJ specimen; (f) Continuous OSB specimen; (g) PWJ specimen; (h) PFJ specimen; (i) Continuous plywood specimen.

3.6. Joint performance analysis

A comparative analysis of joined beams (WJ and FJ) with continuous beams reveals the significant impact of joints on structural performance. Joined beams consistently showed lower maximum load capacities compared to continuous beams of the same material, with reductions ranging from 15% to 30%. Compared with the cor-

responding continuous beams, the joined specimens generally exhibited lower maximum load capacities. The retained load capacity varied depending on both material type and joint location. Web-joined specimens retained approximately 62–79% of the continuous beam capacity, whereas flange-joined specimens retained approximately 46–67%. This confirms that joint location has a pronounced influence on ultimate load capacity,

with flange joints generally producing a more detrimental effect.

Energy absorption showed a more configuration-dependent trend than maximum load capacity. While most joined specimens exhibited lower energy absorption than the corresponding continuous beams, the PWJ group showed higher energy absorption due to its larger deformation capacity. This indicates that energy absorption is governed not only by peak load but also by post-peak deformation behavior and failure progression (Ozdemir 2025).

The effect of joints on stiffness was less uniform than their effect on maximum load capacity. While flange-joined specimens generally showed lower stiffness in the laminated and plywood groups, some web-joined specimens exhibited comparable or slightly higher initial stiffness than their continuous counterparts. This indicates that the presence of a joint does not necessarily reduce the initial elastic stiffness, although it generally affects load-bearing capacity, post-peak response, and failure mode.

Joints often served as initiation points for failure, particularly in flange-joined beams, suggesting that joint design and placement are critical factors in overall beam performance, as interfacial bond quality directly affects stress-transfer efficiency (Tarhan et al. 2025). It should be noted that the tested beam types were compared as commercially available I-beam configurations rather than as geometrically identical laboratory-manufactured specimens. Therefore, the observed differences in maximum load, stiffness, and energy absorption are influenced not only by the web material and joint configuration, but also by differences in cross-sectional geometry and the corresponding sectional properties, such as the moment of inertia. For this reason, the results should be interpreted as a comparative assessment of practical beam configurations under identical testing conditions, rather than as an isolated comparison of material properties alone.

The influence of shear forces was particularly evident in the joined specimens. In web-joined beams, the web joint was located in the component responsible for shear transfer, and this promoted local web damage, adhesive debonding, or progressive web tearing depending on the material type. In flange-joined beams, although the joint was located in the flexural stress-carrying component, shear transfer through the web-flange interface also contributed to local separation and crack propagation near the joint region. Thus, the observed failures should be interpreted as the result of combined bending and shear actions rather than pure flexural failure alone.

Overall, this comprehensive analysis of various timber I-beam configurations provides valuable insights into the complex interplay between material properties, joint design, and overall structural performance. These findings not only contribute to our understanding of engineered timber structures but also offer practical guidance for optimizing beam design in real-world applications. Future research should focus on developing material-specific joint designs that address the observed failure modes, potentially incorporating local reinforcement at critical points identified through this analysis.

4. Conclusions

The extensive investigation into the structural behavior of various engineered wood I-beams has yielded significant insights into the performance characteristics of different materials and joint configurations. Our study, encompassing OSB, laminated timber, and plywood beams with various joining methods, has revealed complex interactions between material properties, joint designs, and load distribution mechanisms that profoundly influence beam performance.

The plywood-web I-beam configuration exhibited the highest load-bearing capacity and stiffness among the tested beam configurations. However, the unexpected competitiveness of OSB beams in terms of strength-to-weight ratio highlights the potential for utilizing this more cost-effective material in applications where weight reduction is crucial. The varied performance of laminated timber beams across different configurations emphasizes the critical role of manufacturing processes and joint designs in determining overall structural integrity.

The observed variations in stiffness and failure modes between continuous beams and their joined counterparts highlight both the challenges and opportunities in joint design. While joined beams generally exhibited lower stiffness and more complex failure mechanisms, the potential for optimizing joint configurations to enhance performance is evident. This opens avenues for developing more efficient and resilient structural elements that balance performance with material economy.

Furthermore, our study demonstrates the viability of incorporating recycled or repurposed materials, such as construction site plywood and typically unused timber pieces, into high-performance structural elements. This approach not only contributes to sustainable construction practices but also aligns with circular economy principles, offering a pathway to reduce waste and enhance resource efficiency in the construction industry. Since the tested specimens represent practical beam configurations with possible differences in cross-sectional geometry, the conclusions should be interpreted in terms of overall beam configuration performance. A further limitation of this study is that analytical load-carrying capacity predictions were not developed, because the tested beams were not laboratory-manufactured specimens produced under fully controlled and standardized material and bonding conditions. The novelty of the study is based on the comparative experimental evaluation of practical timber I-beam configurations incorporating locally available, unused, and previously used wood-based components. Although nominal adhesive properties may be available, the actual web-flange and joint-interface behavior depends on several application-dependent factors, including surface condition, adhesive thickness, bonding pressure, curing conditions, moisture state, and the previous service history of reused materials. These parameters were not independently characterized in the present study. Therefore, a simplified analytical capacity model would require assumptions that may not reliably represent the actual behavior of the tested beams. For this reason, the results are presented

as an experimental comparison of practical beam configurations under identical testing conditions. Future studies should develop analytical or numerical models calibrated with directly measured material properties, adhesive bond characteristics, and joint-interface parameters.

Overall, this study provides practical insight into the flexural performance of timber I-beam configurations produced with different web materials and joint details. The results show that joint location significantly affects load-bearing capacity, deformation behavior, and failure mechanisms, particularly when discontinuities are introduced in the flange region. Since the tested specimens represent practical beam configurations rather than laboratory-manufactured members with fully controlled material and bonding properties, the findings should be interpreted at the configuration level. Future studies should focus on developing analytical and numerical models calibrated with directly measured material properties, adhesive bond characteristics, and joint-interface parameters.

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Conflict of Interest

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Data Availability

The datasets generated and/or analyzed during the current study are not publicly available but are available from the corresponding author upon reasonable request.

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During the preparation of this manuscript, Google Gemini was used exclusively for language editing and stylistic refinement. The authors take full responsibility for the content, interpretation, and conclusions of the published article.

Author Contributions

The author declares sole responsibility for all aspects of the study, including conceptualization, methodology, formal analysis, investigation, data curation, visualization, writing of the original draft, and writing, review, and editing of the manuscript.

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Research Article

Comparative analysis of timber-framed structural systems across different regions: Seismic energy dissipation and controlled damage mechanisms

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ABSTRACT

This study presents a comparative evaluation of the seismic performance of traditional timber-framed structural systems developed in different geographical regions, including Himiş, Dhajji Dewari, Pombalino, Fachwerk, Casa Baraccata, and Chuandou. A literature-based assessment was conducted focusing on energy dissipation mechanisms, ductility characteristics, and damage development processes. The findings indicate that, despite regional and structural differences, these systems exhibit comparable behavioral tendencies under seismic loading. The observed behavior may be interpreted through a multi-stage response mechanism in which seismic demands are progressively distributed through infill materials, connection details, and secondary structural components. In this process, energy dissipation is achieved through the interaction of these elements rather than through the action of a single structural component. Based on the behavioral trends reported in the literature, this study proposes the concept of a “first seismic defense line” as a conceptual framework for interpreting the contribution of secondary components during the early stages of seismic response. Comparative observations suggest that infill elements play a significant role in the energy dissipation process of Himiş and Dhajji Dewari systems, whereas frictional behavior at connection interfaces is more prominent in Chuandou structures. The Pombalino system demonstrates a balanced interaction between stiffness and ductility, while the behavior of Fachwerk and Casa Baraccata systems is governed by frame–infill interaction and the integrated action of timber and masonry components. Overall, these traditional systems exhibit performance-oriented characteristics that are consistent with fundamental principles of modern earthquake engineering and provide valuable insights for contemporary seismic design.

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1. Introduction

Timber has gained increasing importance in sustainable construction due to its renewable nature, low embodied carbon, and carbon storage capacity. Compared with conventional construction materials such as concrete and steel, timber-based structural systems offer lower life-cycle carbon emissions and contribute to the

reduction of greenhouse gas emissions, leading to growing interest in their application in contemporary building design (Gün and Anıktar 2020; Flanagan 2017). Previous studies have also indicated that the ecological footprint of locally sourced timber materials is generally lower than that of modern industrial construction alternatives (Ozdemir 2025; Yılmaz Karaman and Tanaç Zeren 2015; Dikmen 2010).

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The seismic performance of timber-framed structures depends not only on the characteristics of the primary structural system but also on connection details, the behavior of infill materials, roof loads, soil conditions, and the physical changes experienced by timber over time. In particular, connection discontinuities, geometric irregularities, heavy superstructure elements, and material degradation caused by biological or environmental effects have been identified as key factors influencing seismic response (Dişkaya 2004). Earthquake-related damage is not limited to the direct effects of ground motion, as secondary hazards such as fire, flooding, and geotechnical failures may also threaten structural safety (Aksoy 2003). Consequently, reducing seismic risk requires not only the design of new structures but also the assessment of existing and historic buildings and the planning of appropriate conservation and rehabilitation interventions (Arslan 2006).

Although traditional timber-framed construction systems evolved in different geographical regions and historical periods, they are generally based on similar structural principles. The combined use of timber members and masonry infill dates back to ancient civilizations and has led to the development of various timber-framed construction traditions across Europe, Anatolia, and Asia. Over time, distinct systems such as Pombalino in Portugal, Casa Baraccata in Italy, Fachwerk in Germany, Hımsı in Türkiye, and Dhajji Dewari in South Asia emerged. Despite differences in structural detailing, these systems share a common characteristic in which the timber frame plays an active role in resisting lateral loads while interacting with infill elements to enhance overall structural stability (Vasconcelos et al. 2013).

From a structural engineering perspective, timber-framed systems provide a ductile load-resisting mechanism capable of accommodating lateral forces and deformations induced by earthquakes. Infill components contribute to the mass and stiffness of the structure; however, when adequate interaction with the timber frame is not achieved, they may become vulnerable to brittle damage. Observational studies conducted after major seismic events have demonstrated that properly detailed and well-constructed timber-framed buildings generally exhibit superior seismic performance compared with unreinforced masonry structures. This tendency has been confirmed through post-earthquake damage assessments and field observations following the 1999 İzmit Earthquake, the 2005 Kashmir Earthquake, the 2015 Gorkha Earthquake, and the 2023 Kahramanmaraş Earthquakes (Vieux-Champagne et al. 2014; Qu et al. 2020; Tan et al. 2024). Similarly, Korkmaz (2007) reported that seismic performance varies significantly depending on the structural system and that timber-framed structures generally perform better than conventional masonry buildings. Historical records following the 1509 Great İstanbul Earthquake also indicate that Hımsı structures exhibited better seismic performance and resulted in fewer casualties than masonry buildings (Aktaş 2011). These observations highlight the importance of examining the behavioral characteristics of traditional structural systems and conducting comparative evaluations among different timber-framed construction traditions.

Traditional timber-framed systems represent important examples of indigenous seismic knowledge developed by communities living in earthquake-prone regions. Although known by different names across various geographical contexts, these systems are generally characterized by similar structural principles and by the interaction between a flexible timber frame and infill components (Gani et al. 2021; Vasconcelos et al. 2015; Langenbach 2007) (Fig. 1).

Known as Fachwerk and Pombalino in Europe, Dhajji Dewari in South Asia, Chuandou in China, and Casa Baraccata in the Mediterranean region, these systems have attracted considerable attention due to their adaptable and resilient behavior under seismic loading.

Field observations and experimental studies have shown that these structures may exhibit more controlled damage patterns during major earthquakes compared with modern reinforced concrete systems (Doğangün et al. 2006; Gülhan and Özyörük Güney 2000). These observations suggest that such systems may provide enhanced energy dissipation capacity and improved ductile response.

The seismic performance of these systems has been investigated from various perspectives in the literature. In Pombalino structures, the contribution of bracing members to energy dissipation and their relatively high ductility capacity have been extensively documented (Poletti and Vasconcelos 2015). For Hımsı systems, frame-infill interaction has been identified as a key mechanism governing energy dissipation behavior (Gani et al. 2021; Ozdemir 2025). In Chuandou structures, friction-based mechanisms developing at connection interfaces have been shown to provide considerable energy dissipation capacity (Li et al. 2025; Yang et al. 2021). Similarly, the cellular wall configuration of Dhajji Dewari systems has been reported to enhance structural stability by distributing damage across multiple small panels (Ali et al. 2012).

In addition to these structural characteristics, factors such as topographic conditions and material properties also play an important role in structural response. Geometric irregularities associated with sloping terrain may adversely affect seismic performance (Pan et al. 2023). Furthermore, the type of infill material directly influences both energy dissipation capacity and dominant damage mechanisms (Ozdemir 2025).

Existing research indicates that the successful seismic performance of traditional timber-framed systems is rooted in a set of common structural principles. Nevertheless, the reported behavioral characteristics reveal notable differences among systems in terms of load-transfer mechanisms, deformation capacity, and damage distribution patterns. While bracing elements play a dominant role in maintaining structural stability in some systems, in others the interaction between infill components and the timber frame, or localized mechanisms developing at connection regions, exerts a greater influence on structural behavior. These observations suggest that, despite sharing similar structural principles, the seismic performance of traditional timber-framed systems may vary according to material characteristics, construction techniques, and regional building traditions.

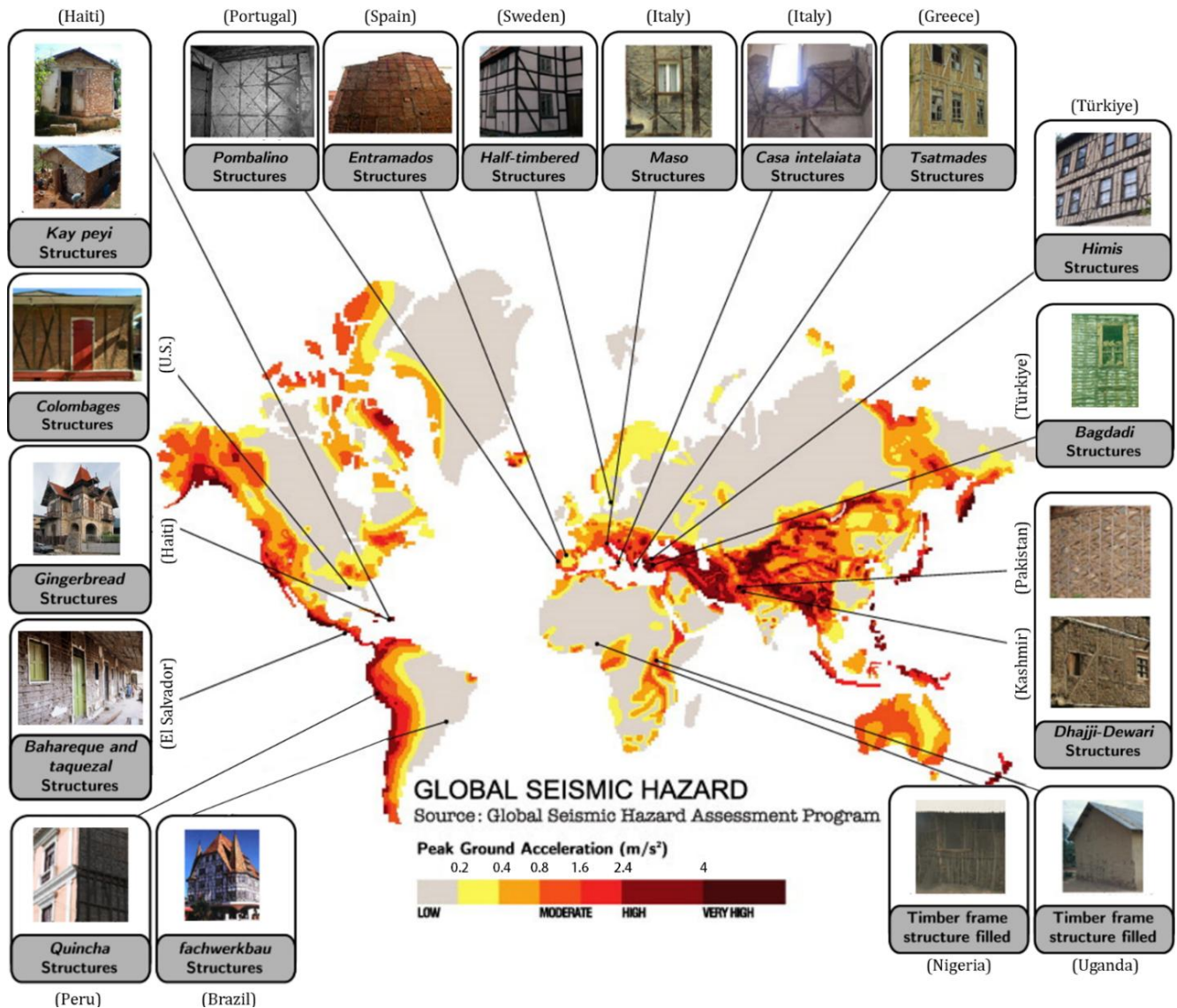


Fig. 1. Examples of timber-framed structures worldwide (adapted from Vieux-Champagne et al. (2014)).

However, most existing studies focus on specific regional construction traditions or individual structural systems, whereas comparative investigations evaluating different timber-framed systems within a common performance framework remain limited. This highlights the need for a comprehensive framework capable of identifying both shared and distinct behavioral mechanisms among traditional timber-framed systems.

Within this context, the present study provides a comparative assessment of Hımiş, Dhajji Dewari, Pombalino, Fachwerk, Casa Baraccata, and Chuandou systems developed in different geographical regions. The primary objective is to examine their seismic behavior within a common analytical framework and to identify their similarities and differences. Accordingly, a conceptual framework referred to as the “first seismic defense line” is proposed to interpret the interaction between infill elements, connection regions, and secondary structural components during the energy dissipation process under seismic loading. This approach offers a conceptual and analytical perspective for understanding the interactive behavior between the primary structural system and secondary components.

2. Investigated Systems and Methodology

2.1. Traditional timber-framed systems

This study evaluates traditional timber-framed structural systems developed in different geographical regions and widely investigated in literature with respect to their seismic behavior. The selected systems exhibit distinct characteristics in terms of structural configuration, infill arrangements, connection details, and energy dissipation mechanisms. Accordingly, Hımiş (Türkiye), Dhajji Dewari (Himalayan region), Pombalino (Portugal), Fachwerk (Germany), Casa Baraccata (Italy), and Chuandou (China) were included in the comparative assessment.

2.1.1. Hımiş system

The Hımiş system is a traditional timber-framed construction technique widely used in Anatolia and neighboring regions. Following the 1509 Great İstanbul Earthquake (Kıyamet-i Süğra), the Ottoman administration restricted the construction of masonry buildings and en-

couraged the use of timber-framed *Himış* structures. The system consists of a load-bearing timber frame composed of vertical and horizontal members, with infill materials placed within the frame bays. Adobe, stone, brick, and other locally available materials are commonly used as infill. While the timber frame constitutes the primary load-resisting system, the infill elements act as secondary components that contribute to overall structural behavior. *Himış* buildings are typically constructed using local materials and are predominantly employed in low-rise residential buildings. Their structural response is governed by a hybrid mechanism defined by the interaction between the timber frame and infill elements (Aktaş et al. 2014; Aktaş 2011).

2.1.2. *Dhajji Dewari* system

Dhajji Dewari is a traditional timber-framed construction system developed in the Himalayan region. It is characterized by a cellular arrangement consisting of small timber-framed panels typically filled with stone or brick fragments. The primary structural system is formed by the timber frame, while the infill elements are divided into smaller compartments. This cellular configuration facilitates load redistribution throughout the structure and limits the concentration of damage in localized regions. Seismic construction traditions in the Himalayan region are not limited to *Dhajji Dewari* and also include timber-laced masonry systems and lightweight infill structures such as *Ikra*, which employ similar strategies based on material flexibility and deformation capacity (Gani et al. 2021; Ali et al. 2012).

2.1.3. *Pombalino* system

The *Pombalino* system is a traditional timber-framed structural system developed in Portugal following the 1755 Lisbon Earthquake to improve seismic resistance. Its principal component is the *gaiola pombalina*, a timber framework reinforced with X-shaped bracing members. This timber frame is integrated within masonry walls to form a composite structural system. The *Pombalino* system relies on the combined action of a braced modular timber frame and masonry walls. The interaction between the internal timber structure and the external masonry components defines its hybrid structural character. Within this arrangement, masonry walls provide protection against environmental effects and fire, while the timber frame contributes to seismic resistance (Aktaş 2011). The interaction between timber and masonry components enables the system to exhibit both stiffness and ductility characteristics (Parrotta 2017; Poletti and Vasconcelos 2015).

2.1.4. *Fachwerk* system

Fachwerk is a traditional timber-framed construction system widely used in Germany. The system consists of a structural skeleton composed of timber posts, horizontal members, and diagonal braces, with infill panels placed within the frame. Brick and adobe are commonly used as infill materials. The timber frame serves as the primary load-resisting system, whereas the infill ele-

ments function as secondary components. The construction technique is characterized by an exposed structural frame visible on the exterior façade, creating a distinctive modular architectural appearance (Parlak Biçer and Şahin 2020).

2.1.5. *Casa Baraccata* system

Casa Baraccata is a traditional timber-masonry hybrid structural system developed in the Calabria region of Italy following the series of destructive earthquakes that occurred in 1783. The system is based on a structural configuration in which timber framing and masonry elements are used together, with timber bracing members positioned on the wall surface rather than embedded within the wall section (Aktaş 2011). The structural arrangement consists of a grid-like timber framework combined with infill components. The interaction between timber and masonry elements constitutes the primary mechanism governing structural behavior. *Casa Baraccata* therefore represents a hybrid structural organization in which timber and masonry components act together to resist seismic actions (Gani et al. 2021; Parrotta 2017).

2.1.6. *Chuandou* system

The *Chuandou* system is a traditional timber structural system widely used in China. It is based on a frame composed of columns and beams connected through mortise-and-tenon joints. The primary load-resisting system consists entirely of the timber frame, while infill components may contribute to structural response without constituting the principal load-transfer mechanism. Traditional construction relies on carpentry techniques without the use of nails. The system is characterized by a modular framing arrangement in which the behavior of connection interfaces plays a critical role in overall structural performance (Li et al. 2025; Yang et al. 2021).

Representative examples of the traditional timber-framed systems investigated in this study are presented in Fig. 2. The selected examples were chosen to illustrate the characteristic architectural appearance and construction features of each structural typology.

Representative schematic drawings of the investigated timber-framed systems were developed within the scope of this study and are presented in Figure 3. These drawings were prepared based on photographs, survey drawings, and structural representations reported in the literature and were redrawn using a standardized graphical format to ensure consistency in the visual presentation of the different structural typologies. Based on the structural characteristics illustrated in Fig. 3, the main descriptive features of traditional timber-framed systems are comparatively summarized in Table 1.

2.2. Methodology

This study adopts a literature-based comparative analysis approach. The primary objective is to evaluate the seismic behavior of traditional timber-framed structural systems developed in different geographical re-

gions within a common analytical framework and to comparatively interpret the behavioral tendencies reported in the literature. The investigated systems were assessed based on criteria including first seismic defense line behavior, damage hierarchy, energy dissipation

mechanisms, connection behavior, preservation of the primary load-resisting system, and ductile deformation capacity. These criteria enable the evaluation of structural response at both the macro scale (system level) and the micro scale (component level).

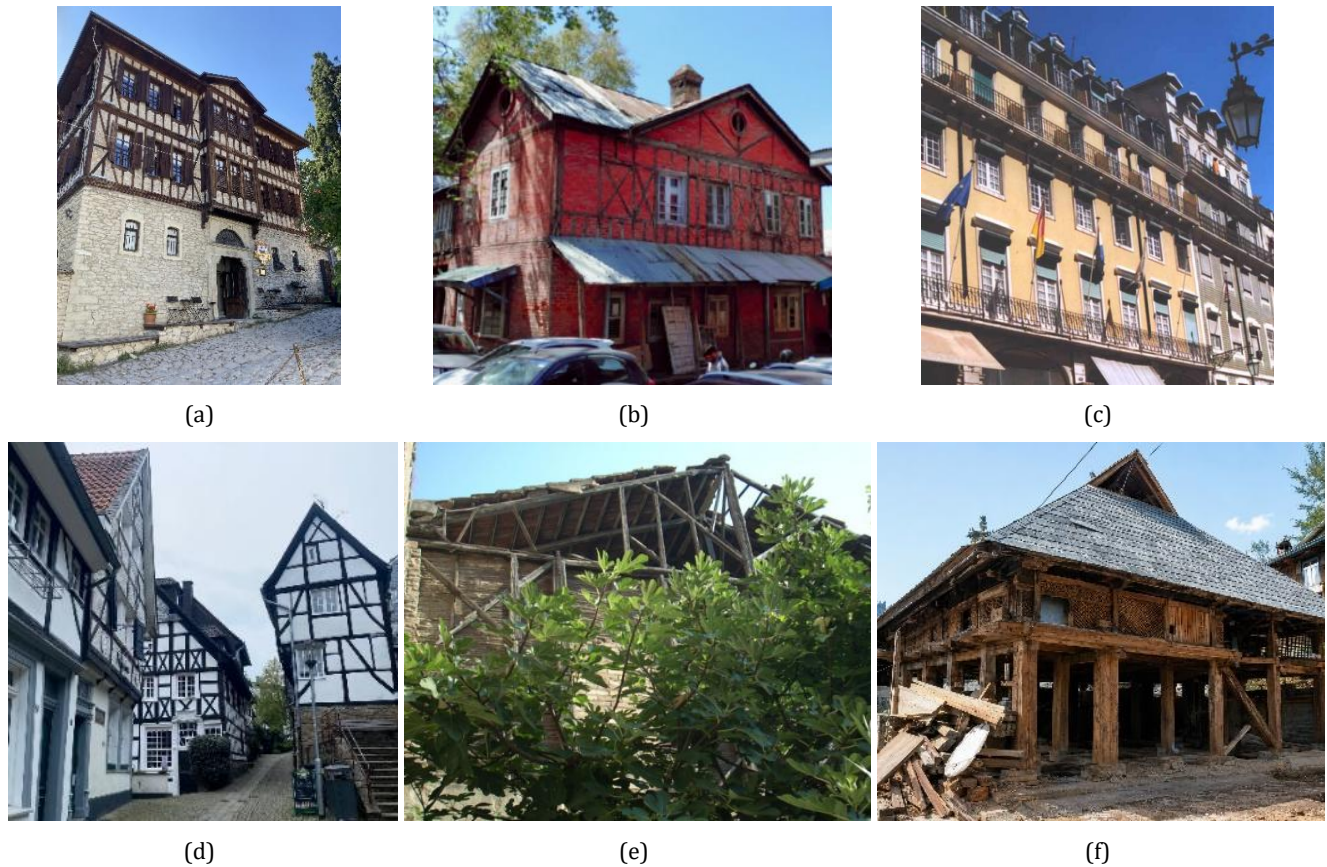


Fig. 2. Exterior views and characteristic structural details of traditional timber-framed systems: (a) Hımiş (author's archive), (b) Dhajji Dewari (Gani et al. 2024), (c) Pombalino (Cardoso et al. 2004), (d) Fachwerk (author's archive), (e) Casa Baraccata (Ruggieri and Zinno 2014), (f) Chuandou (Ren et al. 2018).

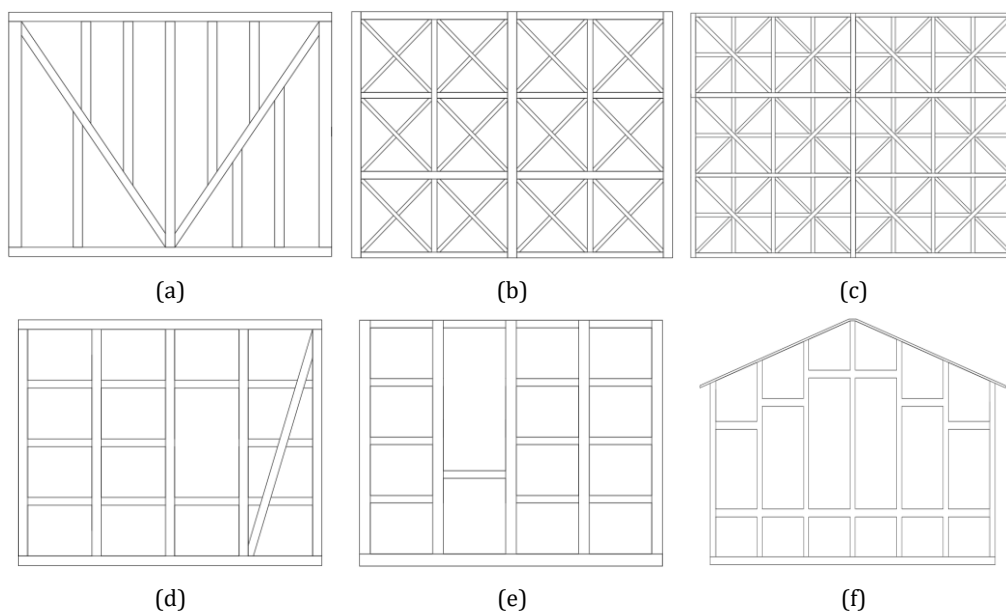


Fig. 3. Representative schematic drawings of traditional timber-framed systems developed based on the structural layouts reported in the literature: (a) Hımiş (Aktaş 2011); (b) Dhajji Dewari (Hicyilmaz et al. 2011); (c) Pombalino (Aktaş 2011); (d) Fachwerk (Perrin 1960); (e) Casa Baraccata (Niglio 2011); (f) Chuandou (Qu et al. 2020).

Table 1. Descriptive characteristics of the investigated traditional timber-framed systems.

System	Region	Primary Structural Feature	Load-Bearing System	Infill Type	Connection Type	Structural Organization	References
Hımsı	Türkiye	Timber frame with infill	Timber frame	Adobe, stone, or brick infill	Nailed timber joints and traditional carpentry details	Infill-frame interaction	Aktaş (2011), Ozdemir (2025)
Dhajji Dewari	Himalaya	Cellular timber frame system	Timber frame	Stone or brick infill with mud mortar	Traditional carpentry joints and nailed connections	Cellular timber frame with distributed infill panels	Hicyilmaz et al. (2011), Gani et al. (2024)
Pombalino	Portugal	Three-dimensional Gaiola Pombalina timber frame system	Hybrid (timber frame + masonry)	Masonry infill	Timber joints with diagonal bracing members	Three-dimensional timber frame embedded within masonry walls	Lopes et al. (2013), Meireles et al. (2012)
Fachwerk	Germany	Timber frame with posts, beams, and braces	Timber frame	Brick, adobe, or wattle-and-daub infill	Mortise-and-tenon and traditional timber joints	Timber frame with independent infill panels	Liu (2022), Gani et al. (2024)
Casa Baraccata	Italy	Timber-reinforced masonry system	Dual system (timber frame + masonry)	Stone or brick infill	Traditional carpentry joints and iron connectors	Integrated timber-masonry structural system	Ruggieri and Zinno (2014), Parrotta (2017)
Chuandou	China	Post-and-beam timber framing system	Timber frame	Timber panels, wattle-and-daub, or masonry infill	Penetrating mortise-and-tenon joints	Post-and-beam frame with infill panels	Qu et al. (2020), Li et al. (2025), Guo et al. (2021)

The literature review included peer-reviewed journal articles, experimental investigations, numerical studies, and other relevant academic publications addressing the seismic behavior of Hımsı, Dhajji Dewari, Pombalino, Fachwerk, Casa Baraccata, and Chuandou systems. The review process was conducted through searches performed in the Scopus, Web of Science, and Google Scholar databases. Searches were carried out using combinations of keywords including “timber-framed structures”, “seismic behavior”, “Hımsı”, “Dhajji Dewari”, “Pombalino”, “Fachwerk”, “Casa Baraccata”, and “Chuandou”. Studies providing technical data or observations related to energy dissipation mechanisms, connection behavior, damage development, ductility characteristics, and overall seismic performance were considered for inclusion. The information used in the comparative assessment was derived from experimental findings, numerical simulation results, and field observations reported in the selected studies.

The data evaluated in this study were obtained from investigations conducted at different scales, under different boundary conditions, and using different experimental methodologies. The reviewed studies included quasi-static tests, shaking-table experiments, numerical analyses, and post-earthquake field observations. Therefore, the comparisons were not intended to provide direct quantitative performance evaluations but rather to support a conceptual interpretation of behavioral tendencies and common structural mechanisms. Consequently, the findings should not be interpreted as normalized performance values obtained under standardized experimental conditions.

Particular emphasis was placed on the behavioral contribution of infill elements, connection regions, and secondary structural components during the early stages of energy dissipation under seismic loading. Within this context, the concept of the “first seismic de-

fense line” is considered a conceptual assessment framework describing the critical stage at which energy dissipation initiates and damage begins to develop in a controlled manner. Comparative analyses were performed through the integrated evaluation of evidence obtained from experimental studies (Poletti and Vasconcelos 2015; Li et al. 2025), numerical investigations (Ozdemir 2025), and field observations reported in the literature. This approach enables a conceptual assessment of the behavioral tendencies and underlying structural principles reported for the investigated systems. The findings contribute to the identification of common behavioral trends and a better understanding of the mechanisms governing structural response. The study presents a conceptual and literature-based framework, while experimental validation is recommended for future research.

3. Findings

3.1. First seismic defense line behavior

In the investigated systems, seismic demands are accommodated through the interaction between the timber structural frame, infill components, and connection regions. However, secondary components appear to play a more prominent role during the early stages of the energy dissipation process. These components, including infill materials, connection details, and auxiliary structural elements, collectively form a behavioral interaction level that may be interpreted as the “first seismic defense line”. This concept is considered a multi-stage behavioral model in which energy dissipation develops progressively through the interaction between infill components, connection regions, and the primary structural frame.

A comparative assessment of this behavior across the investigated systems is presented in Table 2. The table highlights commonalities and differences among traditional timber-framed systems developed in different regions with respect to energy dissipation mechanisms, damage development processes, and in-plane (IP) and out-of-plane (OOP) behavior. The information presented

is compiled from studies employing different experimental methodologies, scales, boundary conditions, and analytical approaches. Therefore, the reported values are not intended to represent standardized performance comparisons but are provided as a conceptual interpretation of the relative behavioral tendencies reported in the literature.

Table 2. Comparative seismic behavior characteristics of traditional timber-framed structural systems.

System	Early-Stage Interaction Component	Predominant Early-Stage Dissipation Component	Energy Dissipation Mechanism	Damage Mechanism	IP–OOP Behavior	References
Hımiş	Infill-frame interaction	Infill wall	Infill cracking, friction, and connection deformation	Infill damage and local connection deformation	Timber frame-supported wall behavior	Aktaş (2011), Özdemir (2025)
Dhajji Dewari	Cellular timber frame and infill panels	Infill panels and bracing members	Panel-frame interaction and connection deformation	Distributed infill damage	Enhanced in-plane and out-of-plane stability through cellular configuration	Hıcıymaz et al. (2011), Gani et al. (2024)
Pombalino	Timber frame–masonry interaction	Timber frame and bracing members	Timber frame deformation and connection behavior	Progressive infill and connection damage	Lateral stability provided by the three-dimensional gaiola system	Lopes et al. (2013), Meireles et al. (2012)
Fachwerk	Timber frame–infill interaction	Infill panels	Timber joint deformation and infill separation	Infill cracking and local frame deformation	Frame–infill interaction-controlled behavior	Liu (2022), Gani et al. (2024)
Casa Baraccata	Timber frame–masonry interaction	Timber frame and masonry infill	Interaction between timber and masonry components	Progressive infill damage and local connection deformation	Box-type behavior resulting from timber–masonry integration	Ruggieri and Zinno (2014), Parrotta (2017)
Chuandou	Mortise-and-tenon joint interaction	Mortise-and-tenon joints	Rotation and friction at timber joints	Joint loosening, local deformation, and infill damage	High deformation capacity governed by joint behavior	Qu et al. (2020), Li et al. (2025), Guo et al. 2021)

The findings indicate that the energy dissipation process is initiated through localized deformations and damage mechanisms developing within different structural components of the investigated systems. Cracking, frictional effects, and connection deformations dissipate a significant portion of the seismic energy, resulting in a gradual and controlled reduction in structural stiffness. However, the dominant components contributing to energy dissipation vary among the investigated systems. In Hımiş and Dhajji Dewari structures, energy dissipation is largely associated with the behavior of infill elements. In Pombalino and Casa Baraccata systems, the interaction between timber and masonry components plays a significant role, whereas in Chuandou structures a considerable portion of the energy dissipation process is associated with rotational and friction-based mechanisms developing within mortise-and-tenon joints. These differences suggest that, despite variations in construction techniques, the investigated systems may exhibit comparable behavioral tendencies under seismic loading.

3.2. Damage hierarchy

The comparative assessment indicates that damage development follows a generally consistent sequence across the investigated systems. Damage is commonly initiated within infill elements or connection regions during the early stages of seismic response. The subsequent distribution of damage varies according to system characteristics, extending into the primary structural

system in some cases while remaining concentrated within connection regions in others.

This progression enables the controlled dissipation of seismic energy throughout the structure. The tendency of infill components to act as the first damaged elements contributes to overall energy dissipation capacity while also assisting in the preservation of the primary load-resisting system. This mechanism appears to be particularly evident in Hımiş and Dhajji Dewari structures.

Nevertheless, the spatial distribution of damage differs among the systems. In Dhajji Dewari structures, damage tends to be distributed throughout the cellular wall configuration, whereas in Pombalino systems it is more likely to concentrate within specific regions. In Chuandou structures, damage is predominantly localized at connection interfaces.

One of the critical stages within the damage hierarchy is the influence of in-plane (IP) damage on out-of-plane (OOP) stability. Damage occurring within infill components may weaken wall integrity and increase the susceptibility to out-of-plane failure. This observation highlights the importance of maintaining connection continuity for preserving overall structural stability.

3.3. System-based behavioral summary

Although the investigated systems were developed in different geographical contexts, they exhibit several common behavioral principles under seismic loading. Nevertheless, the mechanisms through which these responses develop vary among the systems.

In Hımiş structures, cracking within infill components constitutes a primary energy dissipation mechanism, whereas the cellular configuration of Dhajji Dewari systems promotes a more uniform distribution of seismic demands. The Pombalino system exhibits a balanced structural response through the combined contribution of stiffness and ductility. In Fachwerk structures, structural behavior is largely governed by the interaction between the timber frame and infill panels, whereas friction-induced movements at connection interfaces play a dominant role in Chuandou systems. In Casa Baraccata structures, the interaction between timber framing and masonry components is the principal mechanism influencing structural response.

From a comparative perspective, the contribution of infill components to the energy dissipation process appears to be more pronounced in Hımiş and Dhajji Dewari systems, while frictional behavior within connection regions plays a more dominant role in Chuandou structures. The Pombalino system, by contrast, represents a relatively balanced interaction between stiffness and ductility.

3.4. IP–OOP behavior and stability

Among the investigated systems, in-plane (IP) and out-of-plane (OOP) responses constitute key parameters governing structural stability. While in-plane deformations reflect the energy dissipation capacity of the system, out-of-plane behavior plays a critical role in maintaining overall stability.

Damage occurring within infill components reduces structural stiffness and may consequently weaken out-of-plane stability. In particular, the loss of infill integrity represents a significant factor that may increase the risk of wall overturning and out-of-plane collapse. This effect appears to be more pronounced in systems where the behavioral contribution of infill components is greater, whereas it remains relatively limited in systems where connection behavior constitutes the dominant response mechanism. This observation suggests that the mechanisms governing structural stability vary according to the structural configuration of the system.

4. Discussion

The “first seismic defense line” concept proposed in this study should not be interpreted as a structural isolation mechanism. Under seismic loading, the timber structural frame, infill components, and connection regions act together in accordance with the principle of displacement compatibility. Within this context, the concept is considered a conceptual interpretative framework describing the dominant contribution of secondary components during the early stages of the energy dissipation process.

The findings indicate that traditional timber-framed systems developed in different geographical regions exhibit similar fundamental behavioral principles under seismic loading. In these systems, energy dissipation occurs through the interactive deformation behavior between the primary structural frame and the infill and

connection components. This observation suggests that the concept of the “first seismic defense line” may assist in explaining the comparable behavioral tendencies observed across different structural systems. Experimental and numerical studies reported in the literature also support this conceptual interpretation. In Chuandou structures, a significant portion of energy dissipation has been attributed to friction and micro-deformations occurring at connection interfaces (Li et al. 2025). It is well established that neglecting frictional effects in analytical models may lead to substantial inaccuracies (Follesa et al. 2011). Similarly, the damping capacity observed in Hımiş systems highlights the critical role of frame–infill interaction (Gani et al. 2021). Collectively, these findings indicate that energy dissipation is strongly associated with multi-component interaction mechanisms.

All investigated systems exhibit hybrid structural behavior, in which the interaction among infill elements, connection regions, and the structural frame constitutes one of the principal factors governing system performance. Consequently, the systems were evaluated according to the dominant behavioral tendencies reported in the literature rather than through rigid structural classifications. From the perspective of ductility, notable differences can be observed among the systems investigated. Pombalino structures exhibit a balanced structural response characterized by controlled deformation capacity (Poletti and Vasconcelos 2015), indicating that ductile behavior is substantially influenced by connection details. In this study, infill elements are considered not only as potential sources of structural weakness but also as active contributors to the energy dissipation process. In particular, the cellular configuration observed in Dhajji Dewari systems and the frame–infill interaction characteristic of Hımiş structures contribute to a more controlled distribution of damage throughout the system (Ozdemir 2025).

The interaction between in-plane (IP) and out-of-plane (OOP) behavior represents one of the key factors influencing the structural response of traditional timber-framed systems. Damage and loss of integrity within infill components may reduce wall stability and adversely affect out-of-plane behavior (Li et al. 2025). In this context, in-plane damage may be interpreted not only as a deterioration process but also as one of the early stages of the energy dissipation mechanism. Nevertheless, the available literature does not provide the same level of detail and coverage regarding out-of-plane behavior for all investigated systems. Therefore, the interpretations presented herein are based on a comparative assessment of behavioral tendencies reported in the literature rather than on standardized performance comparisons.

Topographical conditions also play an important role in structural response. Variations in column heights on sloping terrain may induce torsional effects and promote the concentration of damage in specific regions (Pan et al. 2023). Such conditions may disrupt the uniform distribution of seismic demands and increase the sensitivity of connection regions.

Material selection constitutes another important factor influencing energy dissipation behavior. While adobe infill may provide greater energy dissipation capacity, stone infill generally exhibits a stiffer but more brittle re-

sponse (Ozdemir 2025). This finding emphasizes the importance of considering infill materials as integral components of controlled damage mechanisms.

The integration of traditional systems into modern engineering practice demonstrates that these principles are compatible with contemporary design approaches (Qi et al. 2023). In this regard, the findings suggest that the interaction-based energy dissipation mechanisms observed between secondary components and the primary structural frame are broadly consistent with the capacity design philosophy defined in Eurocode 8. These mechanisms are also compatible with the principles of performance-based seismic design adopted in modern earthquake engineering practice (Eurocode 8 2004). Furthermore, experimental studies have shown that traditional timber structures may exhibit higher damping capacity than modern rigid systems due to frictional and embedment effects developing between structural components (Xie et al. 2019).

From a comparative perspective, the contribution of infill elements to the energy dissipation process appears to be more pronounced in Himiş and Dhajji Dewari systems, whereas frictional and micro-slip mechanisms at connection regions play a more dominant role in Chuandou structures. The Pombalino system represents a balanced interaction between stiffness and ductility, while the behavior of Fachwerk structures is governed primarily by frame–infill interaction and the response of Casa Baraccata systems is shaped by the integrated action of timber and masonry components.

This study extends the existing literature, which has largely focused on individual structural systems, by providing an integrated comparative framework encompassing multiple traditional systems from different regions. Whereas previous studies have generally examined specific structural typologies, the present research evaluates similarities and differences across systems simultaneously and suggests that energy dissipation and damage development mechanisms may be interpreted within a common behavioral framework. In this respect, the study contributes to the integration of fragmented knowledge into a comprehensive analytical perspective.

Overall, although the investigated systems differ in terms of structural configuration, they share a common performance-oriented approach characterized by controlled energy dissipation and ductile response. Interpreting this behavior through the concept of the “first seismic defense line” provides a new perspective for understanding traditional structural systems within the context of earthquake engineering.

5. Conclusions

This study presented a comparative analysis of traditional timber-framed structural systems developed in different regions with respect to their seismic behavior. The findings indicate that, despite differences in construction traditions, these systems exhibit comparable energy dissipation processes and damage distribution mechanisms under seismic loading.

The analyses suggest that energy dissipation is achieved through multi-component interaction involve-

ing infill elements, connection details, and the primary structural frame. The controlled concentration of damage within specific components enables these systems to develop ductile behavior rather than experiencing brittle failure. Seismic performance indicators and behavioral observations reported in the literature support the comparative interpretation of behavioral tendencies observed across different studies.

The present study further suggests that this common behavioral pattern may be interpreted through the concept of the “first seismic defense line”. This concept is proposed as a conceptual framework for understanding the behavioral contribution of secondary components during the energy dissipation process under seismic loading. The observation of similar behavioral tendencies in systems developed across different geographical regions suggests that traditional construction practices may share common underlying behavioral principles.

At the same time, notable differences were identified among the investigated systems. While infill components appear to play a more pronounced role in the energy dissipation process of Himiş and Dhajji Dewari structures, frictional and micro-slip mechanisms at connection regions are more influential in Chuandou systems. The Pombalino system represents a balanced interaction between stiffness and ductility, whereas the behavior of Fachwerk structures is governed primarily by frame–infill interaction and that of Casa Baraccata systems by the integrated action of timber and masonry components. These observations suggest that different structural strategies may lead to similar behavioral tendencies and energy dissipation mechanisms. The findings also highlight the critical role of connection details in the seismic response of traditional timber-framed systems. In particular, the stiffness, frictional characteristics, and deformation capacity of connection regions directly influence both energy dissipation processes and damage distribution patterns. Consequently, connections should be regarded not only as joining elements but also as active components governing overall structural performance.

The presence of similar behavioral principles in systems developed under different geographical conditions suggests that fundamental earthquake-resistant solutions emerged through empirical knowledge and accumulated construction experience. In this respect, traditional structural systems should be considered not only as elements of cultural heritage but also as valuable sources of engineering knowledge for performance-oriented design.

Overall, traditional timber-framed systems offer significant seismic advantages due to their low mass, high deformation capacity, and effective energy dissipation characteristics. These attributes provide valuable principles that may inform contemporary structural design. Integrating the behavioral principles observed in traditional systems with modern engineering approaches may contribute to the development of safer and more performance-oriented structures. Nevertheless, because the studies reviewed in this research differ in terms of experimental methodologies, scales, boundary conditions, and analytical approaches, the findings should not be interpreted as standardized quantitative performance comparisons. Rather, they should be regarded as

a conceptual interpretation of behavioral tendencies and common structural mechanisms reported in the literature.

An additional advantage of traditional timber-framed systems is that damage is often concentrated within specific components that can be repaired or replaced. However, the long-term performance of timber structures depends on addressing durability-related issues such as biological deterioration, moisture exposure, and environmental weathering. Therefore, maintenance, durability, and repair strategies should be considered alongside seismic performance when evaluating the long-term sustainability of these systems.

Future research should focus on examining the proposed “first seismic defense line” concept through experimental validation and advanced numerical modeling. Furthermore, investigating the integration of traditional structural principles with modern materials and hybrid structural systems may provide new directions for earthquake-resistant design.

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Data Availability

The datasets generated and/or analyzed during the current study are not publicly available but are available from the corresponding author upon reasonable request.

AI Assistance

No AI-based tools were used in the preparation of this manuscript.

Author Contributions

The author declares sole responsibility for all aspects of the study, including conceptualization, methodology, formal analysis, investigation, data curation, visualization, writing of the original draft, and writing, review, and editing of the manuscript.

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Research Article

Thermal regulation and mechanical performance of cement mortars for building thermal energy storage applications

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ABSTRACT

This study investigated the effects of microencapsulated phase change material (mPCM) with a phase transition temperature of approximately 25 °C on the physical, mechanical, thermal, and microstructural properties of cement mortars. mPCM was incorporated into mortar mixtures at 2.5%, 5%, and 7.5% of the cement weight. The results showed that mPCM decreased consistency and unit weight while increasing porosity and water absorption. The 28-day compressive strength decreased from 49.9 MPa for the reference mortar to 46.7, 44.2, and 38.8 MPa for mortars containing 2.5%, 5%, and 7.5% mPCM, respectively. The 28-day flexural strength decreased by 9.7-26.6%. SEM analyses revealed that void content and microcapsule-matrix interface discontinuities increased with increasing mPCM content. During heating-cooling cycles (0–40 °C), the maximum temperature differences between the reference mortar and the mPCM2.5, mPCM5, and mPCM7.5 mixtures were 1.1, 1.2, and 2.1 °C during heating, and 1.0, 2.0, and 2.8 °C during cooling, respectively. Increasing mPCM content enhanced the thermal buffering capacity, delayed heat transfer, and reduced internal temperature fluctuations. The findings demonstrate that mPCM-modified cement mortars have potential for thermal energy storage and indoor temperature regulation in building applications. The highest thermal buffering effect was achieved with 7.5% mPCM, but this mixture also showed the greatest reduction in mechanical strength. Overall, the study provides valuable information on the combined thermal and mechanical performance of mPCM-modified cement mortars for building thermal energy storage applications.

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1. Introduction

The dramatic increase in energy demand and the resulting intensity crises continue to be a focal point for energy in the building sector. In particular, the share of energy spent on climate control (heating-cooling) in buildings is quite critical. Therefore, minimizing indoor temperature fluctuations and reducing heat loads in sustainable architectural design has led to the dissemination of regional research (Sun et al. 2025; Yu et al. 2026). In this context, intensive studies are being conducted on passive energy storage solutions to both increase ther-

mal comfort and save energy (Cui et al. 2017; Ren et al. 2021).

Limiting thermal fluctuations in the indoor environment through passive systems is one of the most effective ways to increase efficient energy use. The methods used for thermal energy storage (TES) purpose are examined in three basic categories: sensible, latent, and thermochemical storage (Cunha et al. 2016; Jin et al. 2017). Among these methods, latent heat storage is particularly advantageous in the building sector due to its ability to absorb a large amount of energy during phase change. Phase Change Materials (PCMs) balance

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heating and cooling loads by delaying temperature changes with their latent heat capacity during melting and freezing cycles (Alakara et al. 2025a; Alkan et al. 2023).

While the integration of PCMs into building materials offers great potential for thermal management, it also introduces technical limitations. In particular, the leakage problem seen when PCM is directly incorporated into the mixture can negatively affect both the mechanical resistance and durability of the material (Li et al. 2026). To eliminate these risks, various application techniques have been developed, such as direct addition (Li et al. 2013), form stabilization (Fang et al. 2014), impregnation (Zhang et al. 2021a), and encapsulation (Xu et al. 2023). Microencapsulation is a prominent method in cement-based systems because it prevents PCM from leaking into the external environment during phase transition (Essid et al. 2019; Pilehvar et al. 2017).

The use of microencapsulated PCMs (mPCMs) in cement-based mortar and concrete systems contributes to indoor temperature stability by increasing the TES capacity of building components. In particular, when selecting mPCMs, it is critical to maximize energy storage efficiency so that the phase transition temperature is close to the ideal indoor comfort range (20–25 °C) (Zhang et al. 2025).

Although numerous studies in literature have investigated the thermal and mechanical performance of cement-based materials containing mPCM, studies that simultaneously evaluate the effects of different mPCM ratios on physical, mechanical, thermal, and microstructural properties are limited. Furthermore, studies that examine the effects of different usage ratios of a commercial mPCM with a phase change temperature of approximately 25 °C on the physical, mechanical, and thermal performance of cement mortars, along with microstructural observations, are also limited. Therefore, in this study, mortars containing mPCM at 0%, 2.5%, 5%, and 7.5% of cement weight were produced, and the effects of mPCM addition on physical, mechanical, thermal, and microstructural properties were investigated. For this purpose, spread diameter, unit weight, porosity, water absorption, flexural strength, and compressive strength tests were performed. Additionally, SEM analyses were conducted to examine the changes in mortar microstructure caused by mPCM addition. To evaluate the effects of different mPCM ratios on thermal behavior, the internal temperature changes of all mixtures were monitored in real-time in a heating-cooling chamber. Using the obtained results, the effects of mPCM content on physical, mechanical, thermal, and microstructural performance were evaluated comparatively.

2. Materials and Method

2.1. Materials

Cement mortars were produced using CEM I 42.5 R type Portland cement as the primary binder, in accordance with TS EN 197-1 (2012). Mixing water was supplied from the municipal water network. Standardized CEN sand was preferred as fine aggregate. The physical

and chemical characteristics of the Portland cement are summarized in Table 1.

Table 1. Chemical compositions and some physical properties of PC.

Chemical composition	(%)
SiO ₂	21.23
Al ₂ O ₃	5.54
Fe ₂ O ₃	3.79
CaO	62.76
MgO	1.04
Na ₂ O	0.18
K ₂ O	0.25
SO ₃	3.09
Physical properties	
Specific gravity (unitless)	3.15
Blaine fineness (cm ² /g)	3445
Loss on ignition	1.19

The mPCM used in this study is a commercial product produced by microencapsulating a paraffin-based core material with a phase transition temperature of approximately 25 °C within a polymeric shell (melamine formaldehyde). The phase change behavior and heat storage properties of mPCM were determined by differential scanning calorimetry (DSC) analyses, and the results are given in Table 2. The average particle size of the mPCM is 90 µm. The mPCM used in this study is shown in Fig. 1

2.2. Preparation and mixing process of cement mortars

Mortar mixtures were prepared in accordance with Turkish legislation (TS EN 196-1 2016). To directly determine the effects of mPCM addition on the physical, mechanical, thermal, and microstructural properties of cement mortars, the water/cement ratio was kept constant in all mixtures, and no additional chemical additives were used. In the mixing process, PC and water were mixed first. Then, sand was added and mixing was continued. mPCM was added in the final stage to minimize the risk of damage to the microcapsules by the mixer. After the addition of the microcapsules, the mixture was thoroughly mixed to ensure homogeneity. The prepared fresh mortars were poured into molds measuring 40 × 40 × 160 mm for mechanical tests and 100 × 100 × 20 mm for thermal measurements. Thermocouples were placed in the thermal test samples. After 24 hours, the samples were removed from the molds and cured in water at 20 ± 2 °C until testing after 3, 7, and 28 days. Microcapsules were added to cement mortars at 2.5%, 5%, and 7.5% by weight of cement. These ratios were selected considering the additive levels commonly used in cement-based composites in literature (Cao et al. 2019; Djamai et al. 2019; Ren et al. 2021).

Table 2. Thermal performance characteristics of mPCM after DSC analysis.

	Initial Temperature (°C)	Final Temperature (°C)	Enthalpy (J/g)
Melting point	25.8	36.1	136.8
Freezing point	25.3	16.01	138.4

**Fig. 1.** Photograph of the mPCM used in this study.

Thus, the effects of increasing mPCM content on the physical, mechanical, thermal, and microstructural properties of the mortars could be systematically evaluated. The mixture compositions are given in Table 3. The $100 \times 100 \times 20$ mm cement mortar samples used in thermal experiments were prepared to represent the heating and cooling effects that mPCM-modified cement-based mate-

rials may be subjected to in building applications under controlled laboratory conditions, and to comparatively evaluate the thermal performance of different mixtures. Sample dimensions and experimental procedure were determined by considering similar studies in literature (Alakara et al. 2025a). The mixing, compaction, and molding process steps of the cement mortars are shown in Fig. 2.

2.3. Characterization of mortars

The consistency of the freshly prepared mortar was determined according to the TS EN 1015-3 (2000) standard. Also, some of the physical properties, like density, porosity, and water absorption, were evaluated following the procedures specified in ASTM C642-21 (2021). Experiments to determine the fresh mortar and its physical properties were performed in triplicate for each mixture, and the results were reported as average values. Mechanical performance was evaluated on prismatic specimens with dimensions of $40 \times 40 \times 160$ mm. Flexural strength tests were performed on three prismatic specimens for each mixture. The fractured fragments obtained from the flexural tests were used in compressive strength tests, and the compressive strength results were reported as the average of six measurements for each mixture. Flexural and compressive strength tests were determined in accordance with TS EN 196-1 (2016) by applying loading rates of 50 N/s and 2.4 kN/s, respectively. The fresh and hardened state properties of cement mortars were measured using the experimental equipment shown in Fig. 3.

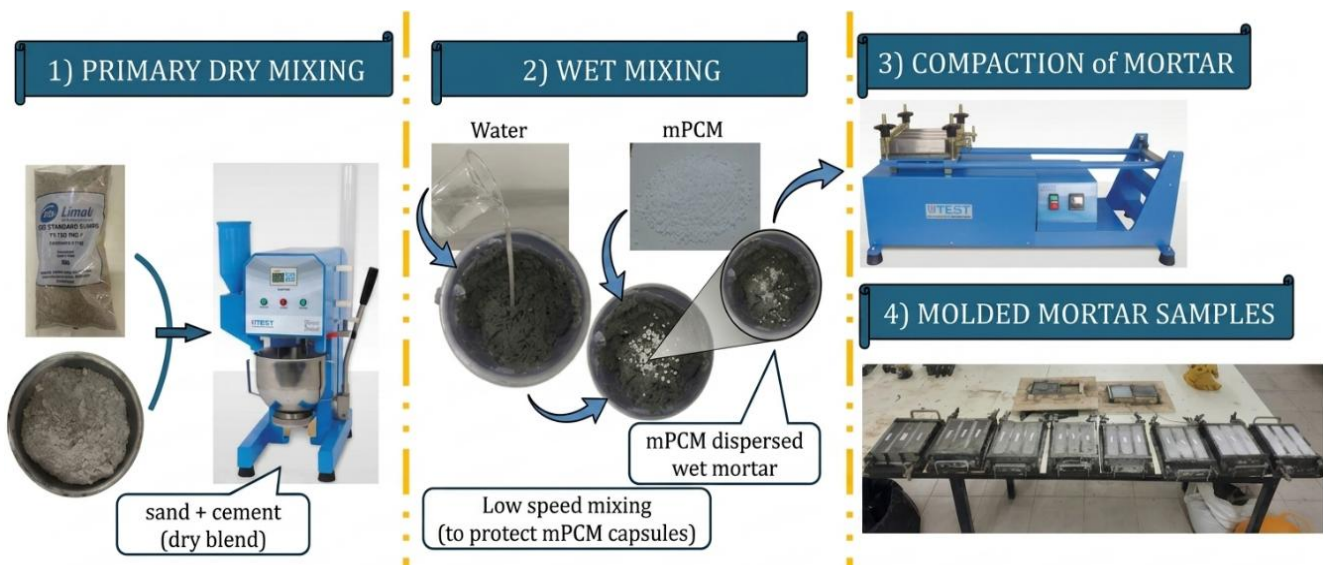
**Fig. 2.** Preparation stages, mixing sequence, and molding process of mPCM-modified cement mortar specimens.

Table 3. Quantities of materials for mortar mixtures

Sample Codes	Cement (g)	Sand (g)	Water (g)	mPCM (g)	mPCM ratio (wt.% of cement)
Ref	450	1350	225	0.0	0.0
mPCM2.5	450	1350	225	11.3	2.5
mPCM5	450	1350	225	22.5	5.0
mPCM7.5	450	1350	225	33.8	7.5



Fig. 3. Experimental test setups used for evaluating mortar properties:
(a) Flow table apparatus; (b) Archimedes balance; (c) Flexural/compressive strength testing machine.

In thermal experiments, one sample measuring $100 \times 100 \times 20$ mm was used for each mixture. For internal temperature measurements, a thermocouple was placed at the geometric center of the sample 10 mm deep from the top surface. The experiments were carried out in a programmable laboratory-type heating-cooling chamber where temperature and holding times could be controlled. The samples were tested without any insulation, and all surfaces were left exposed to environmental conditions. Heating-cooling cycles were applied twice in the temperature range of 0 – 40 °C. To ensure that the samples reached the target temperature and thermal equilibrium was achieved, they were held at the maximum and minimum temperatures for 6 hours. Temperature data were recorded every minute with an accuracy of ± 0.1 °C using a four-channel Testo 176 T4 data logger. Fig. 4 shows an image of the experimental setup and the placement of the thermocouples within the mold.

3. Results and Discussion

3.1. Consistency

The consistency of mortar mixtures was evaluated using the flow table test, and the results are presented in Fig. 5. The spread diameter of the reference mixture was determined to be 155 mm. With the addition of mPCM, a gradual decrease in flow diameter occurred, indicating a

reduction in the workability of the mortars. Compared to the reference mixture, the spread diameters of mixtures containing 2.5%, 5%, and 7.5% mPCM decreased by 3.2%, 9.7%, and 19.4%, respectively. The high R^2 value (0.9524) obtained from the regression analysis shows a strong correlation between mPCM content and spread diameter. This indicates that the addition of mPCM has a significant effect on the fresh-state properties of the mortars.

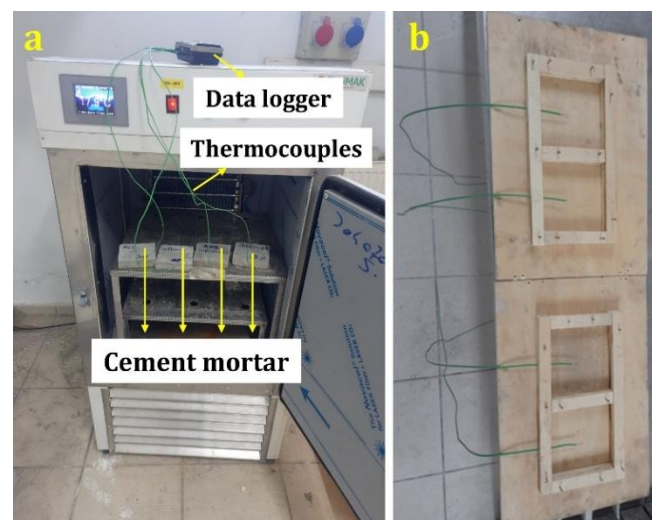


Fig. 4. Thermal performance test setup for cement mortars: (a) Climatic chamber for heating-cooling cycles; (b) Location of thermocouples inside mortar samples.

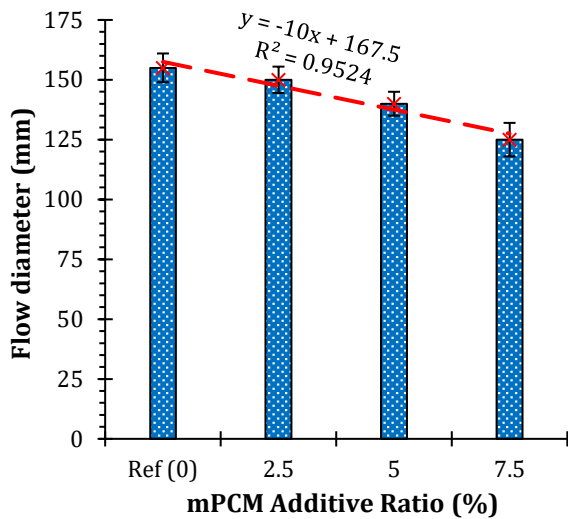


Fig. 5. Flow diameters of fresh mortar.

The workability loss observed with the addition of mPCM can be explained by the fact that microcapsules have a high specific surface area, increasing the water demand in the mixture and limiting the flowability of the cement paste (Cunha et al. 2015). Furthermore, the inclusion of mPCM as an additional fine material in the mixture increases the total solid surface area in the system, leading to a higher water demand. As a result, workability gradually decreased in mixtures prepared with a constant water/cement ratio. The more pronounced

spreading losses observed as the mPCM ratio increases indicate that this effect is dosage dependent. In addition, Zhang et al. (2021b), reported that the workability loss observed in cement mortars may also be due to electrostatic interactions between cement particles and microcapsules.

Similar results have been reported in the literature. Cunha et al. (2015) reported that the addition of 20% mPCM increased the water requirement of the mixtures, and this effect became more pronounced as the mPCM ratio increased. Similarly, Cunha et al. (2013) and Alkan et al. (2023) reported that the use of mPCM reduced the workability of cement-based composites. The results obtained in this study are consistent with these studies and show that the addition of mPCM negatively affects the fresh mortar properties. In particular, the approximately 20% spread loss observed in the mixture containing 7.5% mPCM reveals that workability can become a critical design parameter at high mPCM contents.

3.2. Unit weight, porosity, and water absorption

The unit weight, porosity, and water absorption results of cement mortars are presented in Fig. 6. Examination of the results shows that as the mPCM additive ratio increases, the unit weight decreases, while porosity and water absorption values increase. The high R^2 values (0.9083–0.9899) obtained from the regression analyses indicate strong correlations between mPCM content and the physical properties examined.

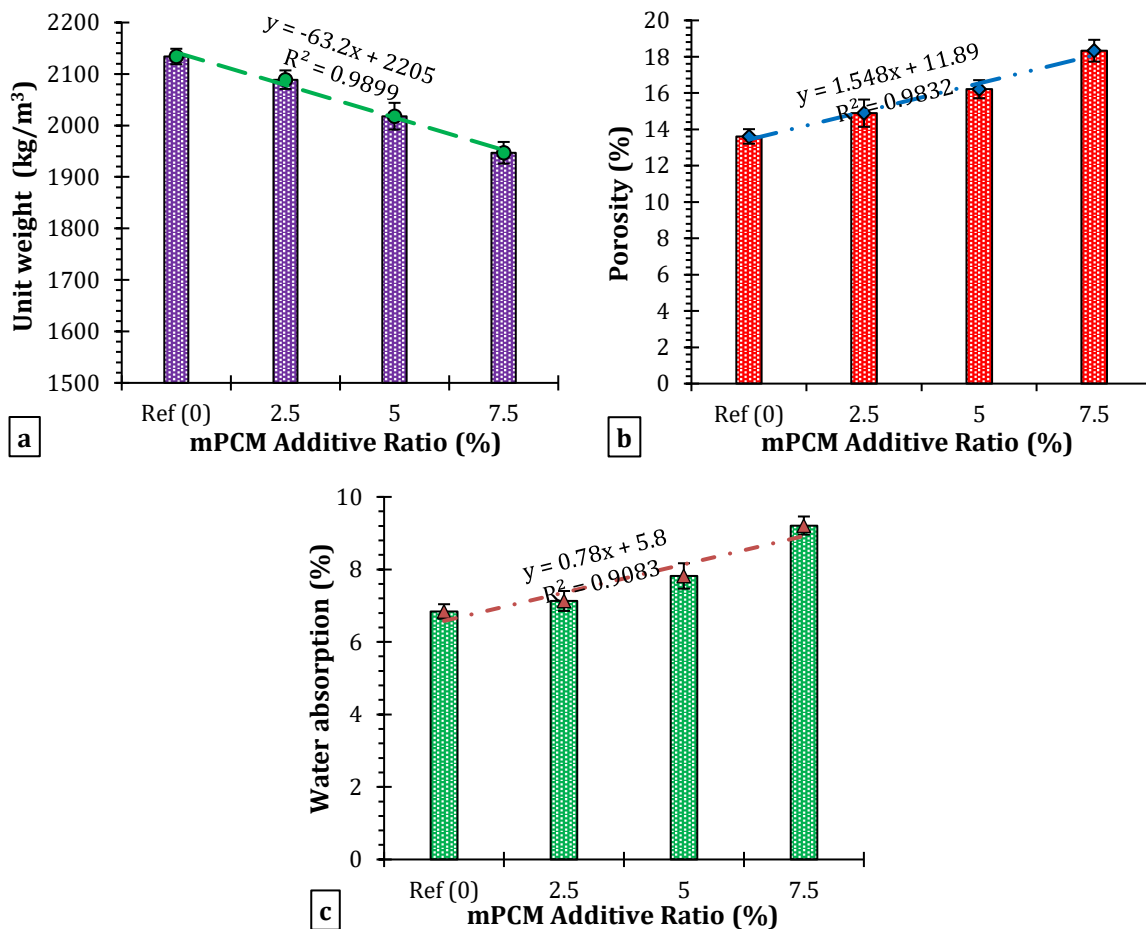


Fig. 6. (a) Unit weight; (b) Porosity; (c) Water absorption values of cement mortars.

The unit weights of the cement mortars ranged from 1947–2134 kg/m³. The unit weights of mortars containing 2.5%, 5%, and 7.5% mPCM additives decreased by 2.1%, 5.4%, and 8.8%, respectively, compared to the mortar with the Ref code. This decrease in unit weight is due to mPCM having a lower density compared to cement. The microcapsules added to the mixture reduced the amount of dense solid phase in the mortar, resulting in lower unit weight values. Similarly, Cao et al. (2017) and Cunha et al. (2015), reported that the addition of mPCM reduced the density of cement-based composites. The results obtained in this study are consistent with these studies and show that the use of mPCM contributes to the lightening of mortars.

Increasing the mPCM additive ratio resulted in significant increases in the porosity and water absorption values of the mortars. Porosity values ranged from 13.61–18.33%, and water absorption values ranged from 6.84–9.21%. Compared to the reference mixture, mixtures containing 7.5% mPCM showed approximately a 35% increase in porosity and water absorption values. This can be attributed to the fact that mPCM, due to its fine-grained structure and high specific surface area, increases the water demand of the mixture. In mixtures prepared with a constant water/cement ratio, the increased mPCM content resulted in insufficient water supply and the formation of additional voids within the matrix (Alkan et al. 2023; Illampas et al. 2021). Furthermore, it is thought that discontinuities formed in the interface regions between the

polymeric shell structure of the microcapsules and the cement matrix also contributed to the increase in porosity. This assessment is supported by the voids observed in SEM analyses and the discontinuities at the microcapsule-matrix interface.

In the literature, it is widely reported that the addition of mPCM negatively affects the porosity and water absorption properties of cement-based composites. For example, Hunger et al. (2009) reported that the porosity of concrete increased from 15.2% to approximately 20.5% with the addition of only 3% mPCM. Similar trends were observed in studies by Alakara et al. (2025a) and Cao et al. (2017). The results obtained in this study are consistent with these studies and show that the decrease in density and the increase in void volume with increasing mPCM content directly affect porosity and water absorption behavior. The significant increases observed, especially in the mixture with 7.5% mPCM additive reveal that the changes in physical properties occurring at high mPCM contents should not be ignored.

3.3. Flexural strength

The flexural strengths of cement mortars were determined on three samples for each mixture, and the results are presented in Fig. 7. Fig. 7(a) shows the flexural strength results, Fig. 7(b) shows the strength losses compared to the reference mixture, and Fig. 7(c) shows the regression relationships between mPCM content and flexural strength.

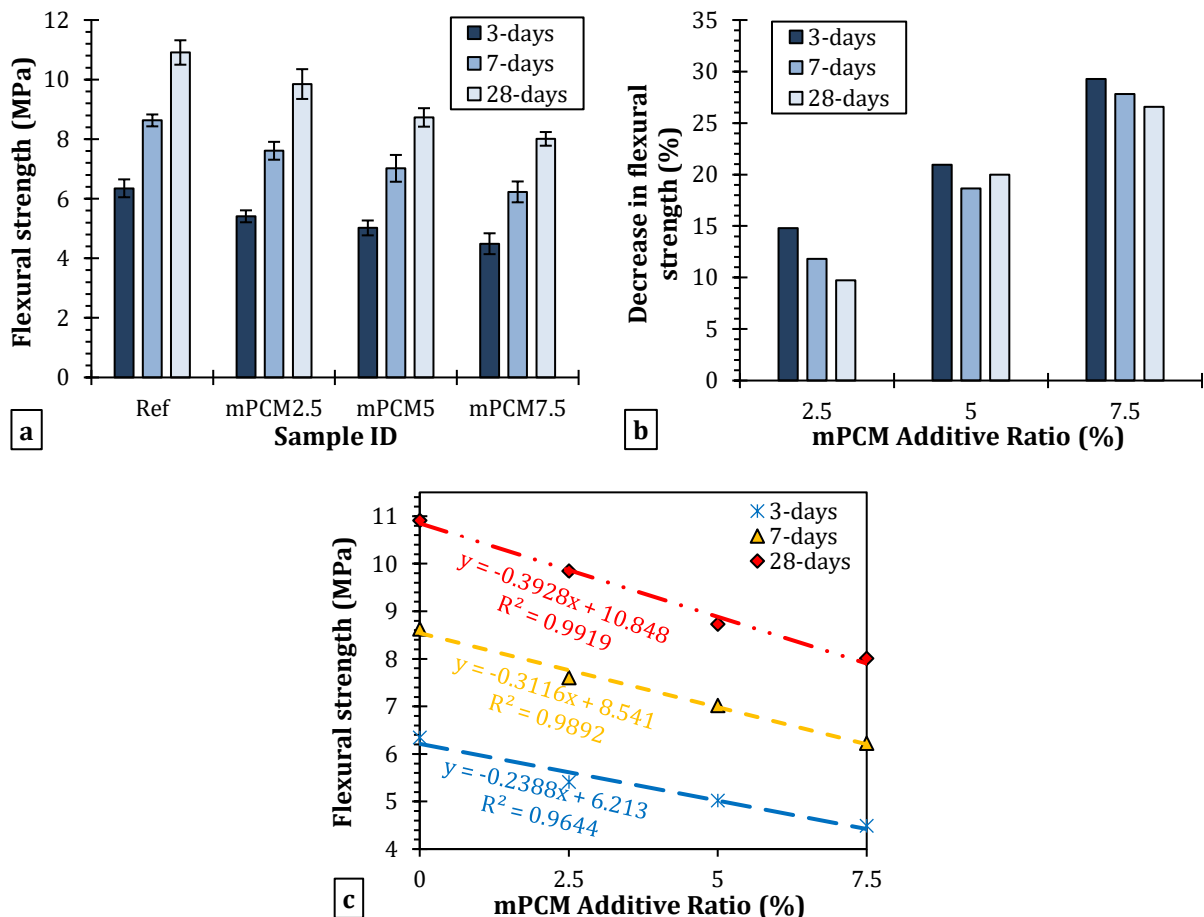


Fig. 7. (a) Flexural strength results; (b) Percentage reduction in flexural strength; (c) Regression analysis of flexural strength.

According to the results obtained, the highest flexural strength was obtained in the reference mixture at all curing times. Flexural strengths increased with increasing curing time in all mixtures depending on the progression of hydration. However, the addition of mPCM negatively affected flexural strength, and the strength loss became more pronounced as the mPCM ratio increased. Specifically, in the 28-day samples, while the flexural strength of the reference mixture was 10.9 MPa, the flexural strengths of the mixtures containing 2.5%, 5%, and 7.5% mPCM were determined as 9.9, 8.7, and 8.0 MPa, respectively. Accordingly, the percentage decrease in flexural strength compared to the reference mixture was calculated as 9.7%, 20.0%, and 26.6%, respectively. Similarly, strength losses occurred in the 3 and 7-day samples due to the addition of mPCM. The regression analyses given in Fig. 6(c) show that there are strong linear relationships between mPCM content and flexural strength. The high R^2 values obtained (0.9644–0.9919) reveal that the decrease in flexural strength is closely related to the mPCM ratio.

The observed flexural strength losses due to mPCM addition can be attributed to changes in the physical properties of the mortars. The results presented in the previous section showed that as the mPCM content increased, the unit weight decreased, while porosity and water absorption values increased. The increased void volume leads to stress concentrations under load, facilitating crack formation and crack propagation. In addition, the lower rigidity of the polymeric shell structure of microcapsules compared to cement hydration products reduces the efficiency of load transfer, negatively affecting flexural strength. Furthermore, the relatively weak structure of the interface zones (ITZ) formed between the microcapsules and the cement matrix may also contribute to strength losses. The voids and interface discontinuities around the microcapsules observed in SEM analyses support this assessment. The fact that cracks primarily start and propagate in these weak zones under load has led to more pronounced flexural strength losses as the mPCM content increases (Alakara et al. 2025b; Cui et al. 2015).

Similar results obtained in flexural strength have also been reported in the literature. Cunha et al. (2015), reported that the addition of 20% mPCM caused a decrease in flexural strength of more than 23%. Studies conducted by Aguayo et al. (2016) and Ren et al. (2021) also found that the addition of mPCM reduced flexural strength. The results obtained in this study are consistent with the aforementioned research and show that although the addition of mPCM contributes to thermal energy storage performance, careful evaluation of mechanical performance is required at high usage rates.

3.4. Compressive strength

Fig. 8(a) shows the compressive strength results of cement mortars, Fig. 8(b) shows the strength losses compared to the reference mix, and Fig. 8(c) shows the regression relationships between mPCM content and compressive strength.

The compressive strength results showed a similar trend to the flexural strength results. The highest compressive strength was obtained in the reference mixture

at all curing times, and the strength values increased in all mixtures with increasing curing time. However, the addition of mPCM negatively affected compressive strength, and the strength loss became more pronounced with increasing mPCM ratio. While the compressive strength of the reference mortar in the 28-day samples was 49.9 MPa, these values were determined as 46.7 MPa, 44.2 MPa, and 38.8 MPa in the mixtures containing 2.5%, 5%, and 7.5% mPCM, respectively. Accordingly, the reduction rates in 28-day compressive strength were calculated as 6.5%, 11.4%, and 22.2%, respectively. Similarly, significant strength losses occurred at early ages as well, and the highest reduction was observed in the 3-day mPCM7.5 mixture with 41.2%. The regression analyses given in Fig. 8(c) show that there are strong linear relationships between mPCM content and compressive strength. The high R^2 values obtained (0.9720–0.9979) reveal that the decrease in compressive strength is closely related to mPCM content.

The decrease in compressive strength can be attributed to the changes in the physical and microstructural properties of the mortars caused by the addition of mPCM. The results presented in previous sections showed that with increasing mPCM content, the unit weight decreased, while porosity and water absorption values increased. In particular, in the mPCM7.5 mixture, an increase of approximately 35% in porosity occurred, while a decrease of approximately 22% in 28-day compressive strength was observed. This indicates that the increase in the void volume within the matrix negatively affects compressive strength by reducing the effective load-bearing cross-sectional area. Furthermore, the fact that mPCM behaves as a low-rigidity phase within the cement matrix and exhibits lower mechanical strength compared to cement hydration products also leads to the less homogeneous distribution of stresses under load. The voids around the microcapsules and matrix discontinuities observed in SEM analyses support the microstructural origin of the decrease in compressive strength, indicating that the compact structure of the mortars is disrupted. The higher strength losses observed particularly at earlier ages indicate that high mPCM contents affect matrix integrity and strength development more significantly.

Similar results have been reported in the literature. Ren et al. (2021), reported approximately 27% and 35% compressive strength loss for 5% and 10% mPCM additions, respectively. Eddhahak-Ouni et al. (2014) reported approximately 32% strength reduction with 5% mPCM use. Furthermore, studies by Pilehvar et al. (2019) and Yeon and Kim (2018) also reported that mPCM addition negatively affects compressive strength. The results obtained in this study reveal that the decrease in matrix compactness and the increase in void volume at high mPCM contents significantly affect compressive strength.

3.5. Results of cyclic heating-cooling tests on mortar samples

To evaluate the temperature behavior of mortars coded Ref., mPCM2.5, mPCM5, and mPCM7.5, they were subjected to heating-cooling cycles in a temperature

range of 0 °C to 40 °C. The samples were held for 6 hours at the lowest and highest temperatures determined during the cycle. Fig. 9 shows the graph of the internal tem-

perature changes of cement mortar samples with dimensions of 100 mm × 100 mm × 20 mm during two consecutive heating-cooling cycles.

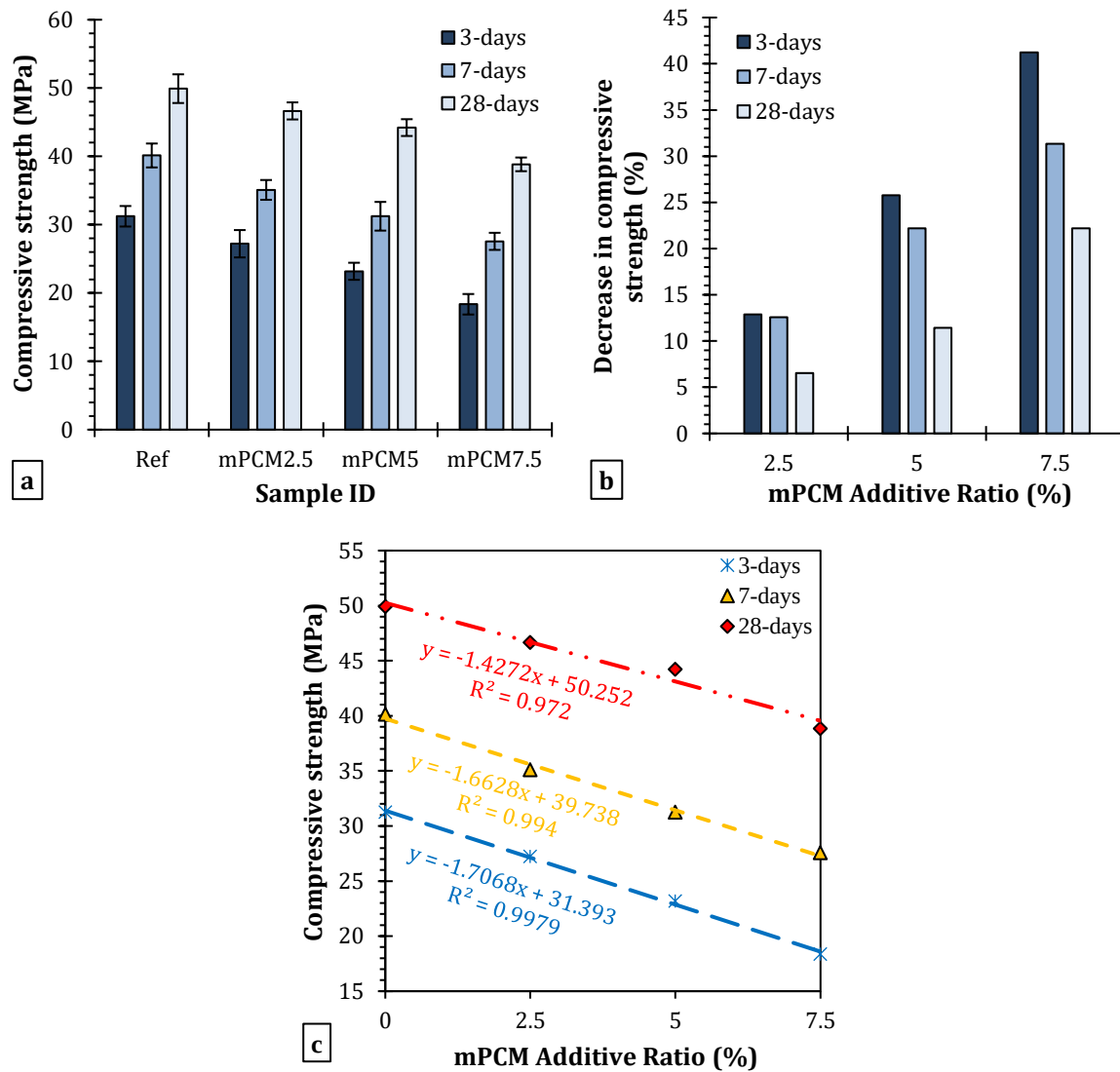


Fig. 8. (a) Compressive strength results; (b) Percentage reduction in compressive strength; (c) Regression analysis of compressive strength.

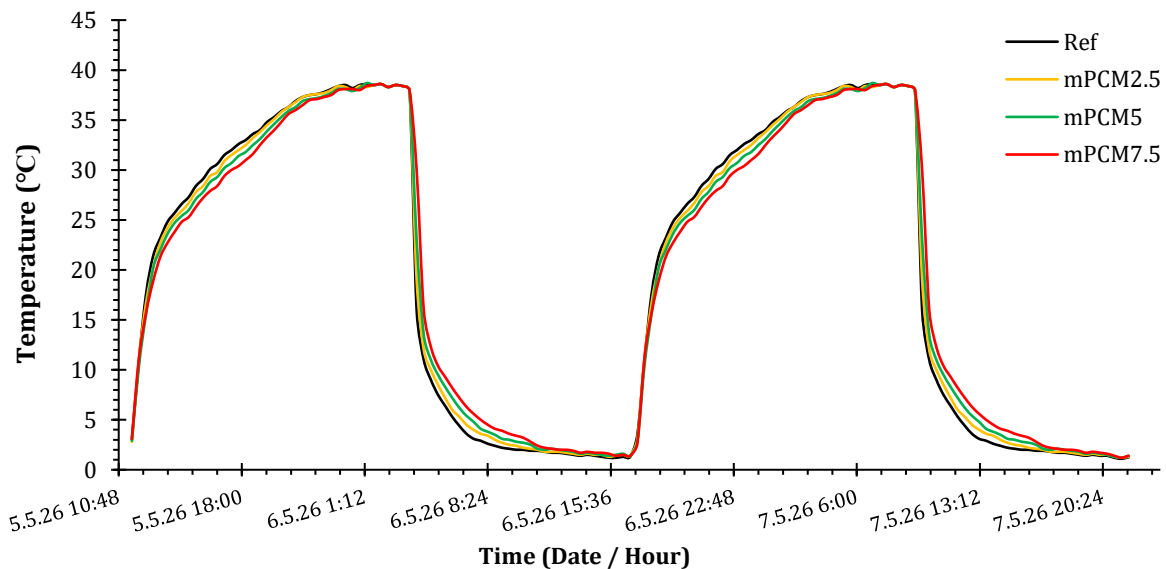


Fig. 9. Internal temperature measurements of mortars after the heating-cooling cycle.

All samples showed similar temperature change trends throughout the cycles. However, it was observed that mPCM-modified mortars heated and cooled more slowly compared to the reference sample. While the reference sample showed a faster temperature increase during the heating phase, the temperature increase in mPCM-modified samples occurred more slowly. This can be explained by mPCM storing some of the heat energy as latent heat during the phase change. Thus, the rate of temperature increase decreases, and a more stable temperature development occurs within the mortar. During the cooling phase, it was observed that mPCM-modified mortars maintained higher temperatures for a longer period compared to the reference sample. This behavior is due to the gradual release of the energy stored during the phase change in the heating process. In conclusion, the addition of mPCM reduced the rate of temperature decrease and increased the thermal inertia of the mortars.

When different mPCM ratios are compared, it is observed that the thermal buffering effect becomes more pronounced with increasing additive ratio. The maximum temperature differences between the reference sample and the mPCM2.5, mPCM5, and mPCM7.5 mixtures during the heating phase were determined to be 1.1, 1.2, and 2.1 °C, respectively. Similarly, the maximum temperature differences during the cooling phase were measured as 1.0, 2.0, and 2.8 °C, respectively. The results show that increasing the mPCM ratio increases the heat storage capacity of the mortars and delays temperature changes more effectively. The higher temperature differ-

ences observed in the mPCM7.5 mixture, in particular, reveal that this mixture exhibits the most significant thermal regulation performance throughout the cycle.

Although the observed internal temperature differences appear limited in absolute terms, temperature buffering effects of approximately 1–3 °C are considered significant for passive thermal energy storage applications (Ren et al. 2021; Wang et al. 2022; Xiong et al. 2021). However, the thermal benefits provided by the addition of mPCM should be evaluated together with the changes in physical and mechanical properties. In this study, while it was observed that internal temperature fluctuations decreased with increasing mPCM content, certain levels of decreases occurred in physical and mechanical properties. Therefore, the practical applicability of mPCM-modified mortars should be evaluated depending on the intended use and performance requirements. While thermal improvements obtained in non-load-bearing structural elements where thermal comfort and energy efficiency are prioritized may provide advantages, the effects of mPCM use in applications requiring high mechanical performance should be evaluated more carefully.

3.6. Microstructural characterization

SEM analysis was performed to evaluate the effects of mPCM addition on the microstructure of cement mortars. Microstructural investigations were carried out on mortars coded Ref and mPCM7.5, which has the highest mPCM content. SEM images are presented in Fig. 10.

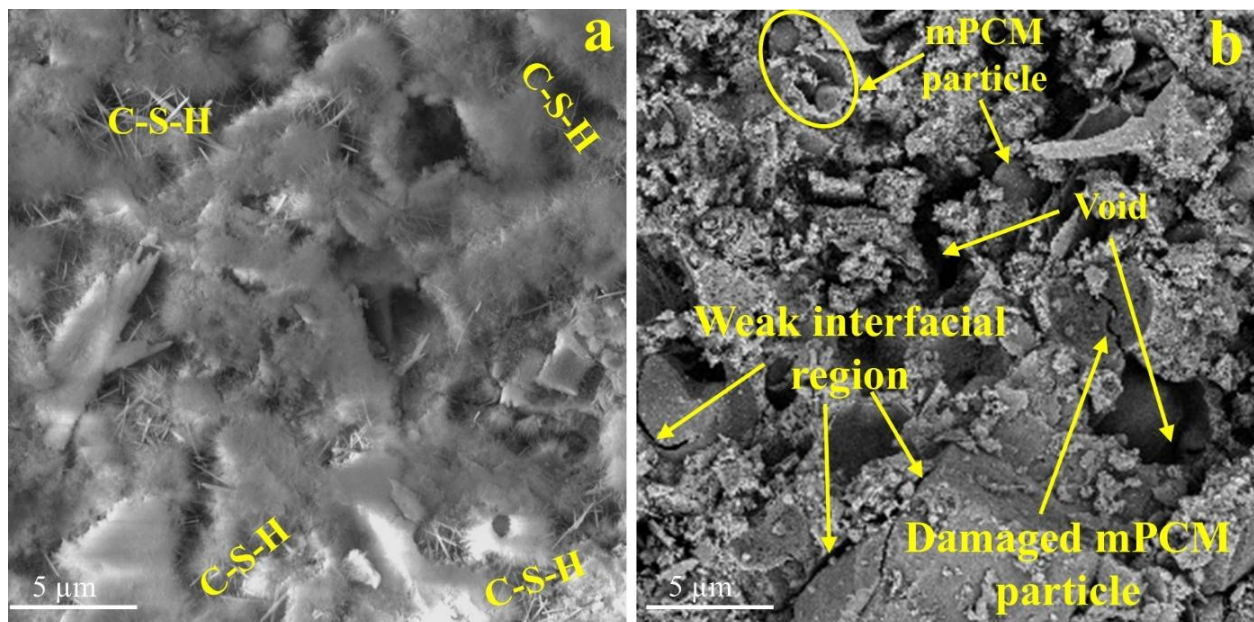


Fig. 10. SEM images of cement mortar samples coded (a) Ref and (b) mPCM7.5.

When the microstructure of the reference mortar is examined in Fig. 10(a), it is observed that the hydration products are well bound to each other, forming a dense and compact matrix structure. While fibrous and needle-like hydration products are commonly observed within the matrix, it is noteworthy that the amount of voids is lower compared to the mPCM-added mortar. The conti-

nunity of the hydration products and the limited voids indicate that the reference mixture has a denser microstructure. This can be considered one of the main reasons why the reference sample exhibits higher compressive and flexural strength. In contrast, in the mortar sample containing 7.5% mPCM, significant microstructural changes are observed in Fig. 10(b). First, the matrix

structure appears to have become more irregular and heterogeneous compared to the reference mixture. The presence of numerous spherical and hemispherical structures in the image indicates that microcapsules are distributed throughout the matrix. However, cracks and deformations are observed on the surfaces of some microcapsules, suggesting that the capsule shells may have been partially damaged during the mixing process or sample preparation. In addition, void formations and weak interfacial regions around the microcapsules are noticeable. This situation indicates that the adhesion between the cement paste and the microcapsule surface is not strong enough. The lower rigidity of the microcapsules compared to the matrix, and the resulting void formation around them, negatively impact the load transfer mechanism. Consequently, matrix continuity is disrupted, and weak regions where stress concentrations can occur are created (Alkan et al. 2023; Djamai et al. 2019).

In general, SEM images reveal that the addition of mPCM increases porosity in the mortar microstructure, reduces matrix integrity, and creates weak regions at the cement paste-microcapsule interface. These microstructural changes can be considered the fundamental factors explaining the mechanical strength losses observed in mPCM-added samples.

4. Conclusions

This study revealed how the addition of an mPCM admixture with a phase transition temperature of around 25 °C at concentrations of 2.5%, 5.0%, and 7.5% altered the physical, mechanical, thermal and microstructural properties of cement mortars. The following results were obtained within the scope of the study:

- The addition of mPCM reduced the consistency of the mortar, resulting in a 3.2%–19.4% decrease in flow diameter compared to the reference mix. The decrease in consistency became more pronounced as the mPCM content increased.
- Increasing the mPCM content reduced the unit weight of the mortars by 2.1–8.8% while simultaneously increasing porosity and water absorption. These changes indicate that the addition of mPCM alters the pore structure of the cement matrix.
- Compared to the reference mortar, the 3, 7, and 28-day flexural strengths decreased by 14.8%–29.3%, 11.8%–27.8%, and 9.7%–26.6% with increasing mPCM content, respectively.
- A similar trend was observed for compressive strength. Although strength losses increased with increasing mPCM content, mortars containing 2.5% and 5% mPCM maintained 28-day compressive strengths above 42.5 MPa. The compressive strength of the mPCM7.5 mixture decreased by 41.2%, 31.3%, and 22.2% on days 3, 7, and 28, respectively, compared to the reference sample.
- SEM observations revealed that increasing mPCM content led to the formation of additional voids and interfacial discontinuities around the microcapsules. These microstructural changes were consistent with

the observed decreases in physical and mechanical performance.

- Thermal cycling tests showed that the addition of mPCM improved the thermal regulation ability of the mortars. The maximum temperature differences between the reference mortar and the mPCM2.5, mPCM5, and mPCM7.5 mixtures reached 1.1, 1.2, and 2.1 °C during heating and 1.0, 2.0, and 2.8 °C during cooling, respectively; indicating that the thermal buffering performance increased with increasing mPCM content.

In summary, the addition of mPCM to cement mortars improved the thermal performance of the mortars by reducing internal temperature fluctuations and delaying heat transfer. However, increasing mPCM content resulted in certain decreases in consistency, unit weight, and mechanical strength values. SEM analyses showed that these changes were related to voids and interfacial discontinuities formed around microcapsules. The results reveal that mPCM-modified cement mortars have significant potential, especially in building applications where thermal comfort and energy efficiency are paramount. Although the highest thermal buffering effect was obtained in the mixture containing 7.5% mPCM, this mixture also showed the highest mechanical strength losses. Therefore, the balance between thermal and mechanical performance should be considered when using mPCM. In particular, mPCM-modified mortars can contribute to increased energy efficiency in non-load-bearing structural elements, interior partition walls, and building envelope applications.

This study is limited to performance evaluations on the material scale. Future studies are proposed to evaluate building-scale energy performance under different climatic conditions using numerical simulations and to investigate the long-term durability properties of mPCM-modified mortars, such as freeze-thaw resistance, sulfate effect, and chloride penetration.

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Conflict of Interest

The author declares no potential conflicts of interest with respect to the research, authorship, and/or publication of this manuscript.

Data Availability

The datasets generated and/or analyzed during the current study are not publicly available but are available from the corresponding author upon reasonable request.

AI Assistance

No AI-based tools were used in the preparation of this manuscript.

Author Contributions

The author declares sole responsibility for all aspects of the study, including conceptualization, methodology, formal analysis, investigation, data curation, visualization, writing of the original draft, and writing, review, and editing of the manuscript.

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Research Article

Diameter–embedment depth interaction of post-installed adhesive anchors in low-strength concrete

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ABSTRACT

Türkiye lies across multiple tectonic plates, resulting in numerous active fault lines that generate destructive earthquakes. Given the vulnerability of the existing building stock, effective retrofitting is critical. In line with the Türkiye Earthquake Code (TEC 2018) for C25, minimum reinforcement diameters for adhesive-anchored bars were reduced to 16 mm. However, because many existing buildings in Türkiye exhibit low concrete strengths (<10 MPa), this study comprehensively investigates the tensile performance of post-installed adhesive anchors under weak concrete conditions. Monotonic pull-out tests were conducted on 192 specimens utilizing four reinforcement diameters (Ø8, Ø10, Ø12, Ø14) and four embedment depths (8Ø, 10Ø, 12Ø, 14Ø) in concrete blocks with four compressive strengths (f_c) of 4.51, 5.35, 7.77, and 10.45 MPa. The results show that increasing the embedment ratio from 8Ø to 14Ø substantially enhances tensile capacity—by up to 95% for Ø8 bars at 4.51 MPa. Moreover, a multiple linear regression model calibrated using the dataset demonstrated a high coefficient of determination ($R^2 = 0.91$), confirming that anchorage performance is primarily governed by geometric parameters rather than material strength. Notably, adequate bond performance is severely limited below 10 MPa, often making failure modes bond-controlled. Based on these findings, it is recommended to evaluate anchorage design on a project-specific basis and avoid standard strengthening interventions in structures with concrete strengths below 10 MPa. In addition, it has been observed that under low concrete strength conditions, the anchor diameter must be reduced to ensure that the expected ductile failure mode via steel bar yielding can be achieved.

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1. Introduction

Türkiye is located in one of the world's most seismically active regions, intersected by major fault systems that expose a substantial portion of the country to significant seismic risk. Catastrophic earthquakes in Erzincan, Gölcük, Van, and most recently Kahramanmaraş have repeatedly demonstrated the severe consequences of this exposure, highlighting the urgent necessity for systematic assessment and retrofitting of the existing building

stock. Seismic vulnerability assessment methodologies provide an effective framework for quantifying the expected damage and seismic risk of existing reinforced concrete buildings by considering structural characteristics that influence their earthquake response (Kassem et al. 2022a, 2022b). A critical vulnerability in much of the existing building stock, particularly in structures constructed prior to the implementation of modern seismic design codes, is the widespread use of substandard materials. Post-earthquake reconnaissance studies con-

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sistently reveal that reinforced concrete structures in Türkiye frequently exhibit low compressive strength, often ranging between 10 to 15 MPa and, in some cases, falling as low as 5 MPa due to poor construction practices and construction defects (Kaltakçı et al. 2010; Kaplan et al. 2004; Odacıoğlu et al. 2022; Yurdakul et al. 2021).

Post-earthquake observations following the 2023 Kahramanmaraş earthquakes revealed out-of-plane overturning of reinforced concrete shear walls in some previously retrofitted buildings (Doğan and Karadağlı 2024), as shown in Fig. 1. These observations indicate that post-installed connections anchored into weak existing concrete may exhibit brittle, non-ductile failure. Although TEC (2018) specifies the use of reinforcing bars with a diameter of at least 16 mm for such anchorage applications, smaller-diameter reinforcing bars may enable a more distributed force-transfer mechanism (Çalışkan et al. 2013; Odacıoğlu et al. 2022) and reduce the likelihood of brittle concrete-related failure. Therefore, their potential to provide a more ductile connection response requires further investigation.



Fig. 1. Out-of-plane overturning and separation of an added reinforced concrete shear wall from the existing structure due to anchorage failure following the 2023 Kahramanmaraş earthquakes (Doğan and Karadağlı 2024).

Post-installed adhesive anchors, such as epoxy-based systems, are widely employed to connect newly added structural elements to existing reinforced concrete members. Typical applications include the connection of externally added reinforced concrete shear walls, column jackets, foundation extensions, and other strengthening components to the existing structural system (Arslan 2016; Cihan et al. 2023; Kaltakçı et al. 2010; Mazılıgüney 2007). The effectiveness of these strengthening interventions depends considerably on the ability of the post-installed adhesive anchors to transfer tensile and shear forces safely into the existing concrete.

The strengthening provisions of the Türkiye Earthquake Code (TEC 2018) specify geometric and detailing requirements for post-installed adhesive anchors used in existing reinforced concrete structures. However, many existing buildings in Türkiye contain concrete with compressive strengths considerably lower than those typically considered in anchor qualification procedures and conventional design models. Most current design provisions, including those published by the American Concrete Institute, have predominantly been calibrated using experimental data obtained from normal- and high-strength concrete (ACI 318 2014; ACI 355.4 2019). Consequently, the applicability of these provisions to anchors installed in concrete with compressive strengths close to or below 10 MPa remains uncertain.

The tensile resistance of post-installed adhesive anchors is governed by a load-transfer mechanism involving the steel bar, the adhesive layer, and the surrounding concrete. Tensile forces are transferred from the reinforcing bar to the adhesive and subsequently from the adhesive to the concrete through a combination of chemical adhesion, mechanical interlocking, and friction (Cook 1993; González et al. 2018; Krishnaveni and Rajendran 2024). The resulting anchorage performance depends on several geometric, material, and installation-related parameters, including concrete compressive strength, bar diameter, embedment depth, edge distance, spacing, confinement, hole-cleaning quality, and adhesive properties.

Under monotonic tensile loading, post-installed adhesive anchors may exhibit steel failure, concrete cone failure, bond or pull-out failure, combined concrete cone-bond failure (Cook 1993; Mahrenholtz et al. 2015). Shallow embedment depths commonly promote concrete breakout, whereas longer embedment depths may increase the contribution of bond resistance and shift the governing mechanism toward combined failure or steel failure. In low-strength concrete, brittle splitting and concrete-controlled failures may become more critical because the radial stresses generated around the reinforcement ribs can exceed the limited tensile resistance of the surrounding concrete (Akgül et al. 2022; Boonmee and Rodsin 2019). Edge distance and concrete cover also substantially influence these failure mechanisms (Ding et al. 2022).

Concrete compressive strength is one of the primary material parameters influencing bonded anchorage performance. Previous experimental studies have generally shown that bond strength and ultimate tensile resistance increase with concrete compressive strength, although the degree of improvement depends on the governing failure mechanism (Ahmed et al. 2021). In concretes with compressive strengths below approximately 15 MPa, brittle concrete breakout and bond-controlled failures may govern the response before the tensile capacity of the reinforcing bar is fully developed (Ahmed et al. 2021). Increasing the embedment depth and providing sufficient confinement may improve anchorage resistance; however, the effectiveness of these measures depends on the concrete properties, anchor geometry, and test configuration (Arslan 2016; Yılmaz et al. 2013).

Anchor diameter also plays an important role in determining both ultimate tensile capacity and failure mode. Larger-diameter bars generally provide a greater bonded surface area and therefore a higher total pull-out resistance. However, increasing the diameter may also generate greater radial stresses and more pronounced local stress concentrations within the surrounding concrete. These effects may increase the likelihood of brittle concrete breakout or splitting, particularly in low-strength concrete (Arslan 2016; Çalışkan et al. 2013). Therefore, an increase in anchor diameter does not necessarily produce a proportional improvement in anchorage efficiency in the low-strength concrete (Çalışkan and Aras 2017).

This study aims to identify suitable anchorage parameters for strengthening existing buildings in Türkiye, particularly those with insufficient concrete strength. Considering the low-quality concrete commonly observed in the national building stock, four different concrete strength classes were produced by varying cement and water ratios to represent weak concrete conditions. For each concrete strength level, four different anchor embedment depths and four different anchor diameters were employed. The type of adhesive system was kept constant, and adequate edge distances and spacing were ensured in accordance with previous studies. For each combination of concrete strength, anchor diameter, and embedment depth, three identical anchors were installed to ensure statistical reliability. In total, 192 post-installed adhesive anchors were subjected to pull-out tests. This experimental program enables a comprehensive evaluation of anchorage performance under varying concrete strength, anchor diameter, and embedment depth, and provides essential data for establishing suitable anchorage parameters for the seismic retrofitting of deficient reinforced concrete buildings.

2. Experimental Program

The experimental program was systematically designed to investigate the multi-parametric interactions governing the ultimate tensile resistance and associated failure mechanisms of post-installed adhesive anchors. To accurately replicate the structural deficiencies and concrete compressive strength classes of 10.45 MPa or lower prevalent in the existing building stock, a comprehensive testing matrix was established. This methodology isolates and evaluates three critical design variables: the base concrete substrate strength, the effective anchor diameter, and the structural embedment depth. By rigorously controlling these geometric and material parameters across 192 test specimens, the investigation aims to delineate the operational boundary conditions and performance thresholds of bonded anchoring systems under monotonic tensile loading, thereby providing an empirical foundation for optimized seismic retrofitting designs.

2.1. Materials

In this context, concrete blocks with four different strength levels were cast by varying the amount of cement and water to represent buildings with low concrete strength considering the existing building stock. The mix proportions and compressive strengths of low-strength concretes are given in Table 1. It should be noted that the relatively high w/c ratios used in this study do not represent standard structural concrete mixtures. They were intentionally selected to obtain low compressive strength levels and to simulate poor-quality concrete conditions observed in deficient existing reinforced concrete buildings. No curing was applied to the concrete blocks or to the cube specimens taken for measuring compressive strength, as intended, in order to represent low-strength buildings and actual construction practice.

Table 1. Concrete mix proportions and compressive strengths.

	1. Specimen	2. Specimen	3. Specimen	4. Specimen
Sand (0.00-4.00 mm) - kg	560.03	531.75	520.60	510.75
Gravel (4.00-11.20mm) - kg	560.03	531.75	520.60	510.75
Cement (CEM II 32.5) - kg	190.00	300.00	345.00	375.00
Water - kg	275.04	274.10	273.81	272.32
W/C	1.45	0.91	0.79	0.73
Average concrete compressive strengths (f_c - Mpa)	4.51	5.35	7.77	10.45

The Fischer FIS EM Plus 390 S is a high-performance pure epoxy resin anchoring system developed for demanding structural applications. The material has a density in the range of 1.4–1.6 g/cm³ and a compressive strength of at least 80 MPa. It also exhibits a tensile adhesion strength of at least 20 MPa, while maintaining a bond strength greater than 11 MPa after 20 days, indicating reliable long-term anchorage performance under heavy-duty loading conditions. In addition, the resin exhibits an elongation at break of 1.1% and a curing shrinkage, defined as the volumetric shrinkage occur-

ring during the hardening of the epoxy resin, of less than 0.01% according to ASTM D2566. These properties indicate excellent dimensional stability and effective contact within the drilled hole, making FIS EM Plus 390 S suitable for critical structural and seismic-resistant applications.

B420C steel was selected as the anchorage element, and 48 reinforcing bars with diameters of 8, 10, 12, and 14 mm and a length of 70 cm were prepared. The average yield strengths of the bars, according to their diameters, were found to be 536.30 MPa, 496.42 MPa, 477.16

MPa, and 466.78 MPa, while the average tensile strengths were determined as 656.25 MPa, 600.12 MPa, 558.46 MPa, and 543.36 MPa, respectively. To prevent concrete cracking, Q131/131 welded wire mesh reinforcement was used, consisting of 5 mm diameter bars spaced at 15×15 cm, with an average yield strength of 516.67 MPa and an average tensile strength of 633.67 MPa.

2.2. Test specimens

For each concrete mix, reinforced concrete blocks with dimensions of 75 cm × 195 cm × 25 cm were produced by placing Q131/131 welded wire mesh with a concrete cover of 4 cm from the top surface. In order to simulate low-performance buildings and common con-

struction practice, no curing conditions were applied to either the concrete blocks prepared for compressive strength testing. After 28 days, when the concrete was assumed to have completed hydration and reached its compressive strength, drilling of the anchor holes was initiated, maintaining a center-to-center spacing of 15 cm. The holes were drilled perpendicular (90°) to the concrete surface with a diameter 4 mm larger than the anchor bar and to embedment depths corresponding to 8, 10, 12, and 14 times the respective anchor diameters, as shown in Fig. 2. Subsequently, the holes were cleaned using an air compressor to ensure that no dust or debris remained inside. The chemical adhesive was then injected into the holes to eliminate any voids between the ribbed anchor bar and the concrete, after which the installation of the anchors was completed.

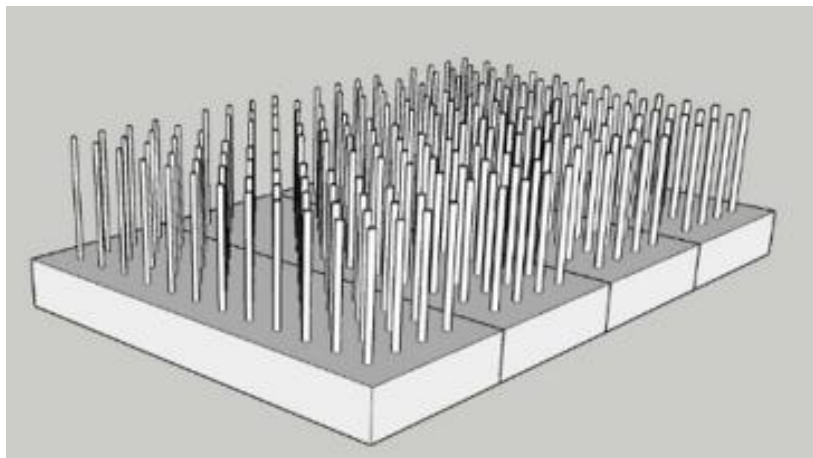


Fig. 2. Three-dimensional drawing of the experimental specimens.

2.3. Pull-out test

The pull-out tests were performed under manually controlled, monotonically increasing tensile loading until failure. The tensile force was applied through a hydraulic cylinder and transferred to the anchor bar by means of a conical pulling head positioned concentrically with the bar axis, as shown in Figs. 3(a–c). The reaction force of the hydraulic cylinder was taken from the concrete block surface around the anchor location. It should be noted that the reaction force of the hydraulic cylinder was transferred to the concrete surface around the anchor location. This contact may have generated a local compression zone beneath the hydraulic jack, producing a confinement effect around the anchor. Such confinement may influence the failure mechanism and may partially restrict the formation of a concrete cone failure. Therefore, the results should be interpreted as ultimate pull-out capacities obtained under the adopted test configuration. Since the same test setup was used for all specimens, the comparative evaluation of anchor diameter, embedment ratio, and concrete compressive strength remains consistent. Care was taken to align the loading axis with the longitudinal axis of the reinforcing bar in order to minimize eccentric loading effects. During the tests, the applied load was monitored using the manometer connected to the hydraulic system, and the maximum tensile force reached before failure was rec-

orded for each specimen. The pull-out tests were initiated 24 hours after anchor installation to allow the chemical adhesive to cure and develop sufficient strength. Since the primary aim of the experimental program was to determine the ultimate pull-out capacity, continuous displacement or slip measurements were not performed. This limitation was considered in the interpretation of the results, and the discussion was restricted to ultimate tensile capacity and observed failure modes.

3. Experimental Results and Evaluation

The experimental dataset presented in Table 2 enables a detailed quantitative assessment of the average tensile performance of post-installed adhesive anchors in low-strength concrete. To improve the interpretation of the triplicate tests, Fig. 4 presents the mean pull-out loads as a histogram, together with the corresponding minimum and maximum ranges. It should be noted that Fig. 4 includes only the specimens in which failure occurred as concrete breakout together with the adhesive anchor; specimens exhibiting steel anchor failure were excluded from the plotted mean values. A clear trend is observed across the test configurations, indicating that the pull-out capacity is influenced by embedment ratio, concrete compressive strength, and anchor diameter.



Fig. 3. Pull-out test: (a) Modeling; (b) Hydraulic cylinder setup; (c) Experimental setup.

In the test groups shown with a dark gray background in Table 2, failure occurred directly in the anchor elements rather than as concrete breakout or pull-out together with the adhesive anchor. Anchor failure became more pronounced particularly in the test groups with increased embedment depth and higher concrete compressive strength. This indicates that, as the embedment depth increases, the bond performance between the adhesive anchor and concrete, as well as the load-bearing capacity of the concrete, improves; consequently, the failure mechanism becomes governed by the tensile capacity of the anchor rod rather than by the concrete or bond interface.

In addition, for the $\varnothing 14$ specimen with a concrete compressive strength of 7.77 MPa, failure occurred in the anchor due to mechanical interlocking with the welded wire mesh reinforcement. The numbers shown inside the circles indicate the number of failed anchors in the corresponding test group. Accordingly, while only one anchor failed in some specimens, the failure of multiple anchors in others demonstrates that, under these

parameters, failure of the anchor element became the governing failure mechanism.

For the $\varnothing 8$, $\varnothing 10$, $\varnothing 12$, and $\varnothing 14$ reinforcing bars, the interaction plots showing the relationship between tensile force, embedment depth, and concrete compressive strength are presented in Figs. 5–8. A parametric evaluation of embedment depth reveals that increasing the embedment ratio from $8\varnothing$ to $14\varnothing$ results in a substantial increase in tensile capacity. For instance, for $\varnothing 8$ anchors in concrete with a compressive strength of 4.51 MPa, the pull-out capacity increases from 13.65 kN to 26.65 kN, corresponding to an increase of approximately 95%. Similarly, for $\varnothing 14$ anchors at 10.45 MPa, the capacity increases from 56.90 kN to 87.38 kN, representing a gain of about 53.6%. These results indicate that while embedment depth is the dominant parameter influencing tensile resistance, its relative effectiveness decreases as both concrete strength and anchor diameter increase. This behavior suggests a nonlinear relationship between embedment depth and pull-out capacity, consistent with bond stress redistribution along the anchorage length.

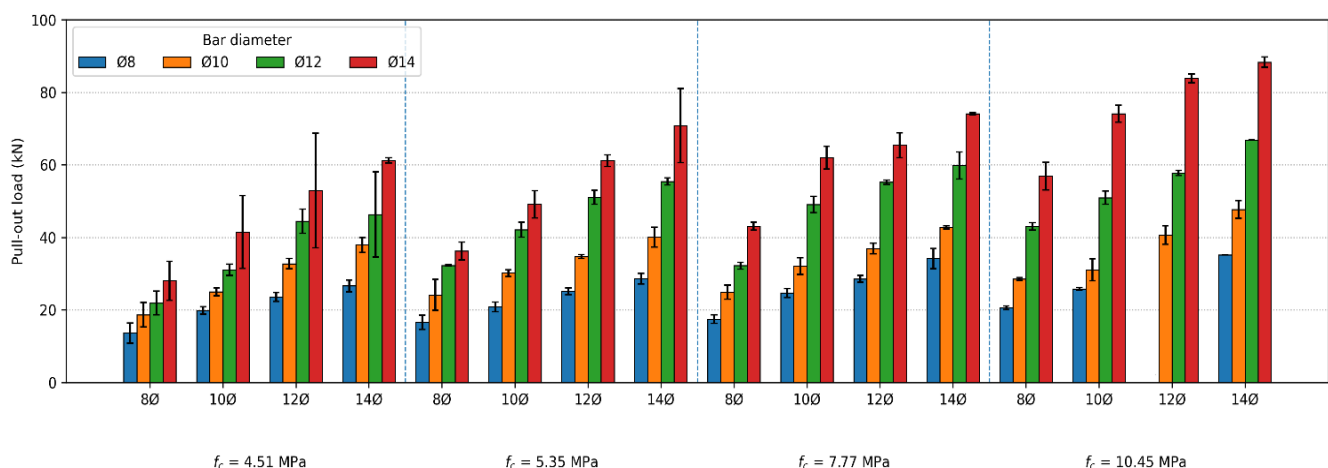


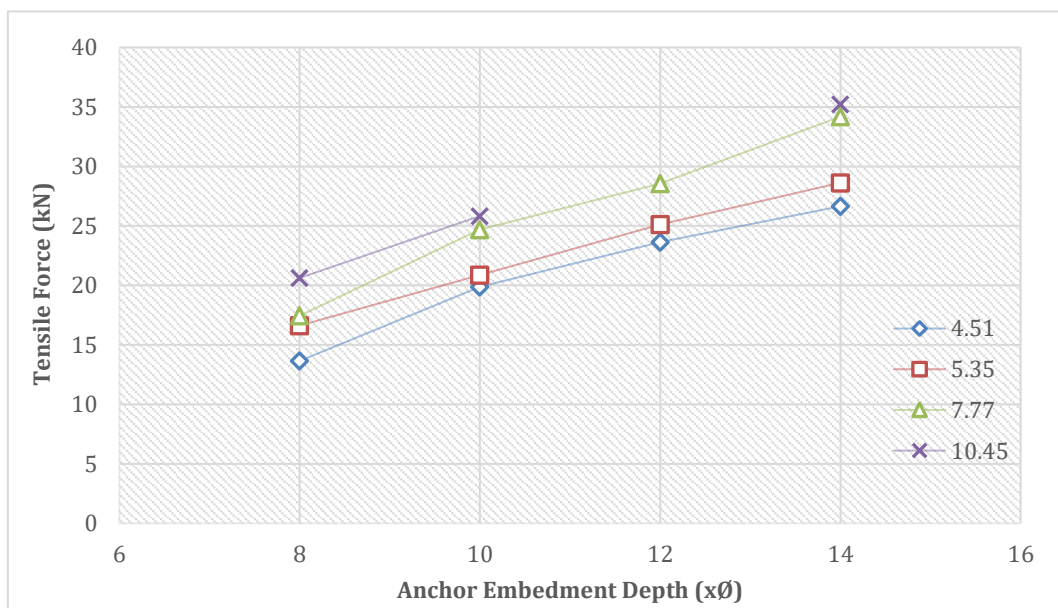
Fig. 4. Mean pull-out loads are presented for different bar diameters and embedment ratios, with error bars showing the minimum and maximum test results.

Table 2. Pull-out test tensile force data obtained for different anchor diameters and embedment depths in low-strength concrete.

Anchor Diameters (mm)	Strength Concrete (f_c) (MPa)	Anchor Embedment Depths (L_d)			
		8Ø (kN)	10Ø (kN)	12Ø (kN)	14Ø (kN)
Ø8	4.51	13.65	19.88	23.63	26.65
	5.35	16.62	20.87	25.13	28.63
	7.77	17.45	24.68	28.57	34.18
	10.45	20.62	25.83 [24.70] ¹	N/A [30.60] ³	35.20 [36.23] ²
Ø10	4.51	18.68	25.00	32.78	37.97
	5.35	24.17	30.17	34.73	40.10
	7.77	24.90	32.13	36.97	42.75
	10.45	28.52	31.10 [40.00] ¹	40.68 [45.00] ¹	43.10 [50.15] ¹
Ø12	4.51	21.93	31.08	44.52	46.33
	5.35	32.28	42.17	51.08	55.43
	7.77	32.23	49.08	55.22	59.85
	10.45	43.07	50.98	57.82	66.90 [66.65] ²
Ø14	4.51	28.05	41.48	52.97	61.25
	5.35	36.3	49.13	61.18	70.82
	7.77	43.10	61.95	65.36 [70.40] ¹	74.08
	10.45	56.90	74.15	83.87	88.33 [85.50] ¹

Note 1: N/A indicates that no concrete-controlled mean value was available because all three specimens exhibited anchor failure.

Note 2: The cells with square brackets indicate test groups in which anchor failure occurred. The values in square brackets represent the average tensile force of the specimens that failed in the anchor itself, and the superscript numbers indicate the number of specimens with anchor failure.

**Fig. 5.** Interaction diagram of pull-out capacity as a function of embedment depth and concrete compressive strength for the Ø8 anchor bar.

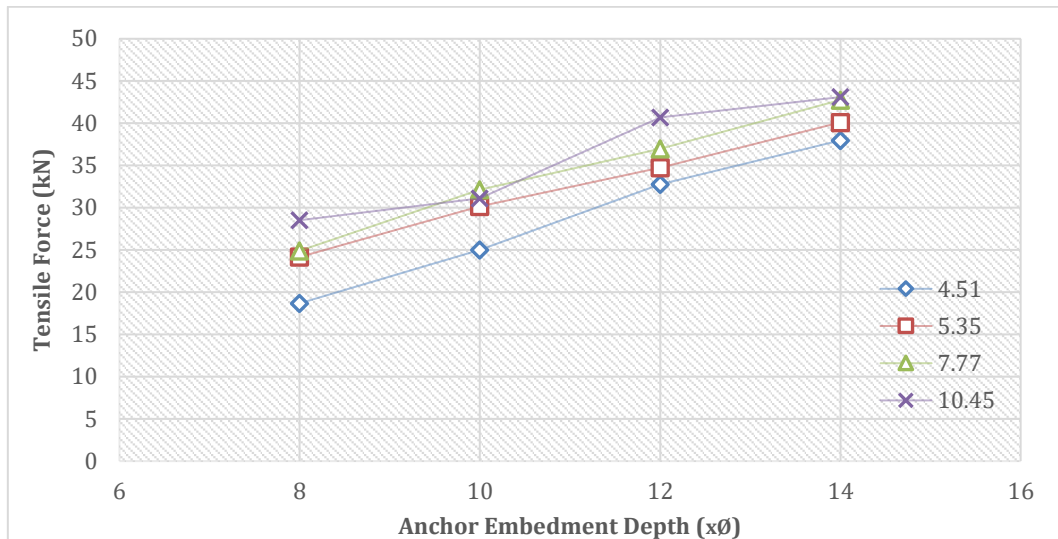


Fig. 6. Interaction diagram of pull-out capacity as a function of embedment depth and concrete compressive strength for the $\phi 10$ anchor bar.

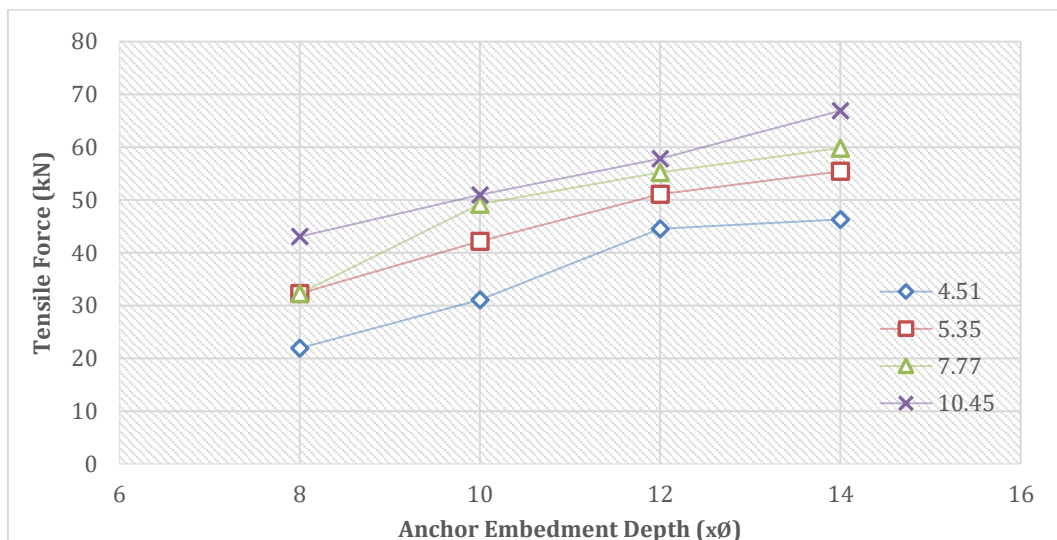


Fig. 7. Interaction diagram of pull-out capacity as a function of embedment depth and concrete compressive strength for the $\phi 12$ anchor bar.

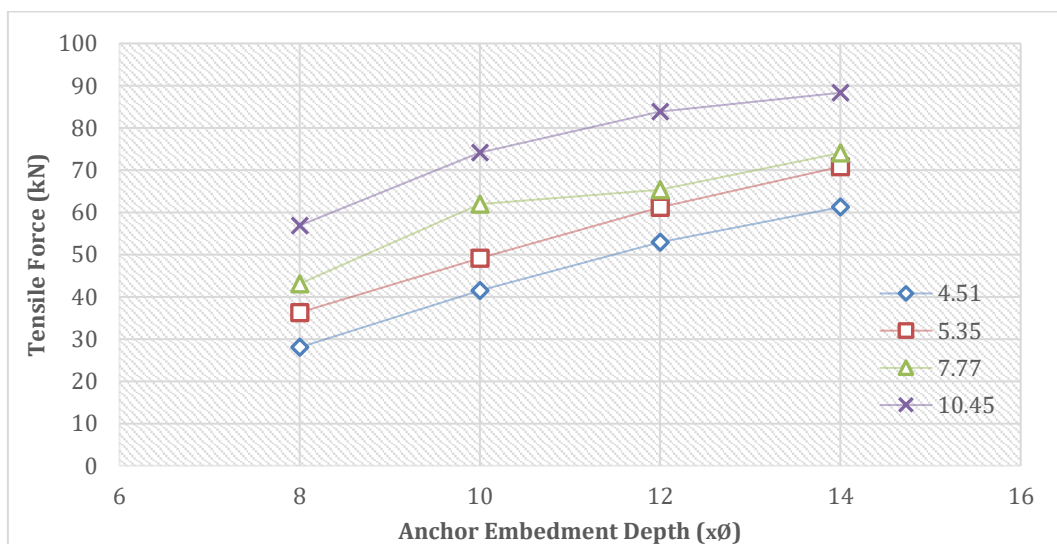


Fig. 8. Interaction diagram of pull-out capacity as a function of embedment depth and concrete compressive strength for the $\phi 14$ anchor bar.

The influence of concrete compressive strength is also evident across all anchor diameters and embedment depths, as shown in Figs. 5–8. It should be noted that these interaction diagrams include only the specimens in which failure occurred as concrete breakout together with the adhesive anchor; specimens exhibiting anchor failure were excluded from the plotted values. Therefore, the values presented in these figures represent the mean pull-out capacities associated with concrete-controlled failure. For example, for Ø10 anchors at an embedment depth of 12Ø, the pull-out capacity increases from 32.78 kN at 4.51 MPa to 40.68 kN at 10.45 MPa, corresponding to an increase of approximately 24%. This trend confirms that higher concrete strength enhances the bond performance by improving the mechanical interlock and confinement provided by the surrounding matrix. However, the rate of increase in pull-out capacity with respect to compressive strength is generally less pronounced than the increase associated with embedment depth and anchor diameter, indicating that bond surface area, governed by both the anchor diameter and embedment depth, plays a critical role in low-strength concrete conditions.

The effect of anchor diameter is similarly significant but exhibits a more complex behavior. At a constant concrete strength of 4.51 MPa and embedment depth of 10Ø, increasing the diameter from Ø8 to Ø14 results in an increase in pull-out capacity from 19.88 kN to 41.48 kN, corresponding to more than a twofold increase. This indicates that larger anchor diameters provide a greater bonded surface area and thus improve the load-carrying capacity. However, this increase may also be accompanied by higher radial stresses in the surrounding concrete, particularly in weak substrates. This observation highlights that while larger diameters enhance load capacity, the failure mechanism and reliability of anchorage performance should also be carefully considered in low-strength concrete.

An important aspect of the experimental findings is the observed transition in failure modes. In the specimens, failure occurred in the form of bond failure or concrete breakout, indicating that the adhesive–concrete interface governs the behavior. However, in selected specimens—particularly those with higher concrete strength (10.45 MPa), larger diameters, and deeper embedment ($\geq 12\text{Ø}$)—failure occurred in the anchor itself. This transition from bond-controlled to steel-controlled failure represents a critical threshold in anchorage performance, indicating that the full tensile capacity of the reinforcement can be mobilized under favorable conditions.

From a comparative perspective, the data suggest that embedment depth is the most effective parameter for improving anchorage performance, followed by anchor diameter and concrete compressive strength. However, the relative contribution of each parameter varies depending on the specific configuration. In ultra-low-strength concrete ($f_c \leq 5$ MPa), bond failure dominates regardless of diameter, emphasizing the limited effectiveness of increasing anchor size under such conditions. In contrast, for higher strength levels, the combined effect of increased embedment depth and diameter can shift the governing failure mode toward a more ductile response.

These findings have direct implications for engineering practice. The results indicate that in low-strength concrete, increasing embedment depth is a more reliable strategy than increasing anchor diameter, as it enhances bond performance without significantly increasing stress concentrations. Moreover, the observed failure mode transitions suggest that achieving steel failure should be considered a desirable design objective, as it ensures ductile behavior and more predictable performance.

Overall, the experimental results demonstrate that anchorage design in low-strength concrete requires a careful balance between geometric parameters and material properties. The conventional design assumptions embedded in current standards may not adequately capture this behavior, particularly in the low-strength regime investigated in this study. Therefore, the development of modified design approaches that explicitly account for these effects is essential for ensuring safe and reliable anchorage performance in existing structures.

A multiple linear regression analysis was performed to evaluate the combined effects of anchor diameter, embedment ratio, and concrete compressive strength on the ultimate pull-out capacity, as given in Eq. (1). The dependent variable was the mean concrete-controlled pull-out capacity (P_u) in kN. The independent variables were anchor diameter ($\emptyset$) in mm, embedment ratio (l_d), and concrete compressive strength (f_c) in MPa. Here, l_d represents the embedment ratios of 8Ø, 10Ø, 12Ø, and 14Ø used in the experimental program. The regression model was calibrated using 63 concrete-controlled mean values. The configuration in which all three specimens exhibited steel anchor failure was excluded because no concrete-controlled mean value was available.

$$F = 5.85\emptyset + 3.68l_d + 2.46f_c - 81.38 \quad (1)$$

The model provided a coefficient of determination of $R^2=0.9123$ and an adjusted $R^2=0.9078$, indicating a strong correlation between the predicted and experimental pull-out capacities. The standard error of the estimate was 5.32 kN. ANOVA results showed that the regression model was statistically significant, with $F=204.60$ and $p=3.90 \times 10^{-31}$. The p-values of all regression coefficients were less than 0.05, indicating that anchor diameter, embedment ratio, and concrete compressive strength had statistically significant effects on the ultimate pull-out capacity. The 95% confidence intervals of the regression coefficients were also evaluated and confirmed the statistical reliability of the proposed model within the investigated parameter range.

4. Conclusions

In this study, considering the characteristics of the existing building stock, pull-out tests were conducted on 192 post-installed adhesive anchors embedded in reinforced concrete blocks with low compressive strengths of 4.51 MPa, 5.35 MPa, 7.77 MPa, and 10.45 MPa. Anchor bars with diameters of 8, 10, 12, and 14 mm were embedded at depths corresponding to 8, 10, 12, and 14

times the respective anchor diameters. Based on the results of these experiments:

- It was observed that the embedment depth of the anchor significantly influences the pull-out capacity. As the embedment depth increases, the tensile pull-out resistance of the anchor correspondingly increases.
- Considering the observed failure modes, it was determined that adequate bond between the reinforcement bar and the concrete was not achieved in structures with a compressive strength below 10 MPa. Therefore, it is recommended that no strengthening interventions be carried out on buildings with concrete compressive strengths lower than 10 MPa.
- It should be emphasized that the use of large-diameter anchors can generate significant stress concentrations within low-strength concrete, potentially resulting in pull-out or crushing-type failure mechanisms. However, Section 15 of TEC (2018), 'Special Rules for the Assessment and Strengthening Design of Existing Building Systems Under Seismic Effects', recommends a single anchor diameter of Ø16 without considering the concrete strength of the building to be strengthened. Furthermore, it states that reinforced concrete buildings with concrete strength below a certain value should not be strengthened due to insufficient anchor performance. Consequently, the diameter of post-installed adhesive anchors should be carefully selected as a function of the concrete compressive strength to ensure reliable anchorage performance and to prevent premature local failures.

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Conflict of Interest

The authors declare no potential conflicts of interest with respect to the research, authorship, and/or publication of this manuscript.

Data Availability

The datasets generated and/or analyzed during the current study are not publicly available but are available from the corresponding author upon reasonable request.

AI Assistance

During the preparation of this manuscript, ChatGPT (OpenAI) was used exclusively for language editing and stylistic refinement. The authors take full responsibility for the content, interpretation, and conclusions of the published article.

Author Contributions

All authors made substantial contributions to the conception and design of the study, acquisition of data, analysis and interpretation of data; drafted or critically revised the manuscript for important intellectual content; and approved the final version to be published.

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Challenge Journal of STRUCTURAL MECHANICS

Research Article

Finite element investigation of the cyclic behavior of bolted extended end plate connections strengthened with longitudinal plates

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ABSTRACT

The cyclic behavior of bolted extended end plate beam-to-column connections strengthened with longitudinal plates parallel to the beam web was investigated through three-dimensional nonlinear finite element analysis. Unlike previously studied methods such as haunches, end plate stiffeners, side plates, and internal reinforcement details, this strengthening configuration places the plates within the beam depth and parallel to the beam web, an arrangement that has not been systematically examined for this connection type. The finite element model was first validated using results from a benchmark experimental test reported in the literature. Following validation, six connection configurations were analyzed under the AISC cyclic loading protocol. These included a four-bolt unstiffened end plate (4E) reference model, a four-bolt stiffened end plate (4ES) model, and four models incorporating longitudinal plates with varying plate lengths and locations. The numerical results were assessed based on hysteretic response, cumulative energy dissipation, bolt force distribution, equivalent plastic strain development, and equivalent viscous damping. It was observed that the addition of longitudinal plates improved the global cyclic response by increasing the moment capacity while maintaining stable hysteresis loops. However, increasing the length of the longitudinal plates resulted in higher tensile force demands on the bolts, indicating the need for careful design to avoid brittle failure modes. Analysis of the equivalent plastic strain showed that the longitudinal plates reduced the peak strain in the beam-end region by up to approximately 64% relative to the unstiffened (4E) reference connection, whereas the conventional stiffener (4ES) shifted the plastic hinge away from the column face but produced the highest strain concentration. The results indicate that the longitudinal plate detail offers a practical strengthening approach for improving the cyclic response of bolted extended end plate connections, provided that the increased bolt tensile demand is explicitly considered in design.

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1. Introduction

Steel moment-resisting frames are commonly utilized in seismic regions due to their inherent ductility and capacity for energy dissipation through stable inelastic mechanisms. In these systems, bolted end plate beam-to-

column connections are frequently selected because they simplify fabrication, minimize field welding, and are suitable for both new construction and retrofit scenarios. The behavior of end plate joints is determined by a combination of factors, including end plate bending, column-flange flexibility, bolt pretension, prying action, lo-

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cal contact conditions, and force redistribution within the joint region. These factors collectively influence the stiffness, strength, rotational capacity, and failure mode of the connection, as established in previous studies (Ba-haari and Sherbourne 1996; Bursi and Jaspart 1998; Troup et al. 1998; Abidelah et al. 2012; Prinz et al. 2014).

A significant body of research has addressed haunch-based strengthening methods. Experimental results by Lee et al. (2003) demonstrated that welded straight haunches are effective in relocating plastic hinging away from the column face and enhancing cyclic performance, provided that detailing is appropriate. Saberi et al. (2019) further showed that welded haunches can rehabilitate weak bolted end plate connections, with their effectiveness being highly dependent on haunch thickness and angle. Zhao et al. (2021) reported that haunch retrofitting increases both moment resistance and initial rotational stiffness, while also modifying the internal force-transfer mechanism within the joint. Collectively, these studies confirm the effectiveness of haunches, but also highlight that their performance is highly sensitive to geometric parameters and may not always achieve an optimal balance between ductility, force redistribution, and constructability.

Stiffeners, side plates, and cover-plate-type reinforcement represent another principal research focus. Herath et al. (2023) emphasized that the performance of bolted stiffened end plate connections is governed not only by resistance but also by the hierarchy of yielding and the specific performance objectives. El-Sa'eed (2024) investigated side-plate strengthening and observed significant increases in moment capacity, though these solutions require additional fabrication and welding. For portal-frame applications, Zhang et al. (2024, 2025a, 2025b) introduced prefabricated retrofit systems utilizing cover plates and channel-type components, demonstrating improvements in strength and initial stiffness, as well as reductions in end plate opening and bolt demand under certain conditions. However, these studies also noted that excessive reinforcement can transfer damage to other components, increase bolt force demand, or compromise the balance between strength enhancement and cyclic performance. Similarly, Yilmaz and Bekiroğlu (2016) showed numerically that reinforcing the panel zone of bolted end plate connections reduces plastic strains in the panel zone but can result in earlier beam flange or web buckling and decreased rotational capacity.

Another group of studies has focused on internal strengthening details for end plate joints connected to hollow or closed-section columns. Zhang et al. (2021) demonstrated that measures such as infilled concrete and facing plates can enhance the stiffness and moment capacity of one-sided bolted end plate connections to hollow square columns. Cai et al. (2022) showed that end plate stiffeners and internal strengthening components improve seismic performance by facilitating force transmission and increasing rotational capacity. Liu et al. (2022) reported that novel internal strengthening structures have a significant impact on failure sequence, bolt-force development, and cyclic energy dissipation. It should be noted that these studies primarily address hollow-section connection configurations, rather than conventional open-section bolted extended end plate beam-

to-column joints. In a related study, Yilmaz and Bekiroğlu (2022) examined the biaxial cyclic behavior of a bolted end plate connection to an I-column modified with box-plates parallel to the column web, and found that the proposed detail exhibited acceptable hysteretic behavior under cyclic loading. However, this investigation focused on a reformed weak-axis connection detail under biaxial loading, rather than on strengthening conventional extended end plate joints using beam-side longitudinal plates.

In addition to strengthening studies, extensive research has addressed the mechanics of end plate connections and their individual components. The behavior of T-stub components, prying action, bolt pretension, and three-dimensional force transfer has been thoroughly investigated through analytical, experimental, and finite element approaches (Yang and Lee 2007; Hu et al. 2011; Peng et al. 2018; Ghamari et al. 2023; Couchaux and Madhouni 2022; Taheripour et al. 2022; Zhang et al. 2023; Xiao et al. 2024; Liu et al. 2025). These studies collectively indicate that strengthening strategies must be evaluated not only for their effect on resistance, but also for their impact on force redistribution, local deformation demand, prying action, and cyclic stability.

Although a wide range of strengthening approaches has been reported, most studies have focused on haunches, end plate stiffeners, side plates, cover-plate assemblies, and internal reinforcement details. In contrast, the application of longitudinal plates welded within the beam depth, between the beam flanges, and oriented parallel to the beam web has not been systematically examined for bolted extended end plate connections. This configuration differs from conventional end plate stiffeners in that it does not introduce an upward projection above the beam top flange, which can be advantageous for architectural and detailing considerations. Furthermore, because the longitudinal plates are aligned with the beam web rather than concentrated in the extended end plate region, this arrangement may provide a more distributed strengthening effect and a more favorable balance among moment resistance, bolt-force development, cyclic stability, and ductile deformation.

In the present study, a novel strengthening concept is introduced, in which longitudinal plates are installed between the beam flanges and parallel to the beam web. The main objective is to determine whether this strengthening configuration can enhance the hysteretic response and the energy dissipation capacity of bolted extended end plate connections, while also evaluating its influence on bolt-force demand and force redistribution within the connection region. For this purpose, detailed three-dimensional nonlinear finite element models were developed and subjected to cyclic loading, and the influence of plate geometry and placement on the seismic performance of the connection was systematically evaluated.

2. Finite Element Models

2.1. Connection configurations

Six finite element models of beam-to-column connections were established and analyzed using Ansys Finite Element Analysis Software (ANSYS 2024) to evaluate the

influence of longitudinal plates on the cyclic performance of bolted extended end plate connections. The general geometry and main dimensions of the models are shown in Fig. 1, and the detailed configurations of the connection models are provided in Fig. 2. Table 1 summarizes the nomenclature of all models.

Each model consisted of an IPE270 beam connected to an HE240B column. Both sections meet the requirements for seismically compact members as specified in ANSI/AISC 341-16 (AISC 2016). The first model is a four-bolt unstiffened extended end plate connection (4E), and the second model is a four-bolt stiffened extended end plate connection (4ES). Both connections were designed

in accordance with ANSI/AISC 358-22 (AISC 2022a). The required end plate thicknesses were determined as 25 mm for the 4E connection and 22 mm for the 4ES connection. M20 bolts were used in both configurations.

The remaining four models were developed by modifying the 4E connection through the addition of longitudinal plates positioned between the beam flanges and parallel to the beam web. These models were established to investigate the effects of plate length and plate location on the connection behavior. In the adopted nomenclature, 'N' indicates that the longitudinal plates are welded near the beam web, while 'O' indicates that the plates are welded to the flange tips.

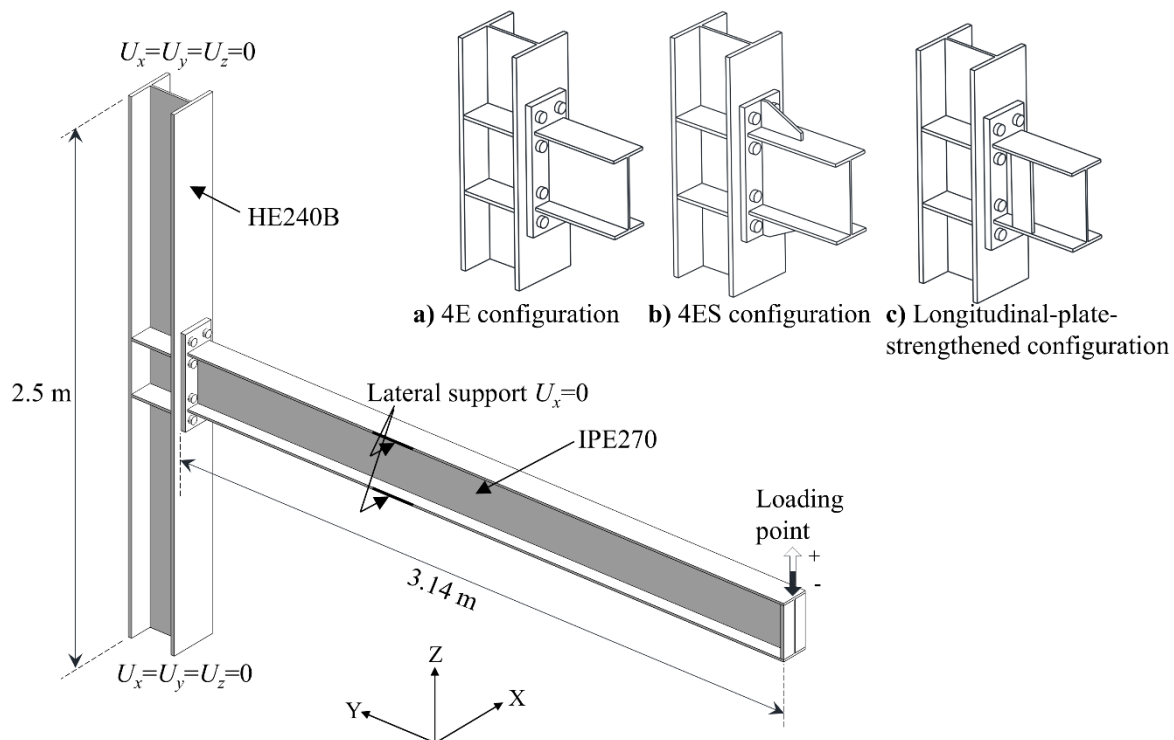


Fig. 1. The overall geometry and principal dimensions of the models.

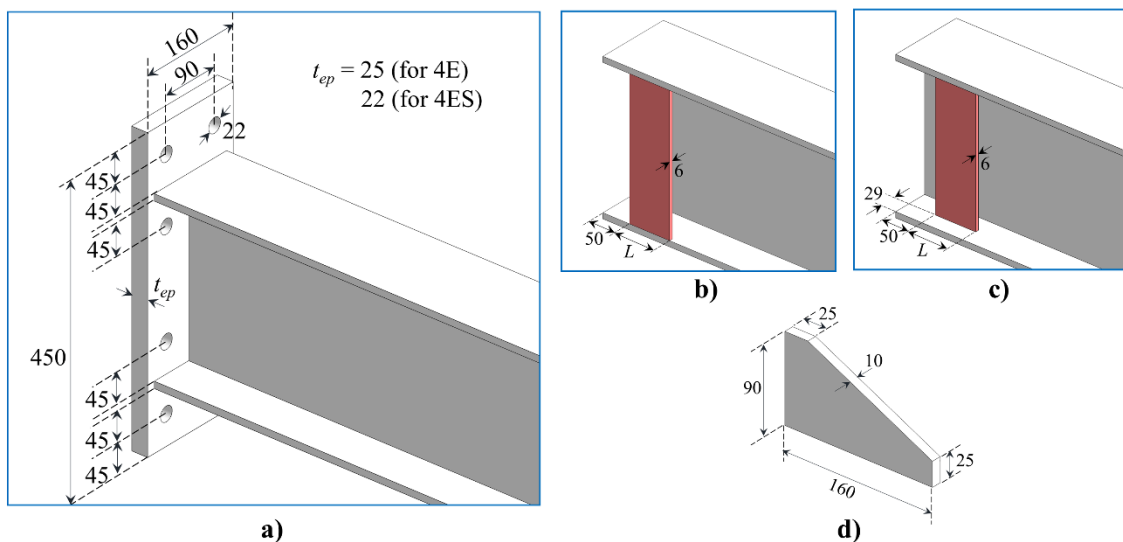
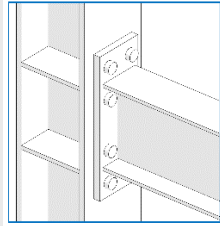
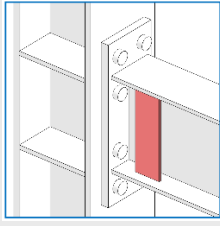
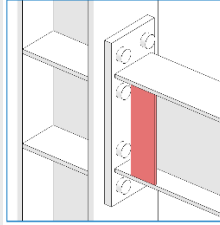
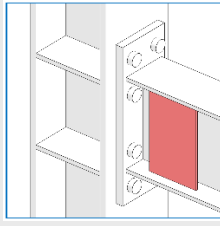
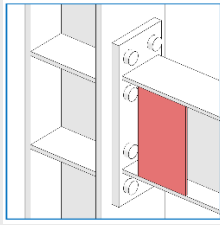
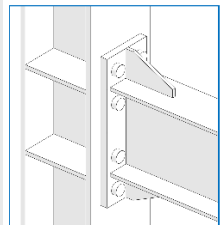


Fig. 2. The dimensions of the beam-to-column connections: (a) end plate dimensions and bolt-hole distances; (b) longitudinal plate dimensions for 4ELP75-O and 4ELP150-O; (c) longitudinal plate dimensions for 4ELP75-N and 4ELP150-N; (d) end plate stiffener dimensions for 4ES.

Table 1. The modeled and analyzed connection configurations.

Model name	Explanation	Geometric view
4E	Four-bolt unstiffened extended end plate connection.	
4ELP75-N	Model strengthened with a 75 mm longitudinal plate placed parallel to the beam web and welded to the middle portions of the beam flanges.	
4ELP75-O	Model strengthened with a 75 mm longitudinal plate placed parallel to the beam web and welded to the end portions of the beam flanges.	
4ELP150-N	Model strengthened with a 150 mm longitudinal plate placed parallel to the beam web and welded to the middle portions of the beam flanges.	
4ELP150-O	Model strengthened with a 150 mm longitudinal plate placed parallel to the beam web and welded to the end portions of the beam flanges.	
4ES	Four-bolt stiffened extended end plate connection.	

2.2. Loading protocol

The cyclic response of the connection models was investigated using a quasi-static, displacement-controlled loading protocol applied at the beam loading point, as illustrated in Fig. 3. The cyclic displacement history was defined according to the AISC 341-16 seismic loading protocol (AISC 2016), and loading was continued up to a maximum story drift angle of 0.06 rad, corresponding to approximately 6% drift.

The analysis was conducted in two sequential steps. Initially, a bolt pretension force of 172 kN was applied to the M20 high-strength bolts, in compliance with the minimum pretension requirements specified in ANSI/AISC 360-22 (AISC 2022b). Subsequently, reversed cyclic vertical displacements were imposed at the beam loading point. This approach allowed the combined effects of bolt pretension and cyclic deformation demands to be captured throughout the nonlinear response of the connection.

The loading history was defined in terms of the story drift angle, θ (rad), which is calculated using Eq. (1).

$$\theta = \frac{\Delta_{CL}}{L_{CL}} \quad (1)$$

where Δ_{CL} represents the vertical displacement at the beam loading point, and L_{CL} denotes the distance from the loading point to the column centerline. Therefore, the target displacement values for the analyses were determined by multiplying the prescribed drift angle by L_{CL} .

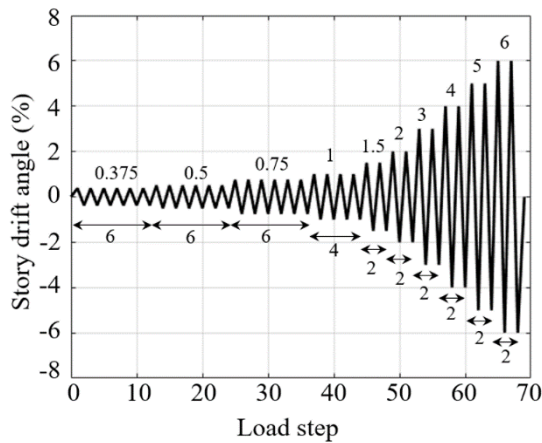


Fig. 3. Cyclic loading history.

2.3. Material properties

In the finite element models, material nonlinearity was addressed by implementing the von Mises yield criterion together with a kinematic hardening rule. The steel components were modeled using multi-linear true stress (σ_{true})-true strain (ϵ_{true}) relationships to accurately represent the inelastic behavior of beam-to-column connections subjected to cyclic loading. The material properties and corresponding true stress-strain values, as detailed in Table 2, were taken from the study of Gerami et al. (2011). In Table 2, for each material, the first point corresponds to the onset of yielding and the final point to the ultimate stress, while the intermediate points define the strain-hardening response. The elastic modulus (E) of each steel was obtained as the initial slope of its true stress-strain curve in Table 2, giving values between 2.03×10^5 and 2.06×10^5 MPa. The multilinear kinematic hardening model in Ansys was defined using the plastic strain (ϵ_{pl}), listed in the last column of Table 2, obtained as $\epsilon_{pl} = \epsilon_{true} - \sigma_{true}/E$. The first point of each curve corresponds to zero plastic strain at the yield stress. A constant Poisson's ratio of 0.3 was used for all materials. This modeling approach allowed the simulations to capture the onset of yielding, subsequent strain hardening, and the development of inelastic regions within the connection zone during cyclic loading.

Table 2. True stress, true total strain, and plastic strain values used in the multilinear kinematic-hardening material model (Gerami et al. 2011).

Material	Component	True stress (MPa)	True total strain	Plastic strain
ASTM A490	Bolt	794	0.003860	0
		1035	0.013500	0.008468
		1035	0.030900	0.025868
		1048	0.200000	0.194905
ASTM A36	End plate	262	0.001276	0
		262	0.014030	0.012754
		476	0.153000	0.150682
ASTM A572 Grade 50	All other components	361	0.001780	0
		361	0.019600	0.017820
		488	0.213400	0.210994

2.4. Element type and mesh

All steel components of the connection, including the beam, column, end plate, stiffener plate, longitudinal plates, and bolts, were discretized using 20-node three-dimensional solid elements (SOLID186). This element type was selected due to its capability to accurately capture material nonlinearity, large deformations, and complex stress distributions in beam-to-column connections. A structured meshing approach was implemented, with mesh refinement applied in regions anticipated to experience high stress gradients and localized inelastic deformations, such as the end plate, bolt region, beam flange adjacent to the connection, panel zone, stiffener

plate, and longitudinal plates. In the beam and column regions distant from the joint, where stress variation is less significant, a coarser mesh was utilized to improve computational efficiency without compromising accuracy. The finite element mesh configuration used in the analyses is presented in Fig. 4.

2.5. Contacts

In the finite element models, the interactions between connected components were represented by contact pairs, with the contact behavior defined as either linear-bonded or nonlinear-frictional according to the anticipated interface response. The corresponding con-

tact element formulations were generated automatically by the program according to the geometry and meshing of each contact region, comprising surface-to-surface contact elements (CONTA174) and target elements (TARGE170). The surface-to-surface contact formulation is appropriate for bolted end plate connections, as it represents the separation, finite sliding, and frictional contact-pressure transfer that occur at the end plate, column flange, and bolt interfaces. Nonlinear-frictional contact pairs were defined between the end plate

and column flange, end plate and bolt shank, column flange and bolt shank, bolt head and end plate, and nut and column flange. For all frictional interfaces, a coefficient of friction of 0.4 was adopted, representing an average value between the 0.30 (Class A) and 0.50 (Class B) coefficients specified in AISC 360-22 (AISC 2022b). All other interfaces were modeled as linear bonded contacts to prevent relative displacement and to maintain full compatibility in both normal and tangential directions.

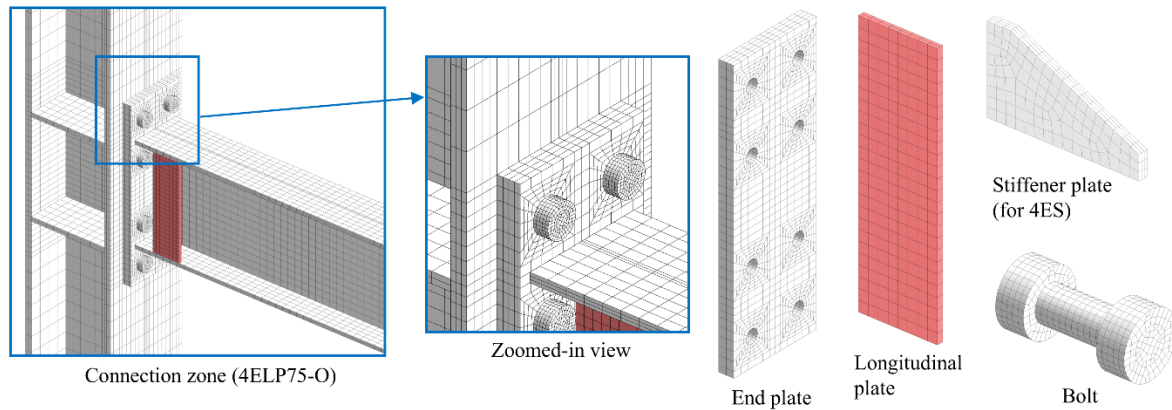


Fig. 4. Mesh of the models.

2.6. Evaluation of the response quantities

The response quantities were obtained from the FE analysis results by post-processing as described below.

2.6.1. Moment

The moment was obtained by multiplying the reaction force at the beam loading point by the distance from the loading point to the column face.

2.6.2. Story drift angle

The story drift angle was obtained from the vertical displacement measured at the beam loading point, divided by the distance from the loading point to the column centerline, in accordance with the definition given in Eq. (1) in Section 2.2.

2.6.3. Dissipated energy

The dissipated energy in the models was determined from the strain energy results of the analysis, evaluated separately for each component, including the beam, column, end plate, continuity plates, bolts, longitudinal plates, and stiffener. For each component, the strain energy was summed over all elements, and the total dissipated energy was calculated as the sum of these component contributions. The strain energy comprises both a recoverable elastic part and the energy dissipated through plastic deformation; at the drift levels examined, the response was dominated by plastic deformation, so that the strain energy predominantly reflects the dissipated energy. To confirm the strain-energy-based values, they were independently checked against the work

done by the applied force, obtained by trapezoidal integration of the force–displacement response, and close agreement was obtained, with differences of approximately 1.5% or less. The accumulated strain energy increased monotonically with ongoing plastic deformation. The cumulative dissipated energy at a specific drift level was obtained at the end of the final cycle for that amplitude, with repeated cycles incorporated throughout the loading history.

2.6.4. Bolt force

The axial force in the bolts was determined by measuring the reaction force at the center of each monitored bolt along the axis of the bolt. Two representative bolts were selected for monitoring: Bolt-1, located above the beam flange, and Bolt-2, located between the flanges. It should be noted that, due to the initial pretension applied to the bolts, the axial force in each bolt remained positive throughout the entire loading history. Therefore, the recorded force histories indicate cyclic variations around the pretension force rather than full tension–compression cycles.

2.6.5. Equivalent viscous damping ratio

The energy dissipation efficiency was evaluated using the hysteretic equivalent viscous damping ratio (ξ_{hyst}), computed for each drift amplitude from the moment–story drift response according to Eq. (2):

$$\xi_{hyst} = \frac{E_D}{4\pi E_S} = \frac{E_D}{2\pi M_{max}\theta_{max}} \quad (2)$$

Here, E_D is the energy dissipated in a full cycle, E_S is the corresponding stored elastic energy taken as

$\frac{1}{2}M_{\max}\theta_{\max}$, $\theta_{\max}$ is the peak story drift angle of the cycle, and $M_{\max}$ is the moment corresponding to $\theta_{\max}$. The damping ratio was evaluated only for the 3–6% drift levels, each of which consists of two cycles in the AISC protocol and was computed for the second of these two cycles. The dissipated energy E_D was obtained directly from the total strain energy of the complete model, summed over all components of the connection, as its increment over this cycle, taken between two instants one full cycle apart at the same peak-displacement state, so that the recoverable elastic strain energy canceled and only the energy dissipated during the cycle remained. The peak drift $\theta_{\max}$ and the corresponding moment $M_{\max}$ were evaluated at the same instant, taken as the final peak of the selected cycle. Because the applied displacement protocol was symmetric, the positive and negative peaks of a cycle corresponded to the same drift amplitude. Although a minor asymmetry developed between the positive and negative peaks at large drift amplitudes, the same peak was used consistently for all models, so that $M_{\max}$ represents the moment attained at that peak.

2.7. Finite element model validation

To verify the reliability of the developed finite element modeling approach, an additional validation was conducted using the experimentally tested EEP 2 specimen as reported by Morrison and Suarez (2022). In the present validation model, all geometric details, material properties, loading protocol, bolt pretension, and boundary conditions were defined to match those of the reference experimental study (Morrison and Suarez 2022). The EEP 2 specimen comprised an unstiffened eight-bolt extended end plate connection, utilizing a W690×140 beam, a W360×382 column, a 38.1 mm thick

end plate, and 31.8 mm diameter ASTM Grade A490 bolts, each pretensioned to 340 kN. Both the beam and column were fabricated from ASTM A992 steel, while the end plate was specified as ASTM A572 Grade 50. The AISC 341-16 loading protocol was applied at the beam tip. The finite element mesh for the EEP 2 specimen is illustrated in Fig. 5.

Fig. 6 presents a comparison of the experimental and numerical moment–story drift responses for the EEP 2 specimen. The finite element model reasonably reproduces the global cyclic behavior, capturing the initial stiffness, the overall shape of the hysteresis loops, the peak moment capacity, and the progressive strength degradation at increased drift amplitudes. While minor discrepancies are observed in certain loading branches, particularly at higher drift levels, the overall correspondence between the experimental and numerical results confirms that the adopted modeling assumptions yield a realistic representation of the connection response.

A qualitative comparison of the damage patterns is provided in Fig. 7, where the experimentally observed failure mode is evaluated against the equivalent plastic strain distribution predicted by the finite element model. Experimentally, yielding initiated in the beam flange and web adjacent to the connection, progressing to local buckling within the beam plastic hinge region, while the column remained essentially elastic throughout the loading protocol. The numerical model successfully captures this sequence, predicting concentrated inelastic deformation and local instability at the beam end. Although there are minor differences in the precise extent and location of local buckling, the overall deformation pattern and the beam-dominated failure mechanism are represented accurately.

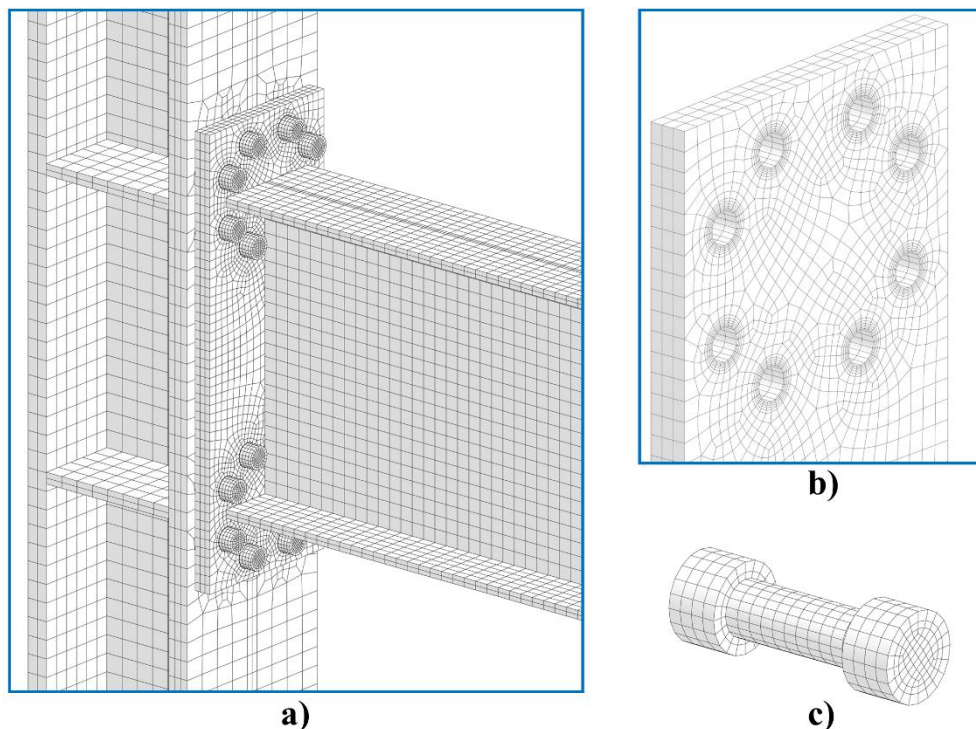


Fig. 5. The finite element mesh of the EEP 2 specimen: (a) Connection region; (b) End plate; (c) Bolt.

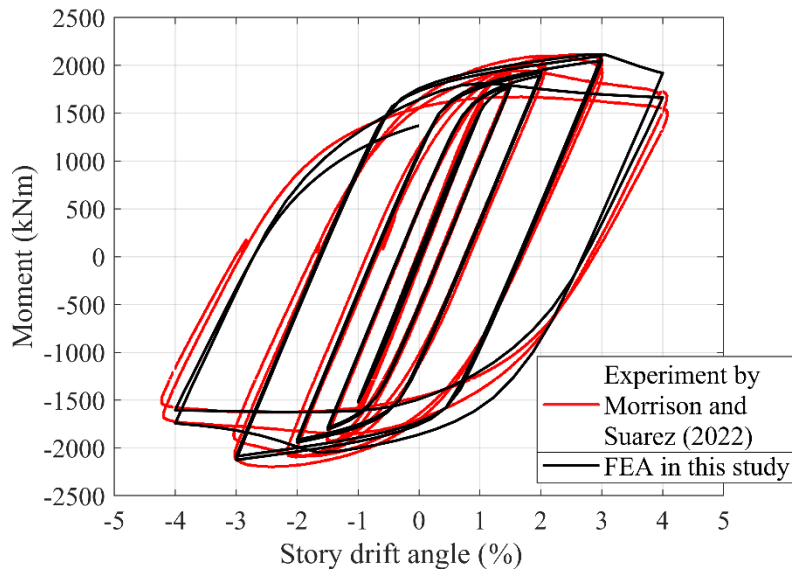


Fig. 6. Comparison of experimental (Morrison and Suarez 2022) and numerical moment-story drift hysteresis curves for the EEP 2 specimen.

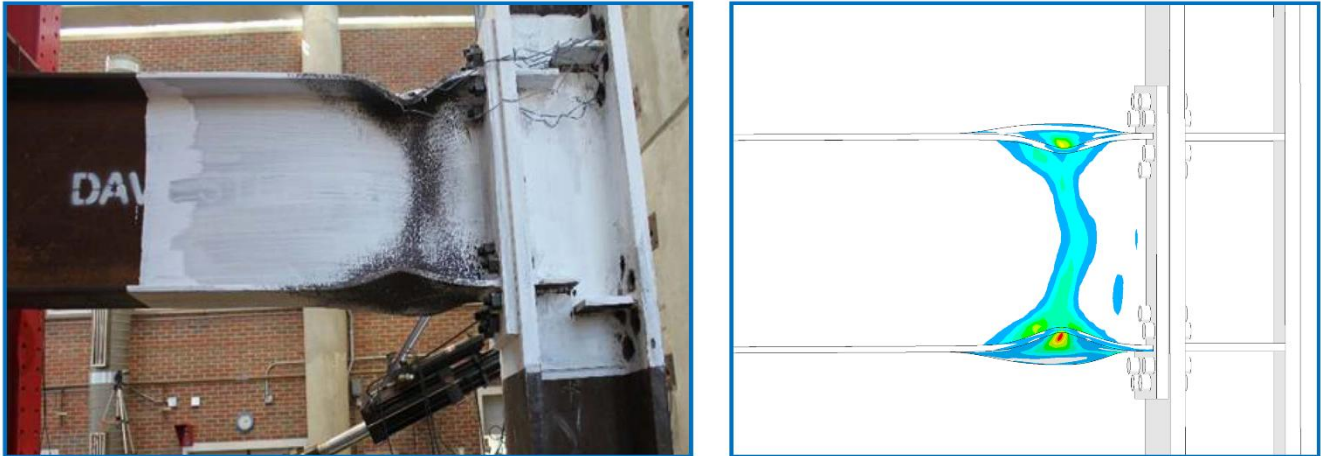


Fig. 7. Comparison of the experimentally observed damage pattern (Morrison and Suarez 2022) and the equivalent plastic strain distribution predicted by the finite element model for the EEP 2 specimen.

The validation results demonstrate that the adopted finite element model is capable of reproducing both the global cyclic response and the local deformation characteristics of the benchmark specimen with acceptable accuracy. Based on these findings, the model was considered appropriate for use in the subsequent parametric investigation of the proposed longitudinal-plate-strengthened end plate connections.

3. Results and Discussion

3.1. Global hysteretic response of the connections

The global cyclic behavior of the beam-to-column connection models was assessed based on the moment-story drift hysteresis curves obtained from nonlinear finite element analyses, as presented in Fig. 8. The comparison includes the unstiffened reference model (4E), four models strengthened with longitudinal plates, and the stiffened model (4ES).

According to ANSI/AISC 341-16, beam-to-column connections in special moment frames (SMFs) are required to sustain a story drift angle of at least 0.04 rad, while maintaining a flexural resistance at the column face of no less than $0.80M_p$ of the connected beam. As shown in Fig. 8, all connections evaluated in this numerical study maintained a flexural moment greater than $0.80M_p$ at a story drift angle of 0.04 rad. Therefore, within the scope of the numerical assessment, the models satisfied the ANSI/AISC 341-16 strength-rotation acceptance limit considered in this study.

The corresponding peak moment values and normalized capacities for all models are summarized in Table 3. The longitudinal-plate models attained higher peak moments than the reference model 4E, with increases ranging from approximately 1.31% to 5.63%, while the stiffened model 4ES reached an increase of approximately 4.49% over 4E.

As illustrated in Fig. 8(a), the reference model 4E exhibits stable and nearly symmetric hysteresis loops at low and moderate drift amplitudes. At larger drift ampli-

tudes, its moment capacity decreases after reaching the peak, approaching $0.80M_p$ at 6% drift, although the connection still satisfies the $0.80M_p$ requirement at 4% drift specified for SMFs. Among the six models, 4E develops the lowest moment capacity.

The addition of longitudinal plates results in a notable improvement in the global hysteretic response. As

shown in Figs. 8(b-c), both 4ELP75-N and 4ELP75-O develop higher moment capacities than the reference model 4E across nearly the entire drift range, while maintaining stable and full hysteresis loops up to large drift amplitudes. Between these two configurations, 4ELP75-N sustains slightly higher moment resistance at higher drift amplitudes.

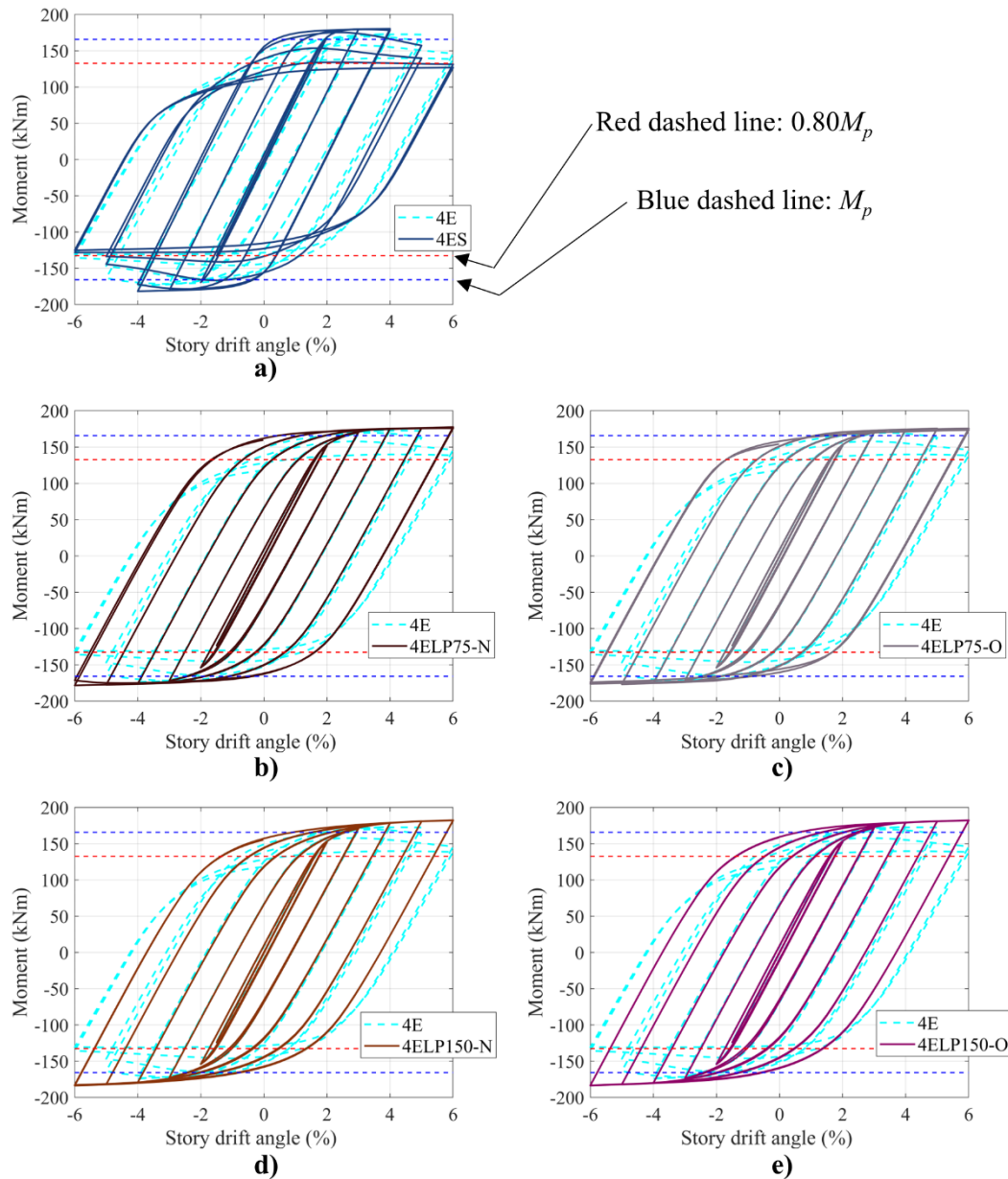


Fig. 8. Comparison of moment–story drift hysteresis responses: (a) 4E and 4ES; (b) 4E and 4ELP75-N; (c) 4E and 4ELP75-O; (d) 4E and 4ELP150-N; (e) 4E and 4ELP150-O.

Table 3. Summary of the peak flexural response of the connection models.

Model	$M_{\max}$ (kNm)	$M_{\max}/M_p$	M/M_p at 4% drift	Increase in $M_{\max}$ relative to 4E (%)
4E	173.81	1.045	1.045	-
4ELP75-N	178.25	1.072	1.060	2.55
4ELP75-O	176.08	1.060	1.053	1.31
4ELP150-N	183.60	1.104	1.079	5.63
4ELP150-O	183.54	1.104	1.082	5.60
4ES	181.62	1.092	1.092	4.49

Further improvement is achieved by increasing the longitudinal plate length to 150 mm. As presented in Fig. 8(d-e), both 4ELP150-N and 4ELP150-O display the most stable hysteresis loops and the highest moment capacities among the longitudinal-plate-strengthened models. Comparison of the 75 mm and 150 mm plate configurations indicates that plate length has a greater effect on the hysteretic response than plate location. The difference between the N and O placements is relatively minor, with the N configuration performing slightly better for the 75 mm plates, while both 150 mm configurations show similar behavior.

The stiffened model 4ES, as shown in Fig. 8(a), also achieves a significant increase in strength compared to 4E. However, its hysteretic response differs from that of the longitudinal-plate-strengthened models. Although 4ES attains high moment levels at moderate drift amplitudes, loop distortion and response asymmetry are observed at larger drift cycles. The deterioration in loop stability is more evident than in the 150 mm longitudinal plate configurations. These results indicate that while the conventional stiffener is effective in increasing connection strength, it does not provide the same level of cyclic stability as longitudinal plate strengthening under severe drift demands.

In summary, the results presented in Fig. 8 show that both strengthening strategies enhance the hysteretic performance of the unstiffened reference connection. The longitudinal plate configurations, in particular, provide a more balanced improvement in strength and loop stability. The 150 mm longitudinal plates yield the most

favorable global response, indicating that increasing plate length is an effective approach for improving the seismic performance of bolted extended end plate connections.

3.2. Energy dissipation capacity

The cumulative dissipated energy of the connection models at story drift angles ranging from 3% to 6% is presented in Fig. 9. In this assessment, the individual contributions of the beam, column, end plate, continuity plates, bolts, longitudinal plates, and stiffener were quantified to determine the distribution of inelastic demand within the connection region. The contributions of the beam, column, and end plate, which together accounted for nearly the entire dissipated energy, are summarized in Table 4 for the 4% and 6% drift levels.

At 4% drift, the total dissipated energy of the longitudinal-plate models was comparable to that of the reference model 4E (34.95 kNm), ranging from 33.76 to 34.75 kNm, while the stiffened model 4ES reached a higher value of 41.09 kNm. At 6% drift, the total dissipated energy of all configurations remained within a narrow range of 114.53 to 119.58 kNm. The longitudinal-plate models exceeded the reference model 4E by approximately 2.2% to 4.4%, and the stiffened model 4ES by approximately 4.0%, confirming that the total energy dissipation was comparable across all configurations and that the principal effect of the strengthening details was a change in the distribution of the dissipated energy rather than a significant increase in its total magnitude.

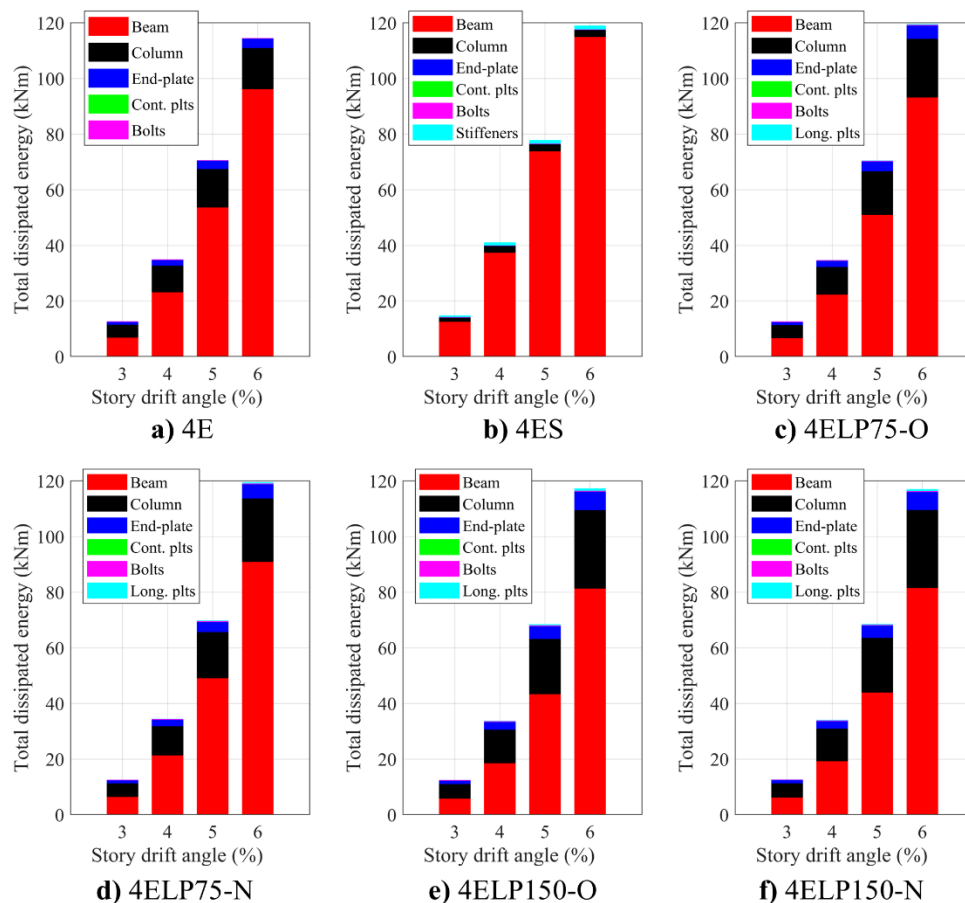


Fig. 9. Comparison of cumulative dissipated energy components versus story drift angle.

Table 4. Summary of the energy dissipation of the connection models.

Model	Total dissipated energy at 4% drift (kNm)	Total dissipated energy at 6% drift (kNm)	Energy dissipation contributions at 4% drift (%)			Energy dissipation contributions at 6% drift (%)		
			Beam	Column	End plate	Beam	Column	End plate
4E	34.95	114.53	66.2	27.3	5.8	83.9	13.0	3.0
4ELP75-N	34.47	119.58	61.9	30.5	6.6	76.0	19.1	4.2
4ELP75-O	34.75	119.57	64.1	28.7	6.3	77.9	17.6	4.0
4ELP150-N	34.03	117.04	56.5	34.6	7.6	69.6	24.0	5.5
4ELP150-O	33.76	117.34	54.8	35.8	8.1	69.3	24.0	5.7
4ES	41.09	119.08	90.8	5.9	0.2	96.5	2.1	0.1

In the reference model 4E, the dissipated energy at 6% drift was concentrated in the beam, which accounted for 83.9% of the total, while the column and end plate contributed 13.0% and 3.0%, respectively. The addition of longitudinal plates progressively shifted the inelastic demand from the beam toward the column and end plate, and this shift became more pronounced as the plate length increased. For the 75 mm models, the beam share decreased to 76.0% (4ELP75-N) and 77.9% (4ELP75-O), with the column contribution rising to 19.1% and 17.6%, respectively. For the 150 mm models, the beam share decreased further to 69.6% (4ELP150-N) and 69.3% (4ELP150-O), while the column contribution increased to 24.0% in both cases and the end plate contribution to 5.5% and 5.7%. The effect of plate location was minor and diminished with plate length, as the two 150 mm configurations exhibited nearly identical distributions. It is also noted that the direct energy contribution of the longitudinal plates themselves remained negligible; their role was therefore to redistribute the inelastic demand toward the column and end plate rather than to dissipate a significant portion of the energy directly.

In contrast, the stiffened model 4ES exhibited the opposite trend, with the dissipated energy becoming even more concentrated in the beam, which accounted for 96.5% of the total at 6% drift, while the column and end plate contributions reduced to 2.1% and 0.1%, respectively. As with the longitudinal plates, the direct energy contribution of the stiffener itself was negligible. This indicates that the conventional stiffener enhances the dissipation capacity primarily by promoting plastic deformation in the beam, whereas the longitudinal plates achieve a more distributed inelastic response by engaging the column and end plate. For all models, the beam share increased between 4% and 6% drift, reflecting the increasing dominance of beam plasticity at larger drift demands. According to the strong-column weak-beam principle, energy dissipation in moment-resisting frames should be concentrated in the beam to maintain the intended seismic hierarchy; therefore, the redistribution observed in the longitudinal-plate models, which was most pronounced for the 150 mm configurations with the column share increasing from 13.0% in 4E to 24.0%, underlines the importance of verifying through component capacity checks that the column and end plate demands remain within acceptable limits.

3.3. Distribution of bolt forces

The distribution of axial force within the tension bolt group was assessed based on the bolt force versus story drift relationships for two representative bolts, as illustrated in Fig. 10.

In the reference model (4E), both bolts display stable cyclic force histories, with axial force increasing as the bolt group is engaged in tension. At higher drift amplitudes, Bolt-1 develops slightly greater axial force than Bolt-2. During load reversal, both bolts unload toward the pretension level. The axial force in Bolt-1 increased by approximately 72 kN and approximately 64 kN in Bolt-2, relative to the initial pretension of 172 kN.

For the 75 mm longitudinal-plate models (4ELP75-O and 4ELP75-N), a modest increase in peak bolt force is observed relative to the reference model 4E. In both configurations, Bolt-1 continues to attract slightly higher force than Bolt-2, and the cyclic force histories remain smooth and stable. The peak force in Bolt-1 increased by approximately 78 kN (4ELP75-N) and 76 kN (4ELP75-O), corresponding to increases of approximately 6–8% over 4E. The difference between the O and N plate placements is minimal, and no significant variation in bolt force distribution is evident at this plate length.

A more pronounced change is evident in the 150 mm longitudinal-plate models (4ELP150-O and 4ELP150-N). In these cases, the peak axial forces exceed those observed in both the reference model and the 75 mm plate configurations, particularly for Bolt-1. The force excursion during load reversal is also more significant, with Bolt-1 experiencing a greater reduction in axial force before increasing again under the opposite loading phase. The peak force in Bolt-1 increased by approximately 83 kN (for both 4ELP150-O and 4ELP150-N), representing increases of approximately 15% relative to 4E, accompanied by a wider cyclic force range. These findings indicate that increasing the plate length intensifies the engagement of the monitored bolts and accentuates the difference between the outer and inner bolts, rather than achieving a more uniform force distribution. The resulting increase in tensile demand on the bolts is a critical consideration for design. If the bolts are not adequately sized for this localized force increase, the failure mode may shift from ductile beam flexural yielding to brittle bolt fracture or thread stripping. The effect of plate placement remains secondary, as both 4ELP150-O and 4ELP150-N show similar trends.

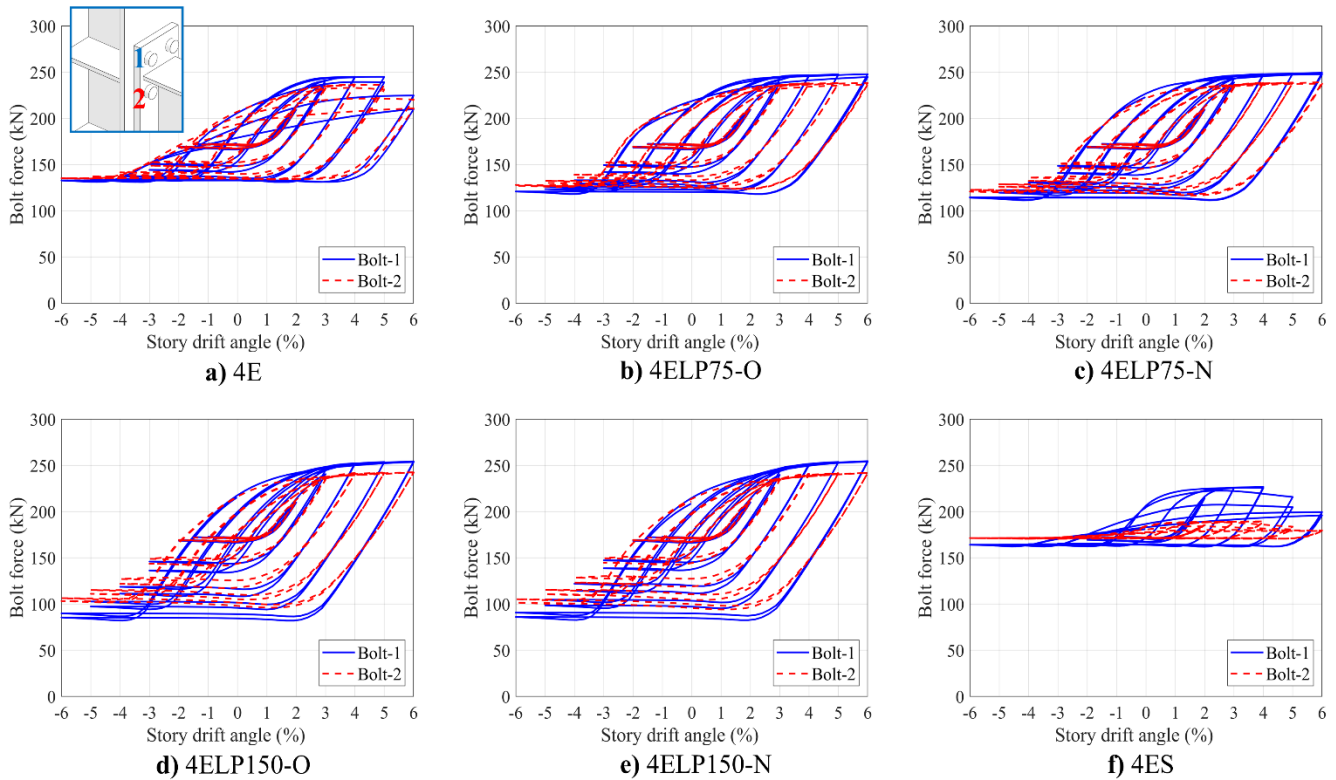


Fig. 10. Variation of bolt axial forces with story drift angle under cyclic loading for different connection configurations: (a) 4E; (b) 4ELP75-O; (c) 4ELP75-N; (d) 4ELP150-O; (e) 4ELP150-N; (f) 4ES.

The stiffened model (4ES) demonstrates a distinctly different bolt force response. The variation in Bolt-2 remains limited throughout the loading history, and the increase in Bolt-1 is also substantially smaller than in the longitudinal-plate-strengthened models. In contrast, the stiffened model 4ES exhibited the lowest bolt demand, with the peak force in Bolt-1 increasing by approximately 55 kN, which is approximately 24% lower than the corresponding increase in 4E. As a result, the bolt force loops are considerably narrower compared to those of 4E and the longitudinal-plate configurations. This behavior indicates that the stiffener modifies the internal force-transfer mechanism and reduces the tensile demand on the monitored bolt pair, especially on the inner bolt.

Fig. 10 shows that longitudinal plate strengthening, particularly with a plate length of 150 mm, increases the tensile demand on the monitored bolts and accentuates the difference between Bolt-1 and Bolt-2. In contrast, the stiffened configuration (4ES) limits the cyclic variation of bolt forces and maintains a lower level of bolt demand. These results demonstrate that longitudinal plate and stiffener strengthening strategies enhance connection behavior through different internal mechanisms.

3.4. Potential failure mechanisms

The potential failure mechanisms of the connection models were evaluated based on equivalent plastic strain (von Mises) contours at the end of the cyclic analyses, as shown in Fig. 11. These contours indicate the intensity and distribution of inelastic deformation in the joint region, allowing assessment of how strengthening details influence the governing failure mode.

In the reference model (4E), shown in Fig. 11(a), plastic strain is concentrated in the beam flange and web near the end plate. The highest strain occurs close to the connection face, indicating that inelastic deformation is localized within a short region adjacent to the end plate. This pattern corresponds to the formation of a plastic hinge at or near the beam-to-column interface.

The longitudinal-plate-strengthened models show a different deformation pattern. In both 75 mm configurations (4ELP75-O and 4ELP75-N), plastic strain near the connection face is reduced compared to 4E, and inelastic deformation is distributed over a wider area of the beam flange and web. Some plastic strain develops in the longitudinal plates, mainly near their ends, indicating their role in redistributing internal forces. Among the 75 mm models, 4ELP75-N has a lower maximum plastic strain than 4ELP75-O.

A similar but more favorable trend is seen in the 150 mm longitudinal-plate models. As shown in Fig. 11(d) and (e), both 4ELP150-O and 4ELP150-N have lower peak plastic strain than the reference model (4E) and show a more distributed inelastic response along the beam flange. The concentration of plastic deformation near the end plate is less severe, and the yielded zone extends farther from the column face. Increasing the plate length improves redistribution of inelastic demand and reduces strain localization at the beam end. Among all longitudinal-plate configurations, 4ELP150-O has the lowest maximum plastic strain, while 4ELP150-N is similar to 4ELP75-N. The results indicate that both plate length and placement influence strain redistribution, and optimal placement depends on plate length.

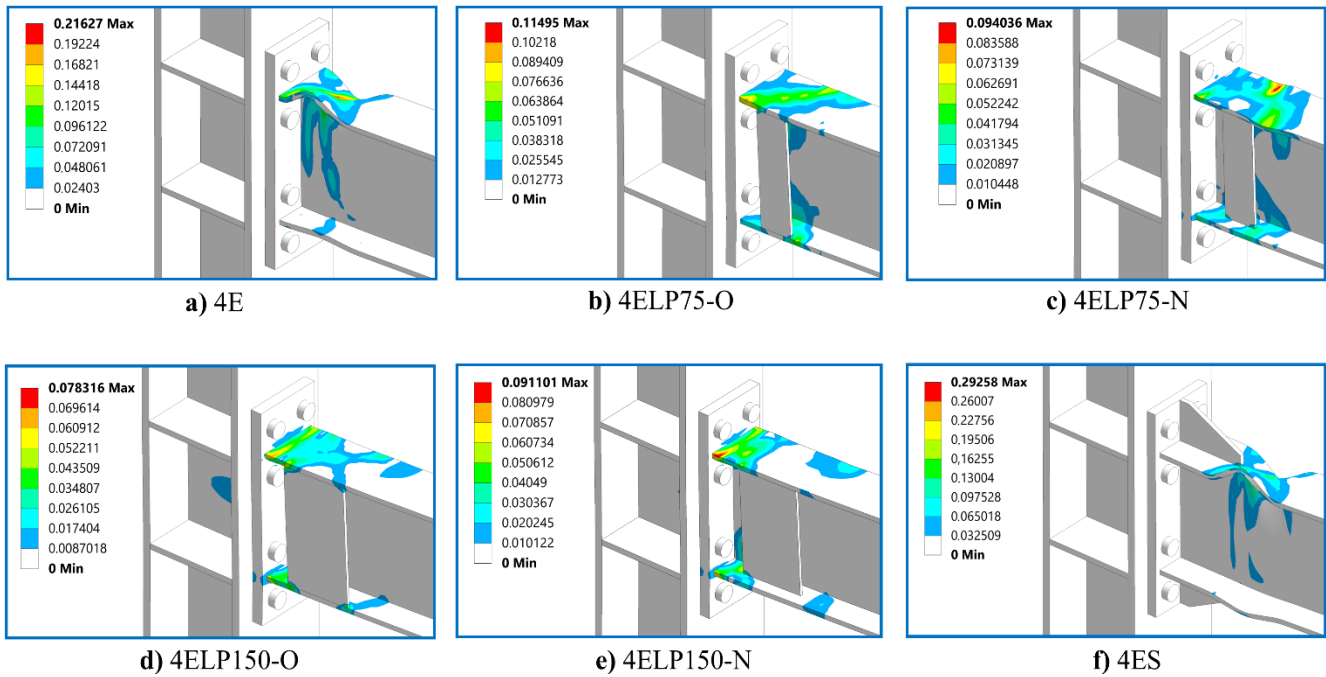


Fig. 11. Plastic strain distribution and dominant deformation patterns in the beam-column connection region for different configurations: (a) 4E; (b) 4ELP75-O; (c) 4ELP75-N; (d) 4ELP150-O; (e) 4ELP150-N; (f) 4ES.

The stiffened model (4ES), shown in Fig. 11(f), has the most severe strain concentration among all models. The highest equivalent plastic strain occurs in the beam flange near the stiffener-beam junction and in the adjacent beam web. This shows that the conventional stiffener moves the plastic hinge away from the column face and weld root, but also increases strain localization at the beam end. Conversely, it is important to ensure that the longitudinal plates do not over-strengthen the beam section, which could shift yielding into the connection elements or the column panel zone.

The maximum equivalent plastic strain in the reference model 4E was found to be approximately 0.216. For the models with 75 mm longitudinal plates, the maximum equivalent plastic strain decreased to approximately 0.115 for 4ELP75-O and 0.094 for 4ELP75-N, which corresponds to reductions of about 47% and 56% compared to 4E. The models with 150 mm longitudinal plates showed the lowest maximum plastic strain values, with approximately 0.078 for 4ELP150-O and 0.091 for 4ELP150-N, representing reductions of about 64% and 58% compared to 4E. On the other hand, the stiffened model 4ES exhibited the highest maximum equivalent plastic strain, reaching approximately 0.293, which is an increase of about 36% relative to 4E. This result is consistent with the most pronounced strain localization observed among all the configurations.

Overall, Fig. 11 shows that longitudinal plate strengthening reduces peak plastic strain in the beam-end region and promotes a more distributed inelastic deformation pattern, especially in the 150 mm configurations. In contrast, the stiffened configuration (4ES) results in the highest local strain demand and the most concentrated failure mechanism. These results indicate that longitudinal plates are more effective than the conventional stiffener in reducing severe strain localization at the connection region.

3.5. Hysteretic equivalent viscous damping ratio

Variation in the hysteretic equivalent viscous damping ratio (ξ_{hyst}) with story drift angle for all six models is presented in Fig. 12.

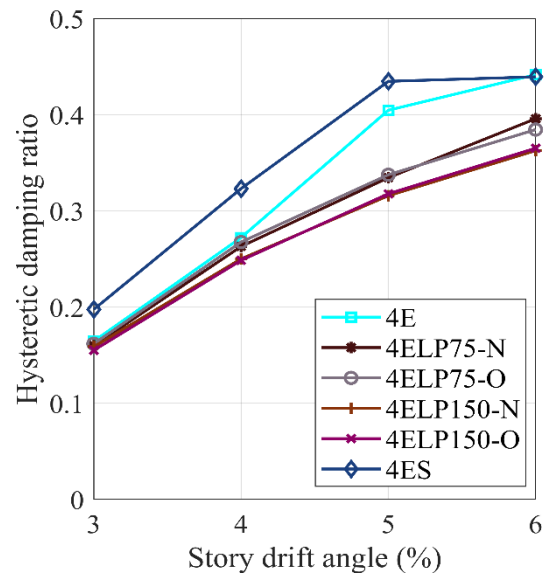


Fig. 12. Variation in the hysteretic equivalent viscous damping ratio with story drift angle for all six models

A consistent increase in ξ_{hyst} was observed for all models as drift demand increased, indicating a greater contribution of inelastic deformation to energy dissipation at higher drift levels. At a drift of 3%, the damping ratios for all configurations were nearly identical, ranging from 0.155 to 0.198. With increasing drift, the stiffened model 4ES demonstrated the highest damping ratio up to 5% drift, reaching 0.323 at 4% and 0.435 at 5%. At 6% drift, both the reference model 4E and the stiffened model 4ES

achieved comparable maximum values of 0.442 and 0.440, respectively. The longitudinal-plate models showed lower damping ratios, and this difference became more significant at larger drift amplitudes. At 6% drift, the 75 mm longitudinal-plate models reached 0.396 (4ELP75-N) and 0.385 (4ELP75-O), while the 150 mm models reached 0.363 (4ELP150-N) and 0.365 (4ELP150-O). For the longitudinal-plate models, increasing the plate length resulted in a slight reduction in damping ratio, whereas the effect of plate location was negligible.

The lower damping ratios observed in the longitudinal-plate models are primarily due to their higher moment capacities, rather than a decrease in total energy dissipation. As indicated in Eq. (2), ξ_{hyst} is normalized by the moment corresponding to θ_{max} . The longitudinal-plate models achieved higher moment capacities (see Section 3.1) while dissipating a similar amount of total energy (see Section 3.2), resulting in lower normalized damping ratios. This outcome is consistent with the more distributed and less concentrated plastic deformation observed in these models (Section 3.4), where higher moments were attained with reduced strain localization at the beam end. Therefore, the lower damping ratios in the longitudinal-plate models reflect their increased moment capacity, not a deficiency in hysteretic performance, since ξ_{hyst} represents the dissipated energy relative to the attained moment. In contrast, the stiffened model 4ES reached its high damping ratio through concentrated and intense plastic deformation in the beam, which is consistent with the highest peak plastic strain among all models (Section 3.4). All configurations, including the longitudinal-plate models, maintained high equivalent viscous damping ratios above 0.36 at 6% drift, demonstrating substantial hysteretic energy dissipation in all cases.

3.6. Component capacity verification

The redistribution of inelastic demand toward the column and end plate regions, as identified in Section 3.2, was evaluated by performing component capacity checks to assess whether the intended yielding hierarchy and component capacity limits were maintained. These checks used the moment delivered to the column face, which was taken as the peak moment from each model as shown in Table 3, along with the panel zone shear forces and peak bolt forces obtained directly from the FE models.

The strong-column weak-beam requirement was evaluated in accordance with ANSI/AISC 341-16, which requires the ratio of the sum of the column plastic flexural strengths to the sum of the beam probable flexural strengths projected to the column centerline to exceed unity. The sum of the column plastic flexural strengths above and below the joint, evaluated as $2 \times Z_c F_y$ with the column axial force neglected, was 733.5 kNm. The sum of the beam flexural strengths projected to the column centerline was determined according to Eq. (3):

$$\sum M_{pb} = \sum (M_{pr} + \alpha_s M_v) \quad (3)$$

where M_{pr} is the probable maximum moment at the plastic hinge, α_s is the force level adjustment factor taken as 1.0 for

load and resistance factor design, and M_v is the additional moment due to shear amplification from the plastic hinge location to the column centerline. The probable beam flexural strength was determined according to Eq. (4):

$$M_{pr} = C_{pr} R_y F_y Z_b \quad (4)$$

Here, $C_{pr} = (F_y + F_u)/(2F_y) \leq 1.2$ is the peak connection strength factor, R_y is the ratio of expected to specified minimum yield strength, and Z_b is the plastic section modulus of the beam. For the ASTM A572 Grade 50 beam, C_{pr} was taken as 1.18 and R_y as 1.1, resulting in a probable beam moment (M_{pr}) of 215.0 kNm. With the shear amplification moment included, ΣM_{pb} is 233.5 kNm, giving a column-to-beam flexural strength ratio of 3.1 for all configurations. This result confirms that the column remained elastic in flexure, no column flexural hinge formed, and the redistribution of dissipated energy toward the column did not affect the intended yielding hierarchy.

The column energy contribution was associated with shear deformation of the panel zone. The panel zone shear force was obtained directly from the FE models as the resultant horizontal force on a section through the mid-height of the panel zone, and the resulting demand-to-capacity ratios are summarized in Table 5. The available panel zone shear strength of the HE240B column was determined according to Eq. (5):

$$R_v = 0.60 F_y d_c t_{wc} \left[1 + \frac{3 b_{fc} t_{fc}^2}{d_b d_c t_{wc}} \right] \quad (5)$$

where d_c and t_{wc} are the depth and web thickness of the column, b_{fc} and t_{fc} are the width and thickness of the column flange, and d_b is the beam depth. This formulation includes the contribution of the column flanges mobilized during inelastic deformation of the panel zone, resulting in an available strength of 687 kN. For all configurations, the panel zone shear demand remained within this strength, with demand-to-capacity ratios between 0.71 and 0.82. The panel zone shear force increased in line with the column energy contribution reported in Section 3.2, from 490 kN for the stiffened model (4ES), which had the lowest column energy share and ratio of 0.71, to 567 kN for the 150 mm configurations, which had the highest. This correspondence shows that the increased column energy participation in the longitudinal-plate models was due to limited panel zone shear yielding, which is an accepted ductile mechanism under cyclic loading and remained within the available panel zone strength.

Table 5. Panel zone shear demand obtained from the FE models and the corresponding demand-to-capacity ratios.

Model	V_u (kN)	V_u/R_v
4E	540.4	0.79
4ELP75-N	551.5	0.80
4ELP75-O	546.0	0.79
4ELP150-N	566.5	0.82
4ELP150-O	564.9	0.82
4ES	490.4	0.71

The end plate thickness was set at 25 mm according to the thick-plate design provisions of ANSI/AISC 358-22, which are intended to keep the end plate essentially elastic and to concentrate inelastic deformation in the beam. The analyses in Section 3.2 showed that the end plate energy contribution ranged from 3.0% to 5.7%, which aligns with this design approach and confirms that the end plate did not control the connection response.

Bolt tension demand was identified as the governing limit state, as summarized in Table 6. The bolt tensile rupture strength was calculated using Eq. (6):

$$R_n = A_b F_{nt} \quad (6)$$

where A_b is the nominal area of the bolt and F_{nt} is the nominal tensile stress, taken as $0.75F_{ub}$ in accordance with ANSI/AISC 360-22, with F_{ub} the ultimate tensile strength of the bolt material. For the M20 ASTM A490 bolts, A_b is 314 mm² and F_{nt} is 786 MPa, resulting in a tensile rupture strength of 247 kN. The peak axial force in the most heavily loaded bolt reached 0.99 of this strength in the reference model 4E, reached or slightly exceeded it in the longitudinal-plate models, with ratios of 1.00 to 1.01 for the 75 mm configurations and 1.03 for the 150 mm configurations, and remained below capacity at 0.92 for the stiffened model 4ES. These results indicate that the M20 ASTM A490 bolts are at or beyond their nominal tensile capacity in the longitudinal-plate configurations, and that a larger bolt diameter or higher grade may be required to develop the strengthened connections without bolt tensile rupture. As the finite element bolt model does not incorporate a fracture criterion, the analysis captures the bolt force demand but not the rupture itself.

Table 6. Peak axial force in the most heavily loaded tension bolt (Bolt-1) and the corresponding demand-to-capacity ratios.

Model	T_u (kN)	T_u/R_n
4E	244	0.99
4ELP75-N	250	1.01
4ELP75-O	248	1.00
4ELP150-N	255	1.03
4ELP150-O	255	1.03
4ES	227	0.92

4. Conclusions

In this study, the cyclic behavior of bolted extended end plate beam-to-column connections strengthened with longitudinal plates parallel to the beam web was investigated through numerical simulation. Six representative connection models were established and analyzed under the AISC cyclic loading protocol. These included an unstiffened reference connection (4E), a stiffened connection (4ES), and four configurations incorporating longitudinal plates. The main conclusions can be summarized as follows:

- The unstiffened reference model (4E) demonstrated stable and nearly symmetric hysteretic behavior at low and moderate drift levels. At larger drift amplitudes, however, it exhibited strength degradation, with the flexural resistance approaching $0.80M_p$ at 6% drift. Among the six models, 4E developed the lowest moment capacity, with its inelastic deformation concentrated in the beam region adjacent to the connection.
- The addition of longitudinal plates enhanced the overall cyclic performance of the connections. All models strengthened with longitudinal plates developed higher moment capacities and exhibited less strength degradation at large drift amplitudes than the reference model (4E).
- Among the parameters investigated, plate length was found to have a more significant effect on connection response than plate location. The configurations with 150 mm longitudinal plates exhibited the most favorable hysteretic behavior, minimal strength degradation, and the most distributed inelastic deformation patterns among the strengthened models.
- The cumulative energy dissipated by the longitudinal-plate-strengthened models was generally comparable to, and at large drift amplitudes slightly exceeded, that of the reference model (4E). The presence of longitudinal plates modified the distribution of dissipated energy by increasing the contribution of the column and end plate regions. The increased column contribution was associated with a higher panel zone shear demand, which component capacity checks confirmed remained within the available panel zone strength and corresponded to limited panel zone yielding.
- The longitudinal plate configurations, particularly those utilizing 150 mm plates, resulted in increased engagement of the monitored bolts and produced higher tensile forces in the bolts compared to the reference model (4E). As a result, the bolt tension demand reached or exceeded the nominal tensile capacity in these configurations, and a larger bolt diameter or higher grade may be required to prevent brittle bolt tensile rupture.
- Analysis of plastic strain contours indicated that the addition of longitudinal plates reduced peak strain demand in the beam-end region and facilitated a more distributed inelastic mechanism. This effect was most pronounced in the models with 150 mm plates. In contrast, the stiffened model (4ES) exhibited the highest local strain concentrations and the most localized failure pattern among all configurations analyzed.
- All configurations maintained high hysteretic equivalent viscous damping, with values above 0.36 at 6% drift. The longitudinal-plate models exhibited slightly lower damping ratios than the reference (4E) and stiffened (4ES) models, reflecting their higher moment capacity rather than a reduction in dissipated energy, since the damping ratio is normalized by the attained moment.

In summary, the findings indicate that longitudinal plates installed parallel to the beam web represent an effective strengthening alternative for bolted extended

end plate connections subjected to cyclic loading. Compared to the conventional stiffener detail (4ES), this approach achieves a more favorable balance between strength enhancement and distributed inelastic response, and does not introduce the upward projection characteristic of end plate stiffeners. More specifically, the longitudinal plates reduced the peak plastic strain in the beam-end region and maintained stable hysteretic behavior at large drift amplitudes, in contrast to the conventional stiffener, which produced the highest local strain concentration among all configurations.

5. Limitations and Future Work

Despite the comprehensive numerical findings presented in this study, several limitations should be acknowledged and addressed in future research.

- The conclusions of this study are derived primarily from three-dimensional nonlinear finite element simulations. Although the modeling approach was validated by comparison with the experimental response of the EEP 2 benchmark specimen, the specific longitudinal-plate configurations introduced in this research have not been subjected to dedicated physical testing. Therefore, experimental studies are required to verify the hysteretic behavior, bolt force development, and local failure mechanisms of the proposed details under cyclic loading conditions.
- The scope of the numerical investigation was restricted to a single beam-to-column configuration and a limited set of geometric parameters. The analyses were conducted using one beam section, one column section, a single end plate configuration, and two longitudinal plate lengths at two plate locations. As a result, the conclusions drawn from this study are applicable only within the boundaries of the investigated models. Further research should extend the parametric study to include a wider range of beam and column sizes, end plate thicknesses, bolt diameters, plate thicknesses, steel grades, and weld details to establish more comprehensive design-oriented trends.
- The loading protocol employed in this study was limited to the AISC quasi-static cyclic loading history. Although this protocol is suitable for assessing connection performance under standardized seismic qualification requirements, it does not capture all characteristics of earthquake loading. Specifically, the effects of loading rate, cumulative damage resulting from irregular loading histories, and record-to-record variability were not addressed. Additional research utilizing nonlinear time-history analysis or experimental dynamic loading is necessary to further evaluate the seismic robustness of the proposed strengthening approach.
- The present research was limited to the evaluation of connection-level behavior. The effects of longitudinal plate strengthening on the global response of steel moment-resisting frames, such as inter-story drift demand, force redistribution at the system level, collapse mechanisms, and residual deformations, were not examined. Incorporating the proposed connection details into frame-scale numerical models is neces-

sary to assess their effectiveness within complete structural systems.

- This study did not provide a quantitative evaluation of constructability, retrofit feasibility, fabrication requirements, or cost efficiency. While longitudinal plates may offer geometric and behavioral advantages compared to conventional stiffener details, their practical implementation should be further investigated with respect to welding accessibility, fabrication sequence, inspection requirements, and economic considerations.

Accordingly, future research should prioritize experimental validation of the proposed connection details, expansion of the geometric and material parameter space, assessment under dynamic earthquake loading, and evaluation of frame-level performance and practical applicability. These studies will contribute to the development of design recommendations for longitudinal-plate-strengthened bolted extended end plate connections.

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Conflict of Interest

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Data Availability

The datasets generated and/or analyzed during the current study are not publicly available but are available from the corresponding author upon reasonable request.

AI Assistance

During the preparation of this manuscript, Google Gemini 3.1 Pro was used exclusively for language editing and stylistic refinement. The authors take full responsibility for the content, interpretation, and conclusions of the published article.

Author Contributions

All authors made substantial contributions to the conception and design of the study, acquisition of data, analysis and interpretation of data; drafted or critically revised the manuscript for important intellectual content; and approved the final version to be published.

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Research Article

Investigation of the behavior of Gezende Dam under seismic excitation

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ABSTRACT

Since structural failure can lead to major loss of life and severe economic damage, evaluating arch dams under strong ground motions is essential. In this study, so as to evaluate the behaviour of the system against strong ground motions, the effect of seismic parameters is examined in terms of determining structural safety. In order to simulate the behavior of arch dams under the seismic excitations, Gezende Dam in Türkiye was selected as the case study. Five earthquake ground-motion records were taken into account to evaluate the seismic response. As a result of the analysis, maximum displacement and stress values were determined through the numerical simulations. The frequency contents of the ground motions were defined by Fast Fourier Transform, while the structural demand was evaluated through spectral acceleration. Spectral Analysis, Fast Fourier Transform (FFT) and Short Time Fourier Transform (STFT) analyses were performed for all the earthquake records. The structural behaviour was comparatively examined by considering the frequency content and spectral acceleration values of seismic motions. The findings obtained highlighted that the dam-reservoir system ought to be considered in the assessment of the performance of the arch dams. Furthermore, it was determined that earthquake characteristics affect the stress and the displacement distributions of arch dams. The results obtained are of great significance for engineering applications in the seismic evaluation of arch dams that exist and the ones that might be constructed.

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1. Introduction

Arch dams are widely employed owing to their numerous advantages. To ensure the structural integrity of arch dams during construction, longitudinal and transverse joints are formed between dam blocks. To enhance impermeability, pressure grouting and other methods are applied, and thus the arc dam acts as a monolithic structure (Huang and Han 2023; Zeinizadeh et al. 2018). Arch dams that are not constructed as single monoliths contain discontinuities due to their construction joints. These expansion joints, which separate the cantilever blocks, also separate the dam body from the concrete

abutment. These contraction joints subjected to shear and tensile stresses act as planes of weakness (Zeinizadeh et al. 2018). As arch dams have thin cross-sections, gate chambers located within the body of the dam may not be supported completely, which results in downstream and upstream regions that demonstrate cantilever behaviour. The interfaces between the dam and the suspended segments play an important role in transferring the load. Hence, the structural response of the interfaces and their adjacent areas remarkably influence the functionality of the water discharge in the dam structure (Zhang et al. 2015). The abutment and the foundation systems of arch dams are highly complex due to geologi-

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cal situations. Therefore, large-scale excavation activities are necessary during the construction process (Zhao and Wu 2020).

The security and the performance of arch dams are determined by geological situations, seismic design effects and important water thrust. All these factors are of vital importance to preserve their structural stability against seismic loads (Liang et al. 2025). Arch dams are constructed to store water for energy production, irrigation, and flood control purposes. When an arch dam is damaged during seismic events, the release of a large amount of water from its reservoir can result in devastating loss of life and property (Jin et al. 2019). However, arch dams can be structured in areas with high seismicity. Thus, evaluating the seismic resistance of arch dams is of great significance owing to the damage resulting from the strong ground motion.

Ainono Dam was influenced by Akita-ken Nantobu earthquake. Makio Dam was slightly damaged during the Nagano-ken Seibu earthquake (Tani 2000). A large number of earth dams were damaged in China during Wenchuan earthquake and in Japan during the Hogo-Ken Nanbu earthquake (Jin and Lee 2024). Numerous dams in Türkiye were damaged during 2023 Kahramanmaraş earthquakes (Hariri-Ardebili and Tosun 2024).

Moghaddam et al. (2026) studied the damage evolution by conducting dynamic analysis of an arch dam. It was determined that close fault seismic records lead to greater structural reactions and damage than the distant fault. Guo et al. (2025) examined the breathing behavior of transverse joints of arch dams through dynamic analyses. The method proposed was applied to a high arch dam in southwestern China. By using this method, it was concluded that the maximum opening depth and duration that might occur in transverse joints during seismic activity could be predicted. Zhang et al. (2025a) evaluated the joint opening and damage conditions by examining the behavior of the GX arch dam with three different reservoir conditions under seismic loads. Zhang et al. (2025b) carried out the nonlinear dynamic analysis by taking different velocity pulse situations into account. Following the artificial simulation methods, the magnitude and duration of nearby fault movement were shown to have a significant impact on the dam's behavior. Li et al. (2024) examined three arch dams with varying degrees of symmetry, considering both extreme and normal load conditions. Since asymmetric foundations might lead to stress and deformation, strengthening measures were investigated. Wang et al. (2024) investigated the effects of near-fault ground motion records on the seismic performance of high arch dams. Yan et al. (2024) performed nonlinear dynamic analyses by considering a high arch dam. Yan et al. (2023) examined the sliding resistance of the foundation-arch dam system by considering uncertainties in the modeling parameters, the ground motions and their combined effects. The findings indicated that these uncertainties significantly affect the sliding resistance of the foundation-arc dam. Jin et al. (2023) performed the fragility analysis of arch dams. It was concluded that the demand level substantially influences the reliability of fragility curves and the effect of material uncertainties is limited. Zhuang et al. (2019) examined the development and the propagation of crack within gallery

structures of arch dams constructed in complex geological regions with high seismic activity, considering the effects of the high arch dams' self-weight and the condition of the reservoir filling. Jin et al. (2020) performed dynamic analyses of the Dagangshan Dam based on the acceleration response spectrum. Türkmen (2010) investigated water leaks and deformations in the energy tunnels of Gezende Dam. Korkmaz and Çarhoğlu (2018) performed a probabilistic seismic assessment of the eight existing arch dams in Türkiye. Çarhoğlu (2012) carried out dynamic analyses of arch dams in Türkiye and examined the effect of the strengthening method.

There are many arch dams in seismically active regions of Türkiye. This study examines the behaviour of Gezende Dam, an arch dam in Türkiye, under the influence of different earthquakes. The dam model examined is based on the models and their analyses from previously published studies (Çarhoğlu 2012; Korkmaz and Çarhoğlu 2018). In recent years, strong ground motions such as Kahramanmaraş (Pazarcik), Kahramanmaraş (Elbistan), Kocaeli (1999), and İzmir (2020) earthquakes occurred. During earthquakes, significant structural damages may occur in arch dams. For this reason, seismic performance assessment is of vital importance in terms of earthquake engineering. Determining the behaviour of the Gezende Dam, located on an active earthquake zone in Türkiye under different seismic characteristics is crucial. PGA, PGV, focus depth and frequency content characteristics of earthquakes significantly affect the dynamic behavior of the structure. Signal and modal analyses provide information about the resonance potential of the structure. Determining the performance of the structure during earthquakes with similar PGA and frequency content is of great importance for dam structures. The main purpose of this study is to evaluate structural performance by considering real earthquake data. However, Investigating the effects of earthquakes with different frequency characteristics on the behaviour of arch dams under seismic loads is of great significance. For this reason, the ground motion records with different frequency content were used to evaluate the seismic behavior of Gezende Dam. The displacement and stress values that were obtained by applying time history method to the dam structure were comparatively investigated. Unlike previous studies; the effects of the frequency contents of earthquake records on the dynamic behaviour of Gezende Dam were assessed. The dam's resonance potential was determined by comparing the dam's natural frequency with the dominant frequency of each earthquake in the analyses.

2. Case Study: Gezende Dam

This study investigated the behavior of Gezende Dam, an arch dam with a height of 75 m from the base, under seismic loading. The dam has a double-curvature geometry. The dam was constructed in Mut district of Mersin province. The dam's volume at normal water level is 91.90 hm³, its body volume is 83,000 m³, and its reservoir area is 3.97 km². The dam's crest elevation is 335 m, its crest width is 7.5m, and its crest length is 120m. (DSI 2010). The concrete class of this arch dam is C30. The

elastic modulus, unit weight (KN/m^3), and poisson ratio of the dam were taken as 32 GPa, 24 KN/m^3 , and 0.2 respectively. The material model is assumed to be linear. The dam was modelled as eight-node solid (brick) elements by using SAP 2000 V14. finite element software, and for the seismic safety assessment of Gezende Dam, a time-history analysis method was used (Wilson 1997). The finite element model of the dam structure consists of 295 solids and 649 points. In the course of time history analysis; the time step was taken to be the same as the sampling interval (Δt) of each earthquake record. The earthquake excitation was applied to upstream–downstream direction (the normal direction to the dam crest). Only one component of the earthquake was applied to the system. Damping ratio of % 5 was assigned to the dam structure. The soil was not modelled in this study. The dam–foundation interface was assumed to be fixed. The finite element model of Gezende Dam as the case

study is shown in Fig. 1. During the analysis, the data of five strong ground motions were selected to examine the seismic performance of Gezende Dam. To determine the structural responses, the values of the displacement and stress were determined.

Reservoir was modelled by utilizing the Westergaard added mass approach (Westergaard 1933). Eqs. (1) and (2) show the water pressure acting on the dam surface and the added water mass. Here; P is the pressure acting on the dam surface, w is the unit weight of water, h is the reservoir height, and y is the water depth from the water surface (Westergaard 1933; Salamon 2018).

$$p = \frac{7}{8} w \alpha \sqrt{h(h-y)} \quad (1)$$

$$m = \frac{7}{8} w \sqrt{h(h-y)} \quad (2)$$

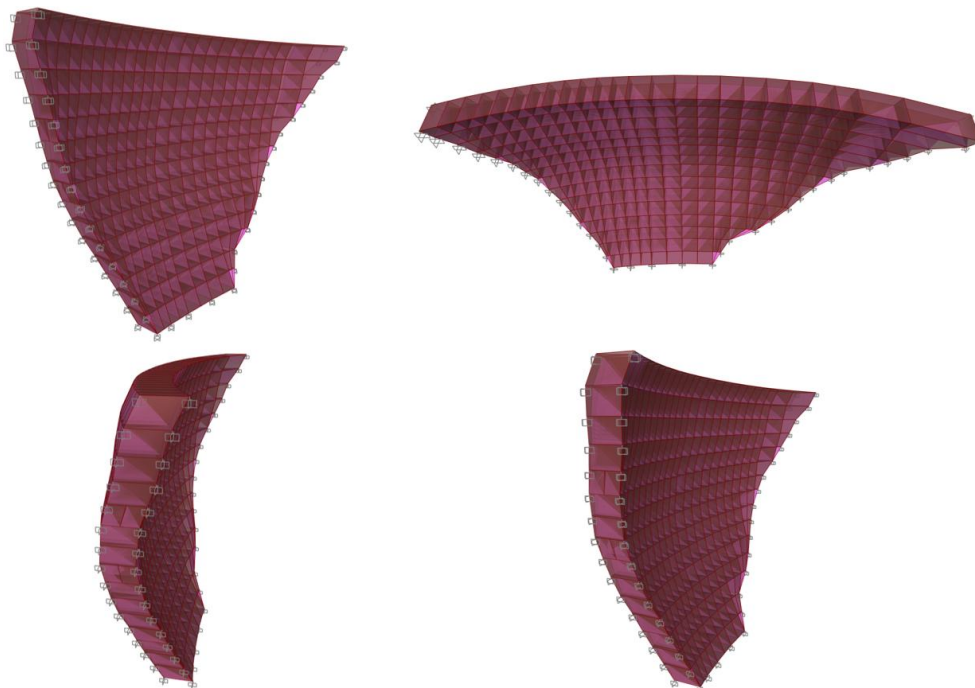


Fig. 1. View of Gezende Dam.

Earthquake records with different characteristics were selected to evaluate the structural behavior. The selected ground motions were determined to have different magnitudes, depths of focus and spectral contents. To determine the relationship between the structure's behavior under seismic loads and the dominant frequencies of earthquakes, recordings with different frequency contents were selected. No ground-motion scaling procedure was performed. Original ground motion records obtained from the PEER database were utilized in the time history analyses. No filtering has been applied to the earthquake records. The time dependent acceleration records obtained by using PEER data are shown in Fig. 2 (PEER 2010). The earthquake records used in the analyses are shown in Table 1 and these records were obtained from the PEER Ground Motion Database (PEER 2010; PEER 2026). The graphics were generated using MATLAB (MathWorks 2024).

The epicenter of the Coyote Lake earthquake ($M_w=5.74$, local magnitude of $M_L=5.9$), which occurred on August 6, 1979, was located on the Calaveras Fault, approximately 100 km southeast of San Francisco. The depth of the fracture surface varied between 4 km and 12 km and the length was 20 km. The breaking surface with such dimensions was rectangular (King et al. 1981). The seismic activity occurred in the form of right-lateral strike-slip faulting (Lee 1979). According to geodesic data, the right lateral shift was approximately 0.33 m. When geodesic data were examined, no prior signs of Coyote Lake earthquake had been seen before the earthquake (King et al. 1981). The spatial distribution of this earthquake is presented in Fig. 3 (USGS 2026). It is seen that the seismic shaking was concentrated along the fault and PGA values were increased in the sections close to the focus area.

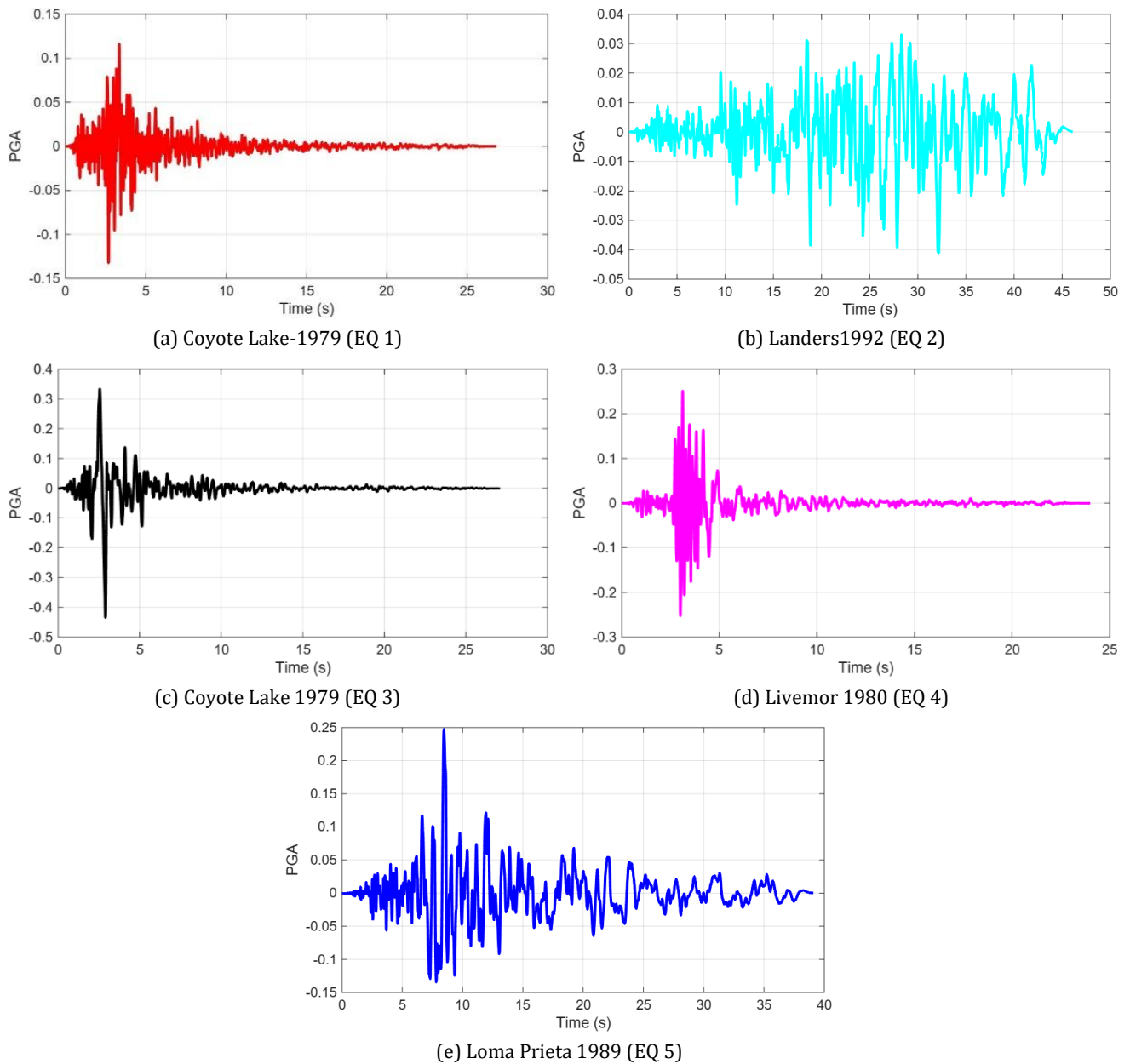


Fig. 2. Time-dependent variation of the earthquakes.

Table 1. Characteristics of the earthquakes used in the analyses.

Earthquake Number	Record Sequence Number	Earthquake Name	Date	Station Name	Magnitude (M_w)	Record Name	NEHRP Site Class (Based on V_{s30})	PGV (cm/s)	PGA (g)	Joyner-Boore Dist. (km)	Fault Type
1	146	Coyote Lake	06/08/1979	Gilroy Array #1	5.74	G01320	B	8.3	0.132	10.21	Strike-Slip
2	889	Landers	28/06/1992	San Gabriel - E Grand Ave	7.3	GRN180	C	14.1	0.041	141.92	Strike-Slip
3	150	Coyote Lake	06/08/1979	Gilroy Array #6	5.74	G06230	C	49.2	0.4339	0.42	Strike-Slip
4	222	Livermore	27/01/1980	Livermore - Morgan Terr Park	5.42	LM0355	C	9.8	0.252	7.94	Strike-Slip
5	777	Loma Prieta	18/10/1989	Hollister City Hall	6.93	HCH090	D	38.5	0.247	27.33	Reverse oblique-slip

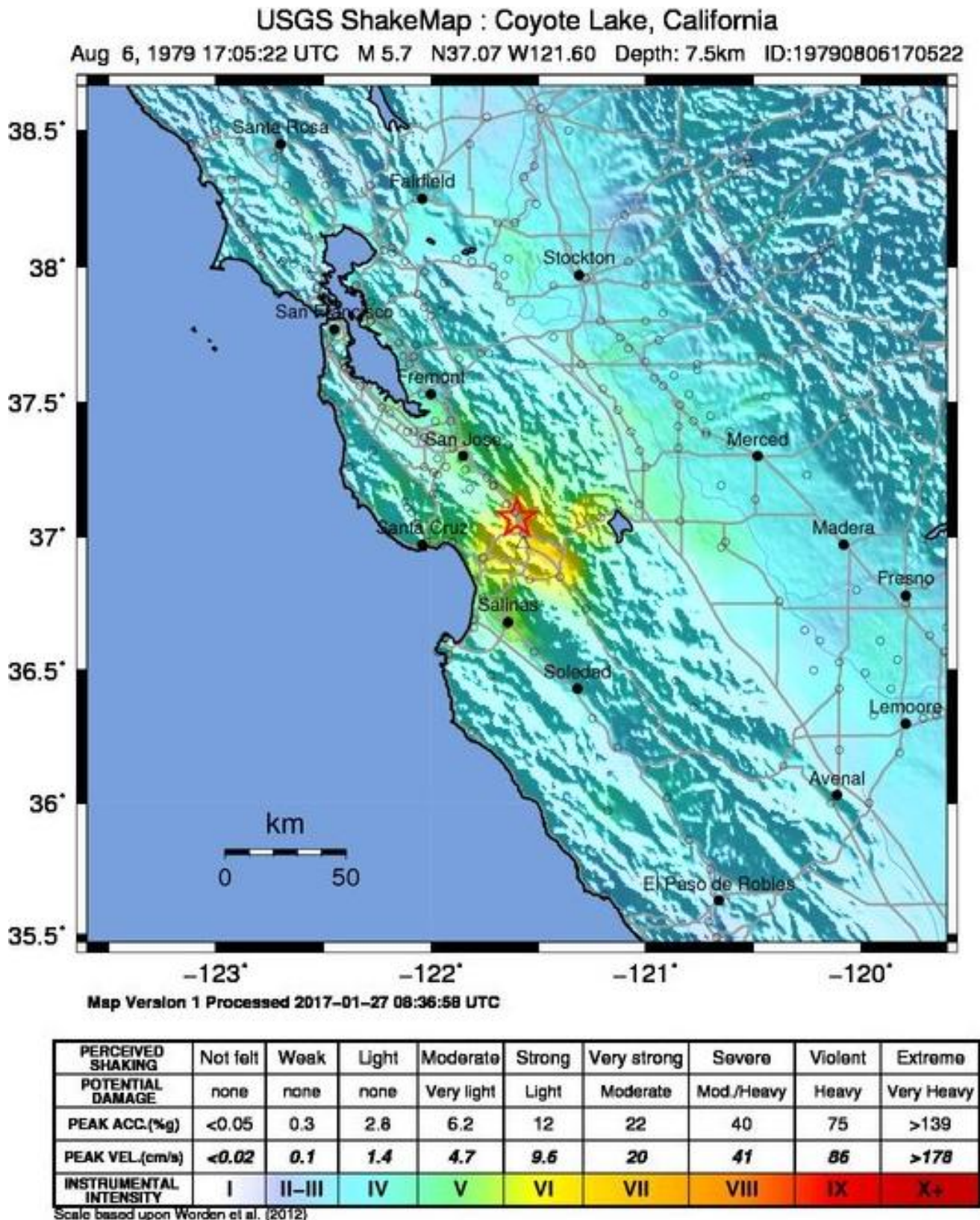


Fig. 3. Coyote Lake Earthquake map (USGS 2026).

3. Modal Analysis of Gezende Dam

The dynamic characteristics of Gezende Dam were examined using modal analysis. Modal analysis was performed by utilizing the Eigenvector method. Fig. 4 shows the first four modes of the dam structure.

The first four period values observed in the system were determined as 0.09 sec, 0.05 sec, 0.03 sec, and 0.03 sec, respectively. The first four frequency values were obtained as 11.739 Hz, 20.520 Hz, 32.846 Hz, and 38.534 Hz. The participating mass ratio for mode 1 was determined as % 68.

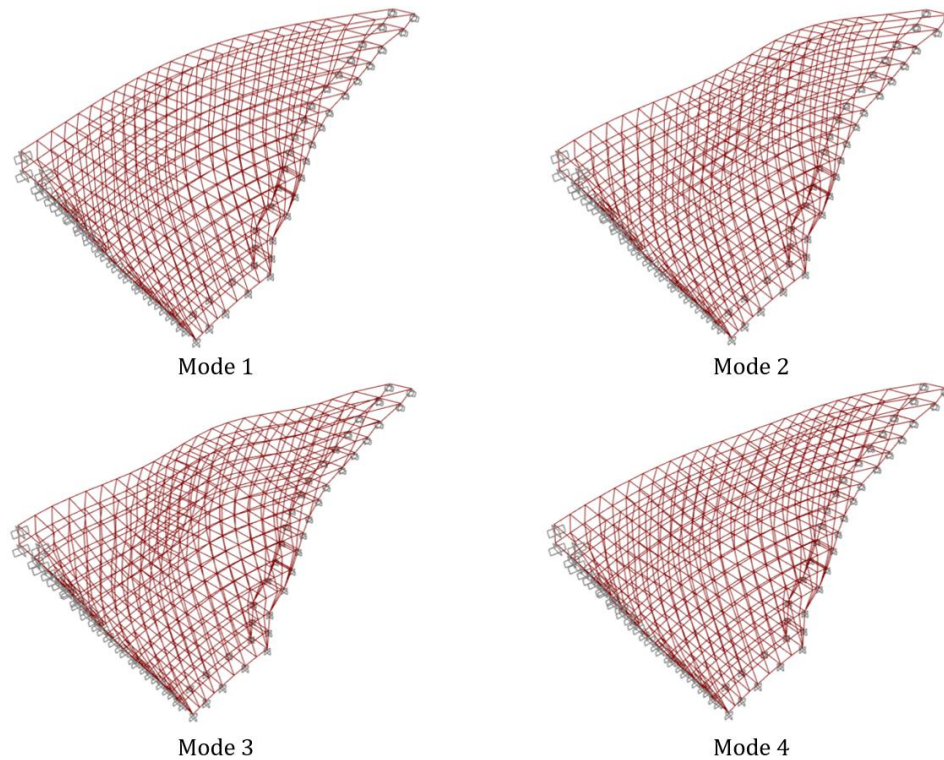


Fig. 4. The first four mode shapes of the Gezende Dam.

4. The Frequency and Spectral Content of Seismic Motions

4.1. The evaluation of the frequency domain analysis

Common methods that are used to determine frequency content include FFT, STFT, wavelet analysis and PSD analysis. FFT analysis (Cohen 1995; Mallat 1999) is carried out using signal records. FFT analysis is an effective method used for estimating dominant frequency and frequency content. While performing these analyses, seismic acceleration records are used to determine the frequency content of the earthquake. Determining frequency content plays an important role in arch dam safety. In order to evaluate the resonance potential, it is of great importance to determine the frequency of the system and the dominant frequency of the signal. Modal analysis is used to determine the frequency values of the system. It is essential to highlight that the identification of frequency content for all ground motions is carried out based on the time series of seismic motions.

In this study, Fast Fourier transform (FFT) analyses were performed using MATLAB software, and the sam-

pling interval (Δt) of the earthquake records was determined using the difference between two consecutive time values. The sampling frequency, f_s , was obtained using the relationship $1/\Delta t$. All five ground-motion records were used in the analyses. The FFT length was taken to be equal to the total number of data points (N) in the earthquake records. The frequency resolution was determined using the expression $\Delta f = f_s/N$. The positive frequency range was considered to be $0 - f_s/2$.

A Fast Fourier Transform (FFT) was performed to analyze the frequency domain characteristics of the time dependent seismic signals. The dominant frequency values were derived from the Fourier spectrum obtained. Frequency spectra of earthquakes are presented in Fig. 5. The frequency components of each ground motion were determined. Table 2 shows the natural frequency of the dam and the first dominant frequency values obtained from the FFT analysis of the ground motions. The ratio of these two frequencies is less than 1, indicating that the dominant frequency of the earthquake records does not match the natural frequency of the dam. The relationship between the dominant frequency and the natural frequency of system is highly influential in determining the resonance potential.

Table 2. Comparison of the dam's first natural frequency with the first dominant frequency of earthquakes.

Earthquake No	Earthquake Name	Record Name	The first dominant frequency of dam (Hz)	Dominant frequency of the ground-motion record (Hz)	$f_{\text{Earthquake}}/f_{\text{dam}}$
1	Coyote Lake	G01320	11.1	4.994	0.44
2	Landers	GRN180	11.1	0.543	0.05
3	Coyote Lake	G06230	11.1	0.960	0.09
4	Livermore	LM0355	11.1	5.542	0.49
5	Loma Prieta	HCH090	11.1	1.100	0.09

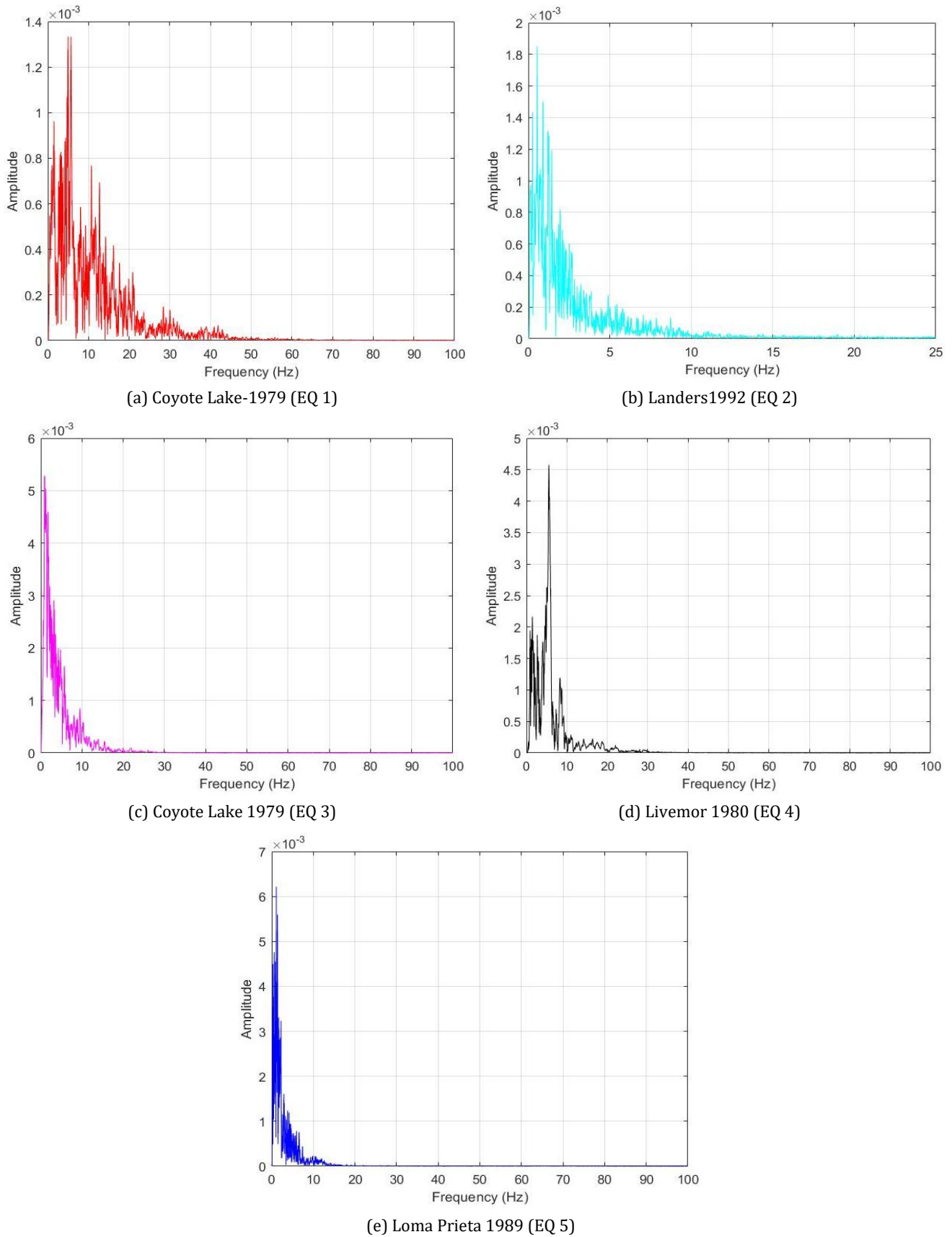


Fig. 5. Fast Fourier transform spectrum of ground motion.

Short-Time Fourier Transform analysis shows the change of frequency content depending on time. Spectrograms of earthquakes are shown in Fig. 6. While FFT indicates frequency content, spectrogram shows energy

concentrations at specific time intervals. The Δt value of the earthquake was used as the STFT sampling interval. The sampling frequency was determined using the relationship $f_s = 1/\Delta t$. For the suitability of time and fre-

quency resolution, a window of length 256 was used in STFT analyses. The number of FFT points was chosen as 512, consistent with the window length, and this value was used to provide a more detailed view of the distribution in the spectrogram. The analyses used a % 78 overlap between windows, a measure that is used to en-

sure an appropriate spectrogram. The spectrograms show that the energy content of the earthquake records varies depending on both time and frequency, with no single continuous frequency band remaining dominant throughout the entire duration. Spectrograms were obtained using MATLAB (MathWorks 2024).

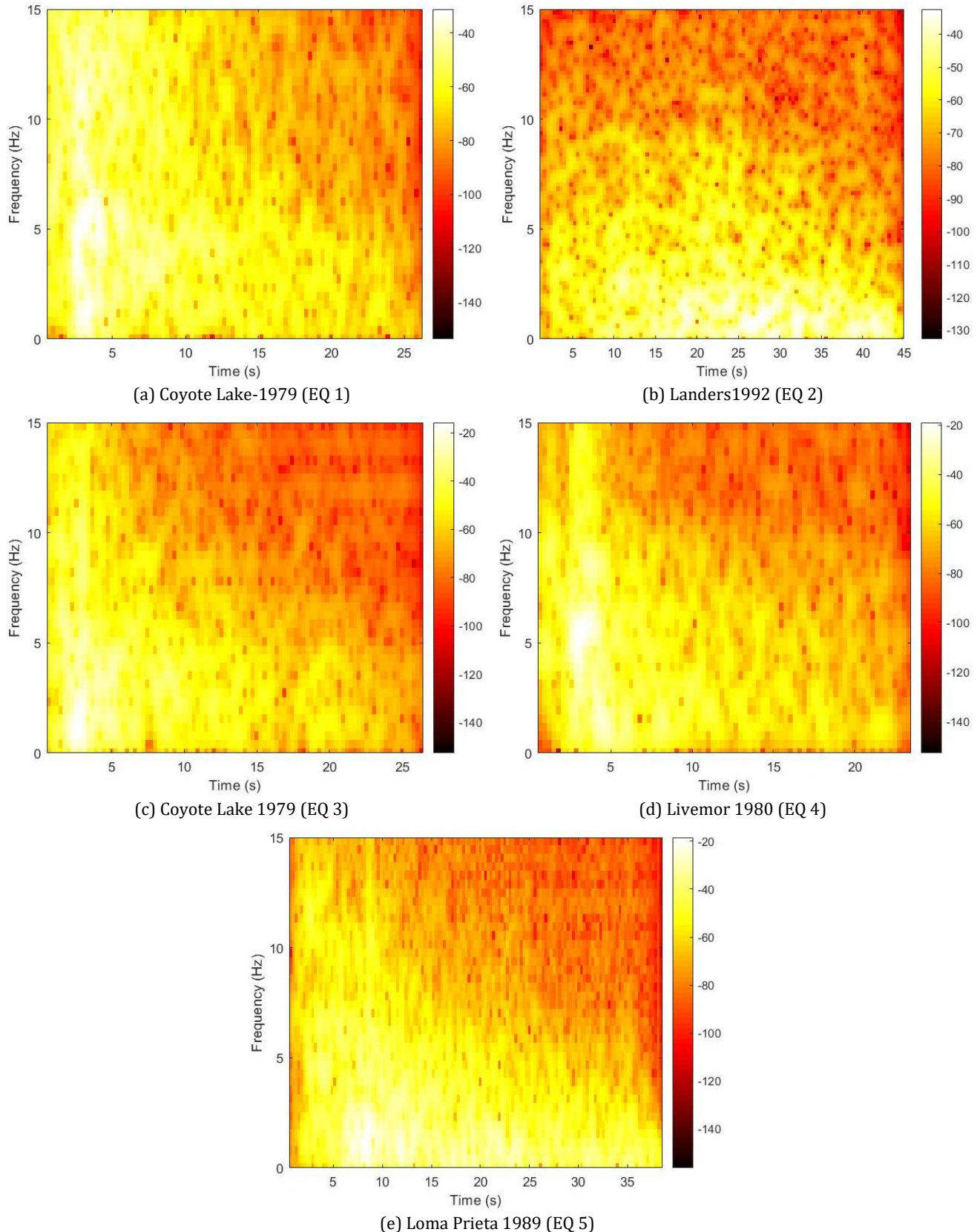


Fig. 6. Spectrograms of earthquakes.

4.2. Spectral analysis assessment

During the seismic analysis of Gezende Dam, the spectral acceleration values associated with the applied ground motion records were obtained using Seismosignal program to evaluate the spectral characteristics of the ground motion records (Seismosoft 2025). The damping ratio used when obtaining the elastic response spectra is 5%. Response spectra were obtained using a period range of 0 to 4 seconds and a period increment of 0.05 seconds.

As illustrated in Fig. 7, these response spectra were used to evaluate the variation of spectral acceleration

with period. For each earthquake record, the dominant period value corresponding to maximum spectral acceleration was determined. The spectrum curves obtained for each earthquake display differences from each other. This variation differs depending on the frequency content and the features of the earthquakes, which significantly affect the dynamic behavior of the system. The first four period values of Gezende Dam are, 0.09 sec, 0.05 sec, 0.03 sec, 0.03 sec, respectively. No correspondence is observed between these values and the period values corresponding to the maximum spectral acceleration.

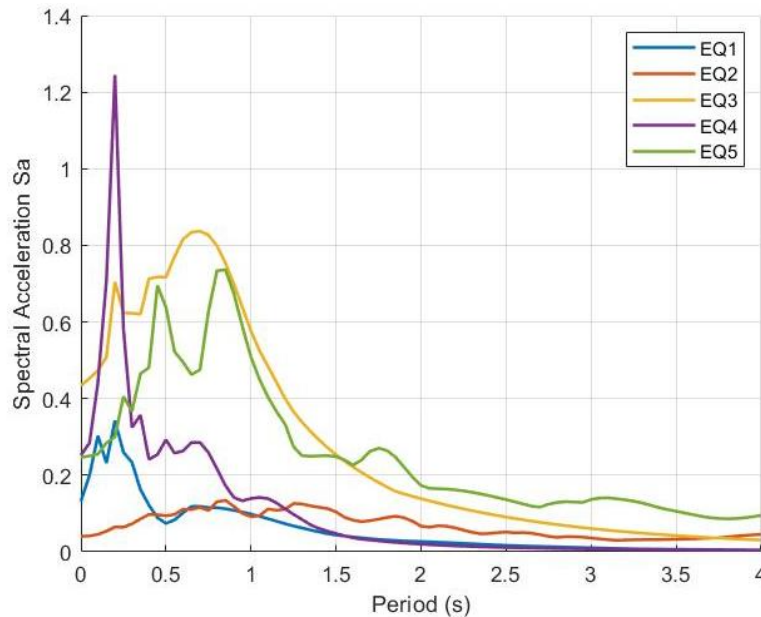


Fig. 7. The comparison of spectral acceleration of ground motions.

Response spectra of the earthquakes used in the analysis were obtained, and for each earthquake, the period values corresponding to the largest spectral acceleration were obtained. Table 3 shows a comparison of the largest period values corresponding to the spectral

acceleration with the first natural period of the structure for each earthquake. In the comparison performed, it was determined that the dominant spectral period values did not match the first natural period value of the structure.

Table 3. Comparison of the dam's first natural period value with the peak spectral acceleration period values of seismic loads.

Earthquake No	Earthquake Name	Record Name	T ₁ value of Gezende Dam (sec)	The period value of peak Sa (sec)
1	Coyote Lake	G01320	0.09	0.20
2	Landers	GRN180	0.09	0.85
3	Coyote Lake	G06230	0.09	0.84
4	Livermore	LM0355	0.09	1.25
5	Loma Prieta	HCH090	0.09	0.74

5. Identifying Structural Responses

The structural responses that resulted from applying seismic loads to the dam structure were compared. The displacement and stress values obtained for the empty and full condition of the dam are shown in Fig. 8(a-b).

The largest displacement values were obtained with values of 35.356 mm and 55.856 mm for the empty and full states of the dam in Coyote Lake (G06230) earthquake respectively. The largest stress values were obtained as 8.7666 MPa and 16.40496 MPa.

A normalization process was performed to facilitate comparison of the displacement and stress values obtained from analyses under seismic loads, considering both empty and full reservoir conditions. This process was carried out by dividing the displacement and stress values by their respective maximum values ($U_{\text{norm}}=U/U_{\text{max}}$, $S_{\text{norm}}=S/S_{\text{max}}$).

Fig. 9(a-b) illustrates the displacement and stress values obtained from the application of earthquakes with different characteristics to Gezende Dam. The strong ground motions induce amplified structural responses, such as increased stress and displacement. It was observed that the values obtained when the dam was full were greater than those obtained when it was empty; in other words, the dam's fill rate significantly affected the system's behaviour.

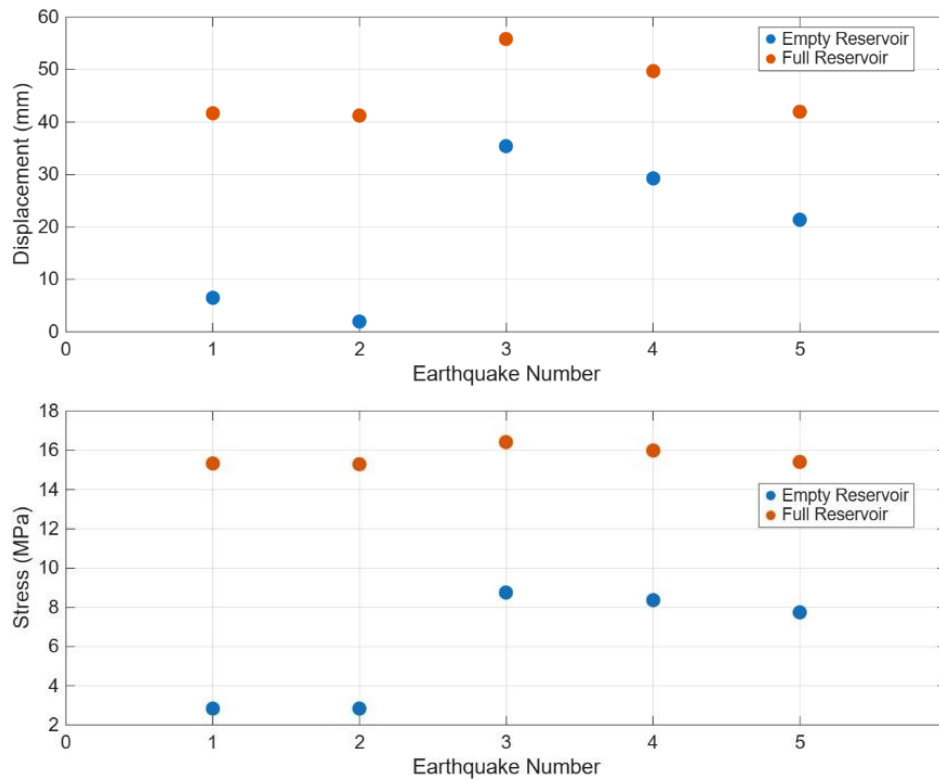


Fig. 8. Displacement and stress values.

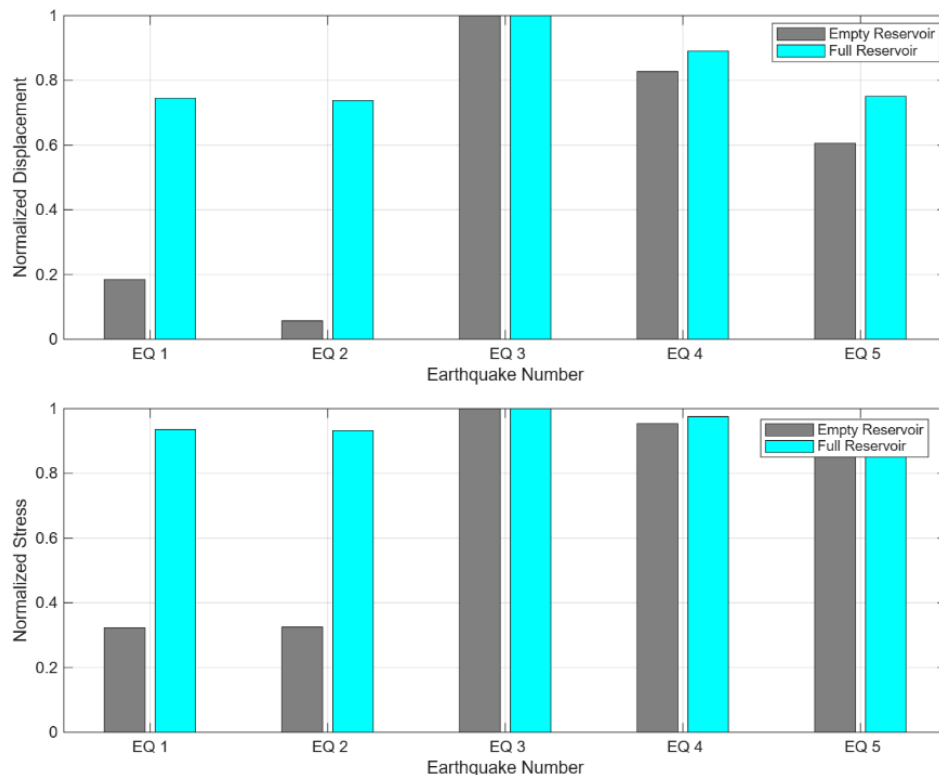


Fig. 9. Normalized displacement normalized stress values.

According to the results obtained; for all the earthquakes, the displacement values reach their maximum level at the top of the dam, while the stress values exhibit the highest values in areas close to the base. Under Coyote Lake (EQ 1), Landers (EQ 2), Coyote Lake (EQ 3), Livermore (EQ 4), Loma Prieta (EQ 5), the normalized displacement and stress values determined for the empty condition were 0.18, 0.05, 1, 0.82, 0.60 and 0.32, 0.32, 1, 0.95, 0.88 respectively. Under the full condition, these values were 0.74, 0.73, 1, 0.89, 0.75 and 0.93, 0.93, 1, 0.97, 0.93 respectively. As seen, the highest structural re-

sponses were observed during Coyote Lake ground motion. Depending on the increase of earthquake effect, inertia forces also increased, which led to the increase of inner forces.

5.1. Model validation

To evaluate the reliability of the numerical model, the results obtained were validated by comparing them with studies in the literature. Table 4 compares the stress values obtained in this study with those in the literature.

Table 4. Comparison of the present study with literature.

	This study	Fan (2015)	Zeinizadeh (2018)	Wang (2018)
Maximum Stress (MPa)	16.40	16.3	15.4	16.59
Deviation	–	% 0.61	% 6.49	% 1.15

Fan (2015) found the maximum compressive stress value at the downstream face of an arch dam as 16.3 MPa in a feasibility study. This value is quite close to the one in the current study. Zeinizadeh (2018) obtained a principal stress value of 15.4 for S1 when the load step was 150. Wang (2018), according to the trial load method, determined the compressive strength values for concrete zones A (C₁₈₀40), B (C₁₈₀35) and C (C₁₈₀30) as 18.96 MPa, 16.59 MPa and 14.22 MPa, respectively. Gezende Dam body has a concrete class of 30 MPa, and the stress value obtained from the analyses is close to 16.59 MPa. The numerical results obtained in this study were compared with values reported in the literature to assess their consistency.

6. Conclusions

In this study, the behaviour of Gezende Dam exposed to various ground motions with differing frequency content was examined.

- Different ground motion records led to the development of different structural responses in the dam. The maximum displacement and stress values were obtained during Coyote Lake earthquake. For empty and full reservoirs, the maximum displacement values were determined to be 35.356 mm and 55.856 mm, and the stress values were 8.7666 MPa and 16.40496 MPa.
- It was observed that the normalized displacement and stress values derived from seismic analyses exhibited an increase in some earthquake conditions, whereas a decline was identified in others. The internal forces demonstrate variability depending on the ground motion amplitude.
- Response spectra for the earthquakes considered were generated, and the periods corresponding to the highest spectral acceleration were determined. The values of spectral acceleration are 0.34264 g, 0.13414 g, 0.83721 g, 1.24503 g, and 0.73637 g, respectively. The peak period values were compared with the dam's first natural period, and it was determined that

the periods did not coincide. Applying the earthquake records to the system showed that no resonance potential was generated in the structure.

- To evaluate the frequency content of the earthquake data, FFT (Fast Fourier Transform) and Short Time Fourier Transform (STFT) analysis were performed. The earthquakes considered in this study were Coyote Lake (G01320), Landers (GRN180), Coyote Lake (G06230), Livermore (LMO355), and Loma Prieta (HCH090). The dominant frequency values obtained from the FFT signal analysis are 4.994, 0.543, 0.960, 5.542, and 1.100 Hz, respectively. The dominant frequency value of the Gezende Dam is 11.1 Hz. The ground motions exhibit variations in terms of their frequency content. The natural frequency value of Gezende Dam and the dominant frequencies of the ground motions were determined to be unmatched.

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Conflict of Interest

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Data Availability

The datasets generated and/or analyzed during the current study are not publicly available but are available from the corresponding author upon reasonable request.

AI Assistance

No AI-based tools were used in the preparation of this manuscript.

Author Contributions

All authors made substantial contributions to the conception and design of the study, acquisition of data, analysis and interpretation of data; drafted or critically revised the manuscript for important intellectual content; and approved the final version to be published.

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Research Article

Analytical free vibration solutions of Euler-Bernoulli beams resting on Winkler foundation using the Elzaki transform method

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ABSTRACT

The natural frequency analysis of a freely vibrating thin beam on Winkler foundation is a significant part of the analysis for design against resonance failures which occur when the natural frequency coincides with the excitation frequency due to loads. This article explores the natural frequency determinations of an Euler-Bernoulli beam on a Winkler foundation using Elzaki transform method (ETM). The main objective is to develop a new analytical solution procedure using the ETM and demonstrate its efficiency for beam foundation vibration problem. The ETM is adopted because of its effectiveness in solving both linear and partial differential equations problems, computational simplicity and versatility. For harmonic vibrations, the governing partial differential equation (GPDE) of motion is simplified to an ordinary differential equation (ODE) in terms of the space variable. The ETM further simplifies the ODE to an algebraic problem whose inverse yields the solution for the modal displacement in the physical domain. The application of boundary conditions gives the characteristic frequency equation as a transcendental equation. The roots give the eigenvalues from which the natural frequencies are found. The boundary conditions considered are the classical boundary conditions (BCs). The natural frequencies for all the classical BCs are found as exact solutions within the assumptions and theoretical framework of Euler-Bernoulli theory, Winkler foundation model, linear elastic behavior, small displacement elasticity theory and classical boundary conditions. They are identical with previous solutions using Generalized Integral Transform Methods (GITM). The present solutions are close to the previous solutions based on semi-analytical methods of Differential Transform Method (DTM). Analytical solutions for the natural frequencies are obtained for the classical BCs considered and for the foundation parameters considered. The effectiveness of the ETM in simplifying the natural frequency analysis of thin beam on Winkler foundation to an algebraic formulation is demonstrated.

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1. Introduction

1.1. Background

Beams are structural members with longitudinal dimensions that are much more than their cross-sectional dimensions. They are parts of building floors, bridges, foundation beams, machines, aircrafts and spacecrafts.

Beams that are placed on the ground are modeled using beam on elastic foundation theories. When such beams are subjected to dynamic forces, they can fail by resonance when the frequency of excitation forces becomes equal to the natural frequency of the beam on elastic foundation (BoEF) system. The calculation of the natural frequencies of vibration of BoEF systems is thus an important aspect of the analysis and design to avert resonance.

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Nomenclature

x	longitudinal coordinate
t	time
$r(x)$	foundation reaction
k	Winkler foundation stiffness
k_1	second parameter of the foundation
$w(x)$	transverse displacement (deflection)
u	Elzaki transform parameter
A	cross-sectional area
ρ	mass density
$q(x, t)$	applied load
T	Elzaki transform
I	moment of inertia
E	modulus of elasticity
ω_n	natural frequency
$4\beta^4$	foundation parameter defined in terms of k and EI
$K_0 = 4\beta^4 I^4$	foundation parameter
Ω_n^4	frequency parameter defined in terms of $\omega_n^2, \rho A, EI$
$W(u)$	Elzaki transform of $w(x)$
T^{-1}	Inverse Elzaki transform
l	length of beam
λ_n	dimensionless frequency parameter
α_n	parameter defined in terms of Ω_n^4 and $4\beta^4$
$M(x)$	bending moment
$Q(x)$	shear force
ETM	Elzaki transform method
DTM	differential transform method
GITM	generalized integral transform method
BCs	boundary conditions
ODE	ordinary differential equation
PDE	partial differential equation
BoEF	beam on elastic foundation
SVIM	Stodola-Vianello iteration method
ADM	Adomian decomposition method
GFEM	Galerkin finite element method
GDE	governing differential equation
FD	finite difference
FEMs	finite element method(s)
VIM	variational iteration method
DQM	differential quadrature method
DQEM	differential quadrature element method
HPM	homotopy perturbation method
EBBoWF	Euler-Bernoulli beam on Winkler foundation
C-free	clamped free
SS	simple supports
SC	simple support on left end and clamped on right
CC	clamped at both ends

The behaviours of beams have been described using several frameworks and models. The Euler-Bernoulli beam theory which is appropriate for slender beams is framed using the Euler-Bernoulli orthogonality hypothesis. This hypothesis is that the straight lines on the cross-section that are originally orthogonal to the middle surface of the beam before deformation remain straight and normal to the middle surface (plane) after deformation. This implies that shear stresses responsible for the change in shape of the cross-section is disregarded, limiting the scope of the resulting formulation to slender beams for which transverse shear deformations are negligible (Ike 2018a, 2018b, 2023).

Other beam theories have been formulated to consider shear deformations by Onah et al. (2020), Krishna Murty (1984), Ghugal (2007), and Sayyad and Ghugal (2016). Other thick beam models include sinusoidal shear deformable beams by Ike (2024a), hyperbolic shear deformable beams by Ike (2022), and first order shear deformable beams by Ike et al. (2019) and Ike (2019).

Kaniğ and Kutlu (2025) utilized a unified higher-order shear deformation theory (HSDT) that considered transverse stretching to analyze laminated composite plates. Kaniğ (2026a, 2026b) studied the stability of laminated composites by using a mixed finite element method based on a HSDT incorporating transverse stretching. The displacement field was defined using higher-order polynomial functions of thickness, leading to accurate expressions of both transverse shear and normal distributions without the need for shear modification factors. Moreover, Kaniğ and Kutlu (2026) developed a two-node C^0 -continuous mixed finite element formulation based on quasi-3D higher-order beam theory and the Hellinger-Reissner principle for the static bending analysis of laminated composite beams, incorporating transverse shear and normal deformation effects. Their work was however focused on beams and plates but with flexural analysis so they did not study on natural vibrations. On the other hand, recently, Kaniğ et al. (2026) developed a mixed HSDT finite element formulation based on the Hellinger-Reissner principle for the flexural analysis of laminated composites on Winkler-Pasternak elastic foundations. Also, Nezami (2026) proposed a compact higher-order shear and normal deformation theory with transverse stretching for the bending analysis of laminated composite plates, achieving accurate displacement and stress predictions with a fixed number of displacement variables independent of the number of layers.

Elastic foundation models have been proposed and developed by Winkler, Vlasov, Hetenyi, Kerr, Pasternak and Filonenko-Borodich. The Winkler foundation model is the simplest representation of the soil interaction with the beam using one-parameter, called the stiffness of the foundation. The model assumes that the foundation reaction $r(x)$ at any point on the beam is directly proportional to the beam deflection $w(x)$ at the point, and the proportionality constant (k) is the soil's stiffness (Ike 2018a, 2018b). Hence,

$$r(x) = kw(x) \quad (1)$$

This is equivalent to a representation of the foundation using a model of closely spaced vertical elastic springs which are unconnected to one another such that interactions do not exist between the springs. The Winkler foundation model is represented in Fig. 1.

The Winkler foundation model however cannot sustain shear stress and causes a discontinuity of deflec-

tions outside the loaded beam. The value of the Winkler stiffness parameter is determined by the geometry, foundation dimensions and soil stratification. The model gave no systematic estimation of the Winkler parameter apart from possibility of empirical determination via loading tests (Balabuſić et al. 2019; Yelce et al. 2023; Arifin et al. 2020; Sadeghian and Ekhteraei Toussi 2011).

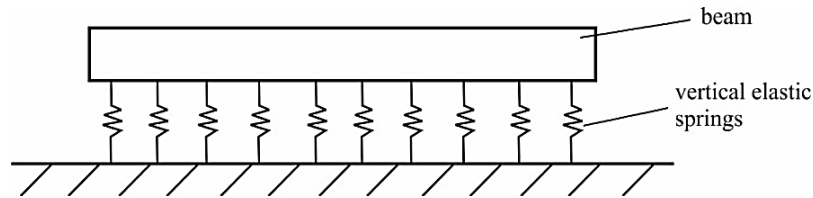


Fig. 1. Winkler foundation model represented as a bed of closely spaced springs.

These obvious limitations of the one-parameter model led to the development of two-parameter models shown in Fig. 2, by Vlasov, Hetenyi, Filonenko-Borodich and Pasternak (Doeva et al. 2022; Akhazhanov et al. 2023a, 2023b). These two-parameter models were developed primarily to resolve the displacement discontinuity issues of the Winkler model by incorporating a second interacting/coupling spring (Aleksandrovskiy and

Zaharova 2018). In the Pasternak foundation, a shear layer is introduced to produce coupling of the vertical springs. The resulting foundation reaction equation is:

$$q(x) = kw(x) - k_1 \frac{d^2w(x)}{dx^2} \quad (2)$$

while k_1 is the second parameter of the foundation.

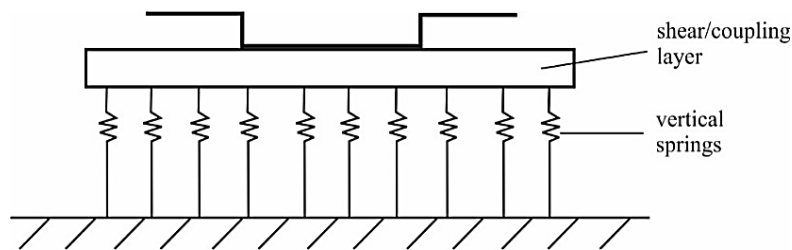


Fig. 2. Pasternak foundation model.

1.2. Previous studies

Ike (2018a) used finite Fourier sine transform method to determine the free transverse vibration frequencies of simply supported thin beam on Winkler foundations. The sinusoidal kernel function of the transformation satisfies the Dirichlet boundary conditions and yielded exact solutions to the eigenvalue problem. The work however did not consider other BCs, and was limited only to simply supported end conditions. Ike (2018b) used point collocation method for the analysis of static bending problems of Euler-Bernoulli beam on Winkler foundation (EBBoWF) but did not consider vibration problems. Ike and Elzaki (2023) explored the Elzaki transform method for the free vibration analysis of thin beams under various BCs, but they did not consider EBBoWF. Ike (2024b) used Stodolla-Vianello iteration method (SVIM) in novel ways for the free transverse vibration solutions of EBBoWF for simply supported BCs. The exact shape functions used in the SVIM gave exact eigenfrequencies for the n th iteration. The work failed to consider other end support conditions for the vibrating thin BoEF problem.

Coşkun (2003) studied the response of an elastic beam on two-dimensional (2D) tensionless Pasternak

foundation which is under a central point sinusoidal load. Erdogan and Coşkun (2020) used semi-analytical and finite element methods to study the free vibrations of a stepped beam on an elastic foundation. Adomian Decomposition Method (ADM) was the semi-analytical method adopted in their work and the finite element solutions were obtained via finite element computational software SAP 2000. Their work considered different support conditions and found favourable comparisons between the ADM results and SAP 2000 results.

Zhou (1993) developed a general solution for transverse vibration analysis of thin beams on variable Winkler elastic foundation. The dynamic response of the beams obtained in the work by considering the foundation reaction as an external force acting on the beam. The four unknown constants in the solution were found using the boundary conditions of the beam. The resulting integration was evaluated approximately using the trapezoidal rule. The frequency equation is a determinant with order that equals the number of discrete nodes. The mode shape functions were expressed as series of analytical functions. Accurate solutions were found using few discrete nodes in the trapezoidal quadrature and the study gave good convergence at lower computation effort.

Kanwal et al. (2024) used Galerkin finite element method (GFEM) and the separation of variables technique to find the eigenfrequencies and modal shapes of beams that are elastically constrained and resting on elastic foundations. They considered Winkler, Pasternak and Hetenyi foundation models and non-classical boundary conditions. They investigated the effects of flexural rigidity, modulus of elasticity and foundation parameters on the free vibration frequencies of beams modeled/idealized using Euler-Bernoulli, Timoshenko, Rayleigh and shear beam models. The GFEM was shown to converge to the closed form solution without shear locking in the stiffness element. They further found that the second foundation parameter in the form of shear layer and foundation constants led to an increase in natural vibration frequencies.

Al-Azzawi and Daud (2020) studied the natural transverse vibrations of non-prismatic, non-homogeneous beam resting on variable Winkler elastic foundations using the finite difference (FD) and finite element methods (FEMs). They considered different end support conditions for the beam. The FD method used in their work demands lesser computing rigour, compared to other methods and is utilized when there is no exact solution to the governing differential equation (GDE), or when closed form solutions are difficult to obtain. In the FD method, the GDE is expressed in finite difference form as algebraic expressions in terms of grid values of the unknowns. For the study, the FD form of the GDE reduced to an algebraic eigenvalue problem which was solved for nontrivial cases for the eigenvalues and hence the natural frequencies. The results showed favourable comparison with finite element method results obtained using ABAQUS.

Kacer et al. (2011) studied the natural transverse vibration analysis of Euler-Bernoulli beam on a variable Winkler foundation using the Differential Transform Method (DTM). Their work considered the Winkler parameter to vary along the beam's major axis, and considered cases of constant, linear and quadratic variations of the Winkler foundation parameter. The study considered three cases of boundaries, namely: simply supported at both ends, clamped at both ends, and cantilevered (clamped at one end and free at the other end). The GDE was solved using DTM which is a transformation technique relying on truncated Taylor series expansions and yielded high accuracy. Recently, Olotu et al. (2021) used DTM for the free vibration analysis of non-prismatic Rayleigh beams on Winkler foundations with varying foundation stiffness. Balkaya et al. (2009) also utilized DTM for the free vibration analysis of thin BoEFs.

Agboola and Gbadeyan (2013) used DTM to obtain accurate free vibration frequencies of thin beams, but they did not consider solutions for thin beam on elastic foundation. Adair et al. (2018) utilized the Variational Iteration Method (VIM) for the natural transverse vibration analysis of Timoshenko beams resting on two-parameter elastic foundation. The VIM is a powerful semi-analytical method for solving linear and nonlinear differential equations, including ordinary and partial differential equations. Their work produced a sequence of approximate solutions which can converge to the exact solution. VIM uses correction functional with Lagrange multipli-

ers to iteratively refine the solution. Yaylı et al. (2014) presented an efficient analytical method for the free vibration analysis of a beam on elastic foundation for non-classical boundary conditions of elastically restrained ends. Franciosi and Masi (1993) integrated the free vibration studies of foundation beams resting on bi-parametric elastic foundations.

Chen (2000) used the differential quadrature element method (DQEM) for the free transverse vibration analysis of beam resting on an elastic foundation. The work combined the ideas of differential quadrature method (DQM) and finite element method (FEM) to solve the governing differential equations of the problem. The DQM uses a numerical method for approximate expressions of derivatives at a point on the solution domain using a weighted form of the function's values at the neighbouring points while the FEM provides a numerical approach for solving the resulting differential equations by discretizing the domain into smaller elements. Accurate eigenfrequency results for various classical BCs were obtained by Chen (2000). Mutman and Coşkun (2013) used Homotopy Perturbation Method (HPM) to develop accurate free vibration solutions for non-prismatic Euler-Bernoulli beams on elastic foundation.

Rahbar-Ranji and Shahbazzabar (2017) used Legendre polynomials and Rayleigh-Ritz method for the eigenfrequencies analyses of beams on Pasternak foundation; and obtained accurate results. Ike (2025a) studied the static flexural analysis of thick beam on bi-parametric elastic foundation, but did not consider vibration analysis. Motaghan et al. (2018) developed a novel Fourier series solution for the natural transverse vibration of non-prismatic beams resting on variable elastic foundation. Soltani and Asgarian (2019) used a new hybrid approach for the free vibration analysis of axially functionally graded (FG) non-prismatic variable cross-section thin beams resting on uniform Winkler-Pasternak foundation. Rao and Raju (2002) developed accurate closed form eigenfrequency solutions for the free transverse vibration of thin beams on two-parameter elastic foundation. Ghannadial and Mofid (2015) presented analytical solutions for freely vibrating elastically restrained Timoshenko beam on variable Winkler foundation. A recent study by Ike (2025b) explored the application of Ritz variational method for the natural transverse harmonic vibration solutions of thin beam resting on biparametric elastic foundations. Satisfactory natural frequencies were obtained for various boundary conditions.

A recent innovative study by Ike (2025a) applied Stodola-Vianello successive iteration method to analyze natural vibration frequencies of thick beams based on Shimpi's shear deformable model. Closed form solutions were obtained in the study but the effect of elastic foundations on the thick beam vibration was not investigated.

Another recent work by Ike (2026) applied the generalized integral transformed method to analyze the natural vibrations of thin beam on two-parameter Filonenko-Borodich elastic foundations. Exact analytical solutions were obtained in the study, which considered various end support conditions, illustrating the accuracy and versatility of the GITM. The frequency results were validated using favorable comparisons with previous results based on other solution methods.

Uzun et al. (2023) investigated the dynamics of functionally graded nano-sized beams supported on Winkler elastic foundations for different boundary conditions; but did not consider the use of Elzaki integral transformations for solving the resulting boundary value problem. Civalek and Demir (2011, 2022) studied bending and buckling beams but did not consider dynamic analysis.

Akgoz et al. (2016) investigated the static bending of beams on an elastic foundation using discrete singular convolution (DSC), but did not consider the effect of elastic foundation and restrained the work to static bending. Akgoz and Civalek (2015) studied the flexural analysis of functional graded micro beams on Winkler elastic foundations using strain gradient elasticity method. Their work however did not consider natural vibration analysis and did not use ETM.

This work explores Elzaki transform method (ETM) for the free harmonic transverse vibration analysis of Euler-Bernoulli beams resting on Winkler foundation. This work aims to apply the ETM in a step by step systematic manner to develop analytical solutions for beam-foundation vibration and illustrate the effectiveness of the ETM for each analysis. The ETM has been satisfactorily used previously by Ike (2020) and Elzaki (2023) and by Elzaki (2011, 2012a, 2012b, 2012c) and also by Elzaki and Elzaki (2011) for solving differential equations. The ETM is adopted in this work because:

- It is efficient for both linear and nonlinear problems, and can be used for partial differential equations (PDEs).
- It reduces the computation effort, compared to rigorous methods for solving differential equations.
- It is applicable to diverse problems described using differential equations in engineering physics.

1.3. Novelty of the work

The novelty of the ETM lies in its ability to transform complex differential equations that govern problems into simple, solvable algebraic equations in the Elzaki space. The novel aspects of this study consists in

- The transformation of the governing differential equations of beam foundation vibration into simple solvable algebraic equations in the Elzaki space.
- The use of step by step, systematic procedure to solve the resulting algebraic equations for the classical BCs considered.
- ETM gives exact analytical solutions without the need for the common assumptions of linearization, perturbation or discretization.
- ETM directly considers the initial conditions of the problem and avoids some of the more complex convolution or expansion processes encountered in other transform methods.

2. Theoretical Framework

2.1. Governing equation

The governing equation for the transverse vibration of thin Euler-Bernoulli beam on Winkler foundation is the non-homogeneous partial differential equation (Ike 2018a, 2026):

$$EI \frac{\partial^4 w}{\partial x^4} + kw + \rho A \frac{\partial^2 w}{\partial t^2} = q(x, t) \quad (3)$$

where $w(x, t)$ is the transverse deflection, k is the Winkler foundation parameter, x is the longitudinal coordinate, t is the time, ρ is the mass density, A is the cross-sectional area, $q(x, t)$ is the applied load, E is the modulus of elasticity, I is the moment of inertia.

For free vibration,

$$q(x, t) = 0 \quad (4)$$

Hence, the governing differential equation (GDE) of free vibration is the homogeneous equation

$$EI \frac{\partial^4 w}{\partial x^4} + kw + \rho A \frac{\partial^2 w}{\partial t^2} = 0 \quad (5)$$

For harmonic vibrations,

$$w(x, t) = W(x) \sin(\omega_n t + \phi) \quad (6)$$

where ω_n is the natural frequency, and $W(x)$ is the modal displacement function.

$$\frac{\partial^2 w}{\partial t^2} = -\omega_n^2 W(x) \sin(\omega_n t + \phi) \quad (7)$$

$$(EI W^{iv}(x) + kW(x) - \rho A \omega_n^2 W(x)) \sin(\omega_n t + \phi) = 0 \quad (8)$$

$$EI W^{iv}(x) + (k - \rho A \omega_n^2) W(x) = 0 \quad (9)$$

$$W^{iv}(x) + \left(\frac{k - \rho A \omega_n^2}{EI} \right) W(x) = 0 \quad (10)$$

$$W^{iv}(x) + (4\beta^4 - \Omega_n^4) W(x) = 0 \quad (11)$$

$$4\beta^4 = \frac{k}{EI}; \Omega_n^4 = \frac{\rho A \omega_n^2}{EI} \quad (12)$$

Let

$$\alpha_n^4 = \Omega_n^4 - 4\beta^4 \quad (13)$$

$$W^{iv}(x) - \alpha_n^4 W(x) = 0 \quad (14)$$

2.2. Elzaki transform method and its properties

The Elzaki transform is an integral transform which was derived from the Fourier integral by Elzaki (2011). The purpose was to simplify the process of solving linear ordinary and partial differential equations with constant or variable parameters. It is now a significant tool in applied mathematics, converting complex differential equations into easier algebraic equations.

The Elzaki transform of a function $f(x)$ is defined as: (Elzaki 2011, 2012a, 2012b, 2012c)

$$Tf(x) = u \int_0^\infty f(x) e^{-x/u} dx = u \lim_{\alpha \rightarrow \infty} \int_0^\alpha f(x) e^{-x/u} dx = \int_0^\infty f(x) e^{-x/u} dx = F(u) \quad (15)$$

where T is the Elzaki transform operator, u is the Elzaki transform parameter.

The Elzaki transformation transforms a function dependent on the physical domain x to another function dependent on the Elzaki transform space (domain) parameter u .

For a function $f(x, t)$ of two independent variables, x and time t , the Elzaki transform can be defined with respect to the parameters x and t as:

$$T_x f(x, t) = T(u, t) = u \int_0^\infty f(x, t) e^{-x/u} dx = u \lim_{\alpha \rightarrow \infty} \int_0^\alpha f(x, t) e^{-x/u} dx \quad (16)$$

$$T_t f(x, t) = T(x, v) = F(x, v) = v \int_0^\infty f(x, t) e^{-t/v} dt = v \lim_{\beta \rightarrow \infty} \int_0^\beta f(x, t) e^{-t/v} dt \quad (17)$$

where T_x denotes Elzaki transform with respect to x , and T_t is Elzaki transform with respect to t .

The Elzaki transform method simplifies the solution of initial value problems by transformation of function derivatives into simple algebraic expressions. The Elzaki transforms of derivatives are obtained using integration by parts as:

$$T f'(x) = u \int_0^\infty \frac{df(x)}{dx} e^{-x/u} dx \quad (18)$$

$$T f'(x) = u [f(x) e^{-x/u}]_0^\infty - \int_0^\infty f(x) \left(\frac{-e^{-x/u}}{u} \right) dx \quad (19)$$

$$T f'(x) = u \left(f(\infty) e^{-\infty} - f(0) e^0 + \frac{1}{u} \int_0^\infty f(x) e^{-x/u} dx \right) \quad (20)$$

$$T f'(x) = \int_0^\infty f(x) e^{-x/u} dx - u f(0) \quad (21)$$

$$T f'(x) = \frac{T f(x)}{u} - u f(0) = \frac{F(u)}{u} - u f(0) \quad (22)$$

$$\text{Let } g(x) = f'(x) \quad (23)$$

$$T g'(x) = T f''(x) = \frac{1}{u} T g(x) - u g(0) \quad (24)$$

$$T f''(x) = \frac{1}{u} \left(\frac{F(u)}{u} - u f(0) \right) - u f'(0) \quad (25)$$

$$T f''(x) = \frac{F(u)}{u^2} - f(0) - u f'(0) \quad (26)$$

$$\text{Let } g(x) = f''(x), g'(x) = f'''(x)$$

$$T g'(x) = T f'''(x) = \frac{1}{u} (T g(x)) - u g(0) \quad (27)$$

$$T f'''(x) = \frac{1}{u} (T f''(x)) - u f''(0) \quad (28)$$

$$T f'''(x) = \frac{1}{u} \left(\frac{F(u)}{u^2} - f(0) - u f'(0) \right) - u f''(0) \quad (29)$$

$$T f'''(x) = \frac{F(u)}{u^3} - \frac{1}{u} f(0) - f'(0) - u f''(0) \quad (30)$$

Similarly,

$$T f^{iv}(x) = \frac{1}{u} (T f'''(x)) - u f'''(0) \quad (31)$$

$$T f^{iv}(x) = \frac{1}{u} \left(\frac{F(u)}{u^3} - \frac{1}{u} f(0) - f'(0) - u f''(0) \right) - u f'''(0) \quad (32)$$

$$T f^{iv}(x) = \frac{F(u)}{u^4} - \frac{f(0)}{u^2} - \frac{1}{u} f'(0) - f''(0) - u f'''(0) \quad (33)$$

2.3. Flowchart for implementing the ETM

1. Define the governing differential equation (GDE) and initial conditions.
2. Apply the Elzaki transformation to the GDE by multiplying the GDE by $u e^{-x/u}$ and integrating with respect to x from $x = 0$ to $x = \infty$. The problem is thus converted from the space (problem) domain to the Elzaki domain, u .
3. Substitute initial conditions by incorporating the given initial conditions into the newly transformed algebraic equation in the Elzaki transform domain.
4. Rearrange and solve algebraically to obtain a solution in the Elzaki domain.
5. Apply the inverse Elzaki transform by using standard inverse Elzaki transform formulas or by using partial fraction decomposition techniques to simplify complex algebraic problems to simpler ones with existing inverses. Thus, the solution in the Elzaki domain is found in the problem domain.
6. Final solution: The final solution gives the analytical solution to the GDE subject to the initial conditions.

3. Methodology

Applying the Elzaki transformation to the governing equation yields:

$$u \int_0^\infty \left(\frac{d^4 W(x)}{dx^4} - \alpha_n^4 W(x) \right) e^{-x/u} dx = 0 \quad (34)$$

wherein, u is the Elzaki transform parameter.

Using the linear properties of the Elzaki transformation, gives:

$$u \int_0^\infty \frac{d^4 W(x)}{dx^4} e^{-x/u} dx - u \alpha_n^4 \int_0^\infty W(x) e^{-x/u} dx = 0 \quad (35)$$

Hence, simplifying Eq. (35) gives:

$$\frac{W(u)}{u^4} - \frac{W(0)}{u^2} - \frac{W'(0)}{u} - W''(0) - u W'''(0) - \alpha^4 W(u) = 0 \quad (36)$$

where

$$W(u) = u \int_0^\infty W(x) e^{-x/u} dx \quad (37)$$

Hence, further simplification gives:

$$\frac{W(u)}{u^4} - \alpha_n^4 W(u) = \frac{W(0)}{u^2} + \frac{W'(0)}{u} + W''(0) + u W'''(0) \quad (38)$$

Further simplification gives:

$$\frac{W(u) - \alpha_n^4 u^4 W(u)}{u^4} = \frac{W(0)}{u^2} + \frac{W'(0)}{u} + W''(0) + u W'''(0) \quad (39)$$

Hence,

$$W(u) \left(\frac{1-\alpha_n^4 u^4}{u^4} \right) = \frac{W(0)}{u^2} + \frac{W'(0)}{u} + W''(0) + uW'''(0) \quad (40)$$

Then,

$$W(u) = W(0) \left(\frac{u^2}{1-\alpha_n^4 u^4} \right) + W'(0) \left(\frac{u^3}{1-\alpha_n^4 u^4} \right) + W''(0) \left(\frac{u^4}{1-\alpha_n^4 u^4} \right) + W'''(0) \left(\frac{u^5}{1-\alpha_n^4 u^4} \right) \quad (41)$$

Inverting,

$$W(x) = T^{-1}W(u) \quad (42)$$

where T^{-1} is the inverse Elzaki transform of $W(u)$. Then,

$$W(x) = T^{-1} \left(W(0) \left(\frac{u^2}{1-\alpha_n^4 u^4} \right) + W'(0) \left(\frac{u^3}{1-\alpha_n^4 u^4} \right) + W''(0) \left(\frac{u^4}{1-\alpha_n^4 u^4} \right) + W'''(0) \left(\frac{u^5}{1-\alpha_n^4 u^4} \right) \right) \quad (43)$$

Using the linearity properties of the inverse transform gives:

$$W(x) = W(0)T^{-1} \left(\frac{u^2}{1-\alpha_n^4 u^4} \right) + W'(0)T^{-1} \left(\frac{u^3}{1-\alpha_n^4 u^4} \right) + W''(0)T^{-1} \left(\frac{u^4}{1-\alpha_n^4 u^4} \right) + W'''(0)T^{-1} \left(\frac{u^5}{1-\alpha_n^4 u^4} \right) \quad (44)$$

By partial fraction decomposition methods and the catalogue of inverse Elzaki transforms, the inverse Elzaki transforms are found by expressing the required inverses using partial fractions.

Let A and B be constants, then,

$$\frac{u^2}{1-\alpha_n^4 u^4} = \frac{Au^2}{1+\alpha_n^2 u^2} + \frac{Bu^2}{1-\alpha_n^2 u^2} \quad (45)$$

$$\frac{u^2}{1-\alpha_n^4 u^4} = \frac{u^2 A(1-\alpha_n^2 u^2) + Bu^2(1+\alpha_n^2 u^2)}{1-\alpha_n^4 u^4} \quad (46)$$

$$\frac{u^2}{1-\alpha_n^4 u^4} = \frac{u^2(A+B) + u^4 \alpha_n^2(B-A)}{1-\alpha_n^4 u^4} \quad (47)$$

Hence, $A + B = 1$ and $B - A = 0$

Solving,

$$A = B = \frac{1}{2} \quad (48)$$

$$\frac{u^2}{1-\alpha_n^4 u^4} = \frac{1}{2} \left(\frac{u^2}{1+\alpha_n^2 u^2} \right) + \frac{1}{2} \left(\frac{u^2}{1-\alpha_n^2 u^2} \right) \quad (49)$$

$$T^{-1} \left(\frac{u^2}{1-\alpha_n^4 u^4} \right) = \cosh(\alpha_n x) \quad (50)$$

$$T^{-1} \left(\frac{u^2}{1+\alpha_n^2 u^2} \right) = \cos(\alpha_n x) \quad (51)$$

Hence,

$$T^{-1} \left(\frac{u^2}{1-\alpha_n^4 u^4} \right) = \frac{1}{2} (\cosh(\alpha_n x) + \cos(\alpha_n x)) \quad (52)$$

Similarly,

$$\frac{u^3}{1-\alpha_n^4 u^4} = \frac{A\alpha_n u^3}{1-\alpha_n^2 u^2} + \frac{B\alpha_n u^3}{1+\alpha_n^2 u^2} \quad (53)$$

$$\frac{u^3}{1-\alpha_n^4 u^4} = \frac{A\alpha_n u^3(1+\alpha_n^2 u^2) + B\alpha_n u^3(1-\alpha_n^2 u^2)}{1-\alpha_n^4 u^4} \quad (54)$$

$$\frac{u^3}{1-\alpha_n^4 u^4} = \frac{A\alpha_n u^3 + B\alpha_n u^3 + (A\alpha_n u^3 - B\alpha_n u^3)\alpha_n^2 u^2}{1-\alpha_n^4 u^4} \quad (55)$$

$$u^3 \equiv (A+B)\alpha_n u^3 + (A-B)\alpha_n^3 u^5 \quad (56)$$

$$A-B=0; (A+B)\alpha_n=1; A=B=\frac{1}{2\alpha_n} \quad (57)$$

$$\frac{u^3}{1-\alpha_n^4 u^4} = \frac{1}{2\alpha_n} \left(\frac{\alpha_n u^3}{1-u^2 \alpha_n^2} + \frac{\alpha_n u^3}{1+\alpha_n^2 u^2} \right) \quad (58)$$

$$T^{-1} \left(\frac{u^3}{1-\alpha_n^4 u^4} \right) = \frac{1}{2\alpha_n} \left(T^{-1} \left(\frac{\alpha_n u^3}{1-u^2 \alpha_n^2} \right) + T^{-1} \left(\frac{\alpha_n u^3}{1+\alpha_n^2 u^2} \right) \right) \quad (59)$$

$$T^{-1} \frac{u^3}{1-\alpha_n^4 u^4} = \frac{1}{2\alpha_n} (\sinh(\alpha_n x) + \sin(\alpha_n x)) \quad (60)$$

Also,

$$\frac{u^4}{1-\alpha_n^4 u^4} = \frac{Au^2}{1-\alpha_n^2 u^2} + \frac{Bu^2}{1+\alpha_n^2 u^2} \quad (61)$$

$$\frac{u^4}{1-\alpha_n^4 u^4} = \frac{Au^2(1+\alpha_n^2 u^2) + Bu^2(1-\alpha_n^2 u^2)}{1-\alpha_n^4 u^4} \quad (62)$$

$$u^4 = (A+B)u^2 + \alpha_n^2 u^4(A-B) \quad (63)$$

$$A+B=0; (A-B)\alpha_n^2=1; A=-B; A-B=\frac{1}{\alpha_n^2}=2A;$$

$$A=\frac{1}{2\alpha_n^2}, B=-\frac{1}{2\alpha_n^2} \quad (64)$$

Hence,

$$\frac{u^4}{1-\alpha_n^4 u^4} = \frac{1}{2\alpha_n^2} \left(\frac{u^2}{1-\alpha_n^2 u^2} - \frac{u^2}{1+\alpha_n^2 u^2} \right) \quad (65)$$

$$T^{-1} \left(\frac{u^4}{1-\alpha_n^4 u^4} \right) = \frac{1}{2\alpha_n^2} \left(T^{-1} \left(\frac{u^2}{1-\alpha_n^2 u^2} \right) - T^{-1} \left(\frac{u^2}{1+\alpha_n^2 u^2} \right) \right) \quad (66)$$

$$T^{-1} \left(\frac{u^4}{1-\alpha_n^4 u^4} \right) = \frac{1}{2\alpha_n^2} (\cosh(\alpha_n x) - \cos(\alpha_n x)) \quad (67)$$

Similarly,

$$\frac{u^5}{1-\alpha_n^4 u^4} = \frac{A\alpha_n u^3}{1-\alpha_n^2 u^2} + \frac{B\alpha_n u^3}{1+\alpha_n^2 u^2} \quad (68)$$

$$\frac{u^5}{1-\alpha_n^4 u^4} = \frac{A\alpha_n u^3(1+\alpha_n^2 u^2) + B\alpha_n u^3(1-\alpha_n^2 u^2)}{1-\alpha_n^4 u^4} \quad (69)$$

$$u^5 \equiv (A+B)\alpha_n u^3 + \alpha_n^3 u^5(A-B) \quad (70)$$

Comparing like terms

$$A+B=0; (A-B)\alpha_n^3=1; A=-B; 2A\alpha_n^3=1; A=\frac{1}{2\alpha_n^3}, B=-\frac{1}{2\alpha_n^3} \quad (71)$$

$$\frac{u^5}{1-\alpha_n^4 u^4} = \frac{1}{2\alpha_n^3} \left(\frac{\alpha_n u^3}{1-\alpha_n^2 u^2} - \frac{\alpha_n u^3}{1+\alpha_n^2 u^2} \right) \quad (72)$$

$$T^{-1} \left(\frac{u^5}{1-\alpha_n^4 u^4} \right) = \frac{1}{2\alpha_n^3} \left(T^{-1} \left(\frac{\alpha_n u^3}{1-\alpha_n^2 u^2} \right) - T^{-1} \left(\frac{\alpha_n u^3}{1+\alpha_n^2 u^2} \right) \right) \quad (73)$$

$$T^{-1} \left(\frac{u^5}{1-\alpha_n^4 u^4} \right) = \frac{1}{2\alpha_n^3} (\sinh(\alpha_n x) - \sin(\alpha_n x)) \quad (74)$$

Hence,

$$W(x) = W(0) \left(\frac{\cosh \alpha_n x + \cos \alpha_n x}{2} \right) + W'(0) \left(\frac{\sinh \alpha_n x + \sin \alpha_n x}{2\alpha_n} \right) + W''(0) \left(\frac{\cosh \alpha_n x - \cos \alpha_n x}{2\alpha_n^2} \right) + W'''(0) \left(\frac{\sinh \alpha_n x - \sin \alpha_n x}{2\alpha_n^3} \right) \quad (75)$$

Differentiating,

$$W'(x) = W(0)\alpha_n \left(\frac{\sinh \alpha_n x - \sin \alpha_n x}{2} \right) + W'(0) \left(\frac{\cosh \alpha_n x + \cos \alpha_n x}{2} \right) + W''(0) \left(\frac{\sinh \alpha_n x + \sin \alpha_n x}{2\alpha_n} \right) + W'''(0) \left(\frac{\cosh \alpha_n x - \cos \alpha_n x}{2\alpha_n^2} \right) \quad (76)$$

$$W''(x) = W(0)\alpha_n^2 \left(\frac{\cosh \alpha_n x - \cos \alpha_n x}{2} \right) + W'(0)\alpha_n \left(\frac{\sinh \alpha_n x - \sin \alpha_n x}{2} \right) + W''(0) \left(\frac{\cosh \alpha_n x + \cos \alpha_n x}{2} \right) + W'''(0) \left(\frac{\sinh \alpha_n x + \sin \alpha_n x}{2\alpha_n} \right) \quad (77)$$

$$W'''(x) = W(0)\alpha_n^3 \left(\frac{\sinh \alpha_n x + \sin \alpha_n x}{2} \right) + W'(0)\alpha_n^2 \left(\frac{\cosh \alpha_n x - \cos \alpha_n x}{2} \right) + W''(0)\alpha_n \left(\frac{\sinh \alpha_n x - \sin \alpha_n x}{2} \right) + W'''(0) \left(\frac{\cosh \alpha_n x + \cos \alpha_n x}{2} \right) \quad (78)$$

4. Results and Discussion

4.1. Results

4.1.1. Results for simply supported EBBowF

The results are presented for simply supported EBBowF shown in Fig. 3.

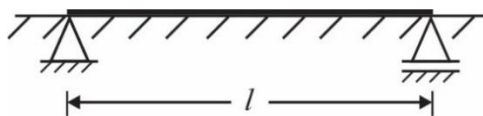


Fig. 3. Simply supported EBBowF.

The BCs for the simply supported EBBowF shown in Fig. 3 are:

$$W(0) = W(l) = 0, \quad W''(0) = W''(l) = 0 \quad (79)$$

Hence,

$$\begin{pmatrix} \left(\frac{\sinh \alpha_n l + \sin \alpha_n l}{2\alpha_n} \right) & \left(\frac{\sinh \alpha_n l - \sin \alpha_n l}{2\alpha_n^3} \right) \\ \alpha_n \left(\frac{\sinh \alpha_n l - \sin \alpha_n l}{2} \right) & \left(\frac{\sinh \alpha_n l + \sin \alpha_n l}{2\alpha_n} \right) \end{pmatrix} \begin{pmatrix} W'(0) \\ W'''(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (80)$$

For nontrivial solutions, the characteristic equation is:

$$\alpha_n \begin{pmatrix} \left(\frac{\sinh \alpha_n l + \sin \alpha_n l}{2\alpha_n} \right) & \left(\frac{\sinh \alpha_n l - \sin \alpha_n l}{2\alpha_n^3} \right) \\ \left(\frac{\sinh \alpha_n l - \sin \alpha_n l}{2} \right) & \left(\frac{\sinh \alpha_n l + \sin \alpha_n l}{2\alpha_n} \right) \end{pmatrix} = 0 \quad (81)$$

Expanding,

$$\left(\frac{(\sinh \alpha_n l + \sin \alpha_n l)^2}{4\alpha_n^2} \right) - \left(\frac{(\sinh \alpha_n l - \sin \alpha_n l)^2}{4\alpha_n^2} \right) = 0 \quad (82)$$

$$\frac{1}{4\alpha_n^2} [(\sinh^2 \alpha_n l + 2 \sinh \alpha_n l \sin \alpha_n l + \sin^2 \alpha_n l) - (\sinh^2 \alpha_n l - 2 \sinh \alpha_n l \sin \alpha_n l + \sin^2 \alpha_n l)] = 0 \quad (83)$$

Simplifying,

$$\frac{1}{4\alpha_n^2} 4 \sinh \alpha_n l \sin \alpha_n l = 0 \quad (84)$$

or,

$$\frac{\sinh \alpha_n l \sin \alpha_n l}{\alpha_n^2} = 0 \quad (85)$$

Hence,

$$\sin \alpha_n l = 0; \text{ or, } \sinh \alpha_n l = 0 \quad (86)$$

$$\alpha_n l = \sin^{-1} 0 = n\pi \quad (87)$$

$$\alpha_n = \frac{n\pi}{l} \quad (88)$$

$$\alpha_n^4 = -4\beta^4 + \Omega_n^4 = \left(\frac{n\pi}{l} \right)^4 \quad (89)$$

Hence,

$$\Omega_n^4 = 4\beta^4 + \left(\frac{n\pi}{l} \right)^4 = \frac{\rho A \omega_n^2}{EI} \quad (90)$$

Thus,

$$\omega_n^2 = \frac{EI}{\rho A} \left(4\beta^4 + \left(\frac{n\pi}{l} \right)^4 \right) = \frac{EI}{\rho A} \left(\frac{(n\pi)^4}{l^4} + 4\beta^4 \right) \quad (91)$$

$$\omega_n^2 = \frac{EI}{\rho A} \left(\frac{(n\pi)^4 + 4\beta^4 l^4}{l^4} \right) = \frac{EI}{\rho A l^4} ((n\pi)^4 + 4\beta^4 l^4) = \frac{EI}{\rho A l^4} ((n\pi)^4 + K_0) \quad (92)$$

where,

$$K_0 = 4\beta^4 l^4 \quad (92a)$$

K_0 is the beam foundation parameter.

Hence,

$$\omega_n = \frac{1}{l^2} \sqrt{\frac{EI}{\rho A} ((n\pi)^4 + 4\beta^4 l^4)^{1/2}} \quad (93)$$

or,

$$\omega_n = \frac{1}{l^2} \sqrt{\frac{EI}{\rho A}} \lambda_n^2 \quad (94)$$

where,

$$\lambda_n^2 = ((n\pi)^4 + 4\beta^4 l^4)^{1/2} = ((n\pi)^4 + K_0)^{1/2} \quad (95)$$

The values of λ_n are calculated for $n = 1, 2, 3, 4, 5$ and for $4\beta^4 l^4 = 1, 10, 100, 1000, 10,000$ and presented in Table 1 along with previous solutions by Rahbar-Ranji and Shahbazzabar (2017), Zhou (1993) and Ike (2023).

Table 1. Dimensionless frequency parameters λ_n for the first five modes of simply supported EBBowF.

$K_0 = 4\beta^4 l^4$	λ_1					λ_2				
	Present study	% Difference between present results and Ike (2023, 2024), Kacer et al. (2011)	Ike (2023, 2024b), Kacer et al. (2011)	Rahbar-Ranji and Shahbazztabar (2017)	Zhou (1993)	Present study	% Difference between present results and Ike (2023, 2024), Kacer et al. (2011)	Ike (2023, 2024b), Kacer et al. (2011)	Rahbar-Ranji and Shahbazztabar (2017)	Zhou (1993)
1	3.1496	(0%)	3.1496	3.1496	-	6.2842	(0%)	6.2842	6.2843	-
10	3.2193	(0%)	3.2193	3.2193	3.2200	6.2932	(0%)	6.2932	6.2932	6.2930
100	3.7484	(0%)	3.7484	3.7484	3.7480	6.3816	(0%)	6.3816	6.3816	6.3820
1000	5.7556	(0%)	5.7556	5.7556	5.7550	7.1121	(0%)	7.1121	7.1121	7.1120
10,000	10.0243	(0%)	10.0243	10.0243	-	10.3687	(0%)	10.3687	10.3687	-

$K_0 = 4\beta^4 l^4$	λ_3					λ_4				
	Present study	% Difference between present results and Ike (2023, 2024), Kacer et al. (2011)	Ike (2023, 2024b), Kacer et al. (2011)	Rahbar-Ranji and Shahbazztabar (2017)	Zhou (1993)	Present study	% Difference between present results and Ike (2023, 2024), Kacer et al. (2011)	Ike (2023, 2024b), Kacer et al. (2011)	Rahbar-Ranji and Shahbazztabar (2017)	Zhou (1993)
1	9.4251	(0%)	9.4251	9.4251	-	12.5665	(0%)	12.5665	12.5665	-
10	9.4278	(0%)	9.4278	9.4277	9.4270	12.5676	(0%)	12.5676	12.5676	12.5680
100	9.4545	(0%)	9.4545	9.4545	9.4540	12.5789	(0%)	12.5789	12.5790	12.5790
1000	9.7102	(0%)	9.7102	9.7102	9.7100	12.6905	(0%)	12.6905	12.6905	12.6900
10,000	11.5652	(0%)	11.5652	11.5652	-	13.6716	(0%)	13.6716	13.6716	-

$K_0 = 4\beta^4 l^4$	λ_5				
	Present study	% Difference between present results and Ike (2023, 2024), Kacer et al. (2011)	Ike (2023, 2024b), Kacer et al. (2011)	Rahbar-Ranji and Shahbazztabar (2017)	Zhou (1993)
1	15.7080	(0%)	15.7080	15.7080	-
10	15.7086	(0%)	15.7086	15.7086	15.7090
100	15.7144	(0%)	15.7144	15.7144	15.7150
1000	15.7721	(0%)	15.7721	15.7721	-
10,000	16.3167	(0%)	16.3167	16.3167	-

4.1.2. EBBowF with clamped ends $x = 0, x = l$

The natural vibration of EBBowF with both left and right ends clamped as shown in Fig. 4 is considered.

**Fig. 4.** EBBowF with clamped ends at $x = 0$ and $x = l$.

The BCs are $W(0) = W'(0) = 0$; $W(l) = W'(l) = 0$
Hence,

$$\begin{pmatrix} \left(\frac{\cosh \alpha_n l - \cos \alpha_n l}{2\alpha_n^2} \right) & \left(\frac{\sinh \alpha_n l - \sin \alpha_n l}{2\alpha_n^3} \right) \\ \left(\frac{\sinh \alpha_n l + \sin \alpha_n l}{2\alpha_n} \right) & \left(\frac{\cosh \alpha_n l - \cos \alpha_n l}{2\alpha_n^2} \right) \end{pmatrix} \begin{pmatrix} W''(0) \\ W'''(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (96)$$

For nontrivial solutions,

$$\begin{pmatrix} \left(\frac{\cosh \alpha_n l - \cos \alpha_n l}{2\alpha_n^2} \right) & \left(\frac{\sinh \alpha_n l - \sin \alpha_n l}{2\alpha_n^3} \right) \\ \left(\frac{\sinh \alpha_n l + \sin \alpha_n l}{2\alpha_n} \right) & \left(\frac{\cosh \alpha_n l - \cos \alpha_n l}{2\alpha_n^2} \right) \end{pmatrix} = 0 \quad (97)$$

Expanding,

$$\left(\frac{\cosh \alpha_n l - \cos \alpha_n l}{2\alpha_n^2} \right)^2 - \frac{1}{4\alpha_n^4} (\sinh^2 \alpha_n l - \sin^2 \alpha_n l) = 0 \quad (98)$$

Expanding,

$$\frac{1}{4\alpha_n^4} (\cosh^2 \alpha_n l - 2 \cosh \alpha_n l \cos \alpha_n l + \cos^2 \alpha_n l - \sinh^2 \alpha_n l + \sin^2 \alpha_n l) = 0 \quad (99)$$

$$\therefore \cosh^2 \alpha_n l - \sinh^2 \alpha_n l + \cos^2 \alpha_n l + \sin^2 \alpha_n l - 2 \cosh \alpha_n l \cos \alpha_n l = 0 \quad (100)$$

Using trigonometric identities,

$$\cosh^2 \alpha_n l - \sinh^2 \alpha_n l = 1; \cos^2 \alpha_n l + \sin^2 \alpha_n l = 1 \quad (101)$$

Eq. (100) simplifies to Eq. (102)

$$2 - 2 \cosh \alpha_n l \cos \alpha_n l = 0 \quad (102)$$

Hence,

$$\cosh \alpha_n l \cos \alpha_n l = 1 \quad (103)$$

The eigenvalues are the roots which are extracted using Newton Raphson's iteration. Eq. (103) is expressed in function form as:

$$f(x) = \cosh x \cos x - 1 = 0 \quad (104)$$

By differentiation,

$$f'(x) = \sinh x \cos x - \cosh x \sin x \quad (105)$$

The Newton-Raphson iterative relationship is the recurrence formula:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad (106)$$

$$x_{n+1} = x_n - \frac{\cosh x_n \cos x_n - 1}{\sinh x_n \cos x_n - \cosh x_n \sin x_n} \quad (107)$$

For first iteration, $x_0 = 4.70$

• Iteration 1

$$x_1 = 4.70 - \frac{\cosh 4.7 \cos 4.7 - 1}{\sinh 4.7 \cos 4.7 - \cosh 4.7 \sin 4.7} = 4.730964$$

• Iteration 2

$$x_2 = 4.730964 - \frac{f(4.730964)}{f'(4.730964)} = 4.730042$$

• Iteration 3

$$x_3 = 4.730042 - \frac{f(4.730042)}{f'(4.730042)} = 4.730041$$

Table 2 shows the Newton-Raphsons iteration calculations presented in table format. The eigenvalues have converged to six decimal places since the next iteration yields.

The first positive non-trivial solution to Eq. (103) obtained using Newton-Raphson's iteration is

$$x_3 = 4.730041 \quad (108)$$

Table 2. Newton-Raphson's Iteration calculation for first natural frequencies from $f(x) = \cosh x \cos x - 1 = 0$.

n	x_n	$f(x_n)$	$f'(x_n)$	x_{n+1}	Error
0	4.70	-1.6811066	54.292922	4.730964	-1.6811066
1	4.730964	0.053270	57.750148	4.730042	0.053270
2	4.730042	7.235313×10^{-5}	57.645663	4.730041	7.235313×10^{-5}
3	4.730041	1.470753×10^{-5}	57.645549	4.730041	1.470753×10^{-5}
4	4.730041	1.470753×10^{-5}			1.470753×10^{-5}

Solving using Mathematica software, or Online Equation Solver of Wolfram Alpha, the first five eigenvalues of Eq. (103) are found as

$$\alpha_1 l = 4.73004; \alpha_2 l = 7.85321; \alpha_3 l = 10.9956$$

$$\alpha_4 l = 14.13717; \alpha_5 l = 17.27876 \quad (109)$$

Hence,

$$\alpha_n^4 = \Omega_n^4 - 4\beta^4 \quad (110)$$

Rearranging,

$$\Omega_n^4 = \alpha_n^4 + 4\beta^4 = \frac{\rho A \omega_n^2}{EI} \quad (111)$$

$$\omega_n^2 = \frac{EI}{\rho A} (\alpha_n^4 + 4\beta^4) \quad (112)$$

$$\omega_n^2 = \frac{EI}{\rho A l^4} (\alpha_n^4 l^4 + 4\beta^4 l^4) \quad (113)$$

$$\omega_n = \frac{1}{l^2} \sqrt{\frac{EI}{\rho A}} \lambda_n^2 \quad (114)$$

$$\lambda_n^2 = (\alpha_n^4 l^4 + 4\beta^4 l^4)^{1/2} = (\alpha_n^4 l^4 + K_0)^{1/2} \quad (115)$$

The values of λ_n are calculated for $n = 1, 2, 3, 4$ and for $4\beta^4 l^4 = 1, 10, 100, 1000, 10,000$ and presented in Table 3 along with previous results by Ike (2023), Kacer et al. (2011) and Balkaya et al. (2009). Table 4 shows the values of λ_n^2 for EBBWF with clamped boundaries for $\rho = A = l = E = 1, 4\beta^4 l^4 = 1$ together with previous results using Homotopy Perturbation method by Mutman and Coşkun (2013), Balkaya et al. (2009), Chen (2000) and Ike (2023).

4.1.3. EBBWF clamped at $x = l$ and simply supported at $x = 0$

The natural vibration of EBBWF with simple support at $x = 0$ and clamped at $x = l$ as shown in Fig. 5 is considered.



Fig. 5. Vibration of EBBWF clamped at $x = l$ and simply supported at $x = 0$.

Table 3. Natural vibration frequency parameters λ_n for the first four vibration modes of clamped-clamped beam on Winkler foundation.

$K_0 = 4\beta^4 l^4$	λ_1				λ_2			
	Present study	% Difference between present results and Ike (2023)	Ike (2023)	Balkaya et al. (2009), Kacer et al. (2011)	Present study	% Difference between present results and Ike (2023)	Ike (2023)	Balkaya et al. (2009), Kacer et al. (2011)
1	4.7324	(0%)	4.7324	4.7324	7.8537	(0%)	7.8537	7.8537
10	4.7535	(0%)	4.7535	4.7535	7.8584	(0%)	7.8584	7.8584
100	4.9504	(0%)	4.9504	4.9504	7.9043	(0%)	7.9043	7.9043
1000	6.2239	(0%)	6.2239	6.2239	8.3251	(0%)	8.3251	8.3251
10,000	10.1229	(0%)	10.1229	-	10.8392	(0%)	10.8392	-

$K_0 = 4\beta^4 l^4$	λ_3				λ_4			
	Present study	% Difference between present results and Ike (2023)	Ike (2023)	Balkaya et al. (2009), Kacer et al. (2011)	Present study	% Difference between present results and Ike (2023)	Ike (2023)	Balkaya et al. (2009), Kacer et al. (2011)
1	10.9958	(0%)	10.9958	10.9958	14.1373	(0%)	14.1373	14.1373
10	10.9975	(0%)	10.9975	10.9975	14.1381	(0%)	14.1381	14.1381
100	11.0144	(0%)	11.0144	11.0144	14.1460	(0%)	14.1460	14.1460
1000	11.1790	(0%)	11.1790	11.1790	14.2248	(0%)	14.2248	14.2248
10,000	12.5260	(0%)	12.5260	-	14.9493	(0%)	14.9493	14.9493

Table 4. Natural frequencies, ω_n for clamped-clamped EB beam resting on Winkler foundation for $\rho = A = I = E = l = 1$, $K_0 = 4\beta^4 l^4 = 1$, $\omega_n = \lambda_n^2$.

Method	ω_1	ω_2	ω_3	ω_4
Present method	22.3956	61.6810	120.9074	199.8621
Ike (2023)	22.3956	61.6810	120.9074	199.8621
HPM (Mutman and Coşkun 2013)	22.3956	61.6809	120.9080	199.8620
DTM (Balkaya et al. 2009)	22.3733	61.6728	120.9030	199.8590
ADM (Mutman and Coşkun 2013)	22.3956	61.6809	120.9080	199.8620
DQEM (Chen 2000)	22.3956	61.6811	120.9100	199.8850

The BCs are $W(0) = 0$, $W''(0) = 0$, $W(l) = 0$, $W'(l) = 0$.
Hence,

$$\begin{pmatrix} \frac{(\sinh \alpha_n l + \sin \alpha_n l)}{2\alpha_n} \\ \frac{(\cosh \alpha_n l + \cos \alpha_n l)}{2} \end{pmatrix} \begin{pmatrix} \frac{(\sinh \alpha_n l - \sin \alpha_n l)}{2\alpha_n^3} \\ \frac{(\cosh \alpha_n l - \cos \alpha_n l)}{2\alpha_n^2} \end{pmatrix} \begin{pmatrix} W'(0) \\ W'''(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (116)$$

For nontrivial solutions,

$$\begin{pmatrix} \frac{(\sinh \alpha_n l + \sin \alpha_n l)}{2\alpha_n} \\ \frac{(\cosh \alpha_n l + \cos \alpha_n l)}{2} \end{pmatrix} \begin{pmatrix} \frac{(\sinh \alpha_n l - \sin \alpha_n l)}{2\alpha_n^3} \\ \frac{(\cosh \alpha_n l - \cos \alpha_n l)}{2\alpha_n^2} \end{pmatrix} = 0 \quad (117)$$

Expanding,

$$\frac{1}{4\alpha_n^3} [(\sinh \alpha_n l + \sin \alpha_n l)(\cosh \alpha_n l - \cos \alpha_n l) - (\sinh \alpha_n l - \sin \alpha_n l)(\cosh \alpha_n l + \cos \alpha_n l)] = 0 \quad (118)$$

Hence,

$$\begin{aligned} &(\sinh \alpha_n l \cosh \alpha_n l + \sin \alpha_n l \cosh \alpha_n l - \cos \alpha_n l \sinh \alpha_n l - \sin \alpha_n l \cos \alpha_n l) - \\ &(\sinh \alpha_n l \cosh \alpha_n l - \sin \alpha_n l \cosh \alpha_n l + \sinh \alpha_n l \cos \alpha_n l - \sin \alpha_n l \cos \alpha_n l) = 0 \end{aligned} \quad (119)$$

Simplifying,

$$2 \sin \alpha_n l \cosh \alpha_n l - 2 \cos \alpha_n l \sinh \alpha_n l = 0 \quad (120)$$

$$\therefore \sin \alpha_n l \cosh \alpha_n l = \cos \alpha_n l \sinh \alpha_n l \quad (121)$$

$$\sin \alpha_n l = \cos \alpha_n l \frac{\sinh \alpha_n l}{\cosh \alpha_n l} \quad (122)$$

$$\frac{\sin \alpha_n l}{\cos \alpha_n l} = \frac{\sinh \alpha_n l}{\cosh \alpha_n l} \quad (123)$$

$$\tan \alpha_n l = \tanh \alpha_n l \quad (124)$$

The equation $f(x) = \tan x - \tanh x = 0$ has a trivial solution at $x = 0$, since $f(x = 0) = 0$, but its first non-trivial positive root is $x \approx 3.926602$. The first non-trivial positive root can be extracted using Newton-Raphson iteration method. The iteration scheme is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{\tan x_n - \tanh x_n}{\sec^2 x_n - \operatorname{sech}^2 x_n} \quad (125)$$

The iteration can start at $x_0 = \frac{5\pi}{4} = 3.927$

Then,

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 3.927 - \frac{0.000988}{1.999393} = 3.926506$$

Similarly,

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}; x_2 = 3.926506 - \frac{(-0.000192)}{1.998214} = 3.926602$$

Then,

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 3.926602$$

The iteration has converged at $x = 3.926602$

The calculations are summarized in Table 5.

Table 5. Newton-Raphson iteration calculations for $f(x) = \tan x - \tanh x = 0$.

Iteration n	x_1	$f(x_n)$	$f'(x_n)$	x_{n+1}	Error
0	3.92700	0.000412	1.9904	3.92679	0.000412
1	3.92679	0.000002	1.9998	3.92660	0.000002
2	3.92660	0.000000	2.0000	3.92660	0.000000
3	3.92660	0.000000	2.0000		0.000000

Solving by Mathematica software, or Online Equation Solver of Wolfram Alpha the first four non-trivial eigenvalues are:

$$\begin{aligned} \alpha_1 l &= 3.92660246; \alpha_2 l = 7.068583; \\ \alpha_3 l &= 10.2107612; \alpha_4 l = 13.3517688 \end{aligned} \quad (126)$$

The natural frequencies are calculated for $n = 1, 2, 3, 4$ and for $4\beta^4 l^4 = 1, 10, 100, 1000$, and presented in Table 6 together with previous results by Ike (2023), Coşkun (2003), Mutman and Coşkun (2013).

Table 6. Natural frequency parameters for first four modes of transverse vibration of SC thin beams on Winkler foundation.

$K_0 = 4\beta^4 l^4$	λ_1				λ_2			
	Present study	% Difference between present results and GITM results	GITM (Ike 2023)	HPM (Coşkun 2003; Mutman and Coşkun 2013)	Present study	% Difference between present results and GITM results	GITM (Ike 2023)	HPM (Coşkun 2003; Mutman and Coşkun 2013)
1	3.9307	(0%)	3.9307	3.9308	7.0693	(0%)	7.0693	7.0693
10	3.9673	(0%)	3.9673	-	7.0757	(0%)	7.0757	-
100	4.2867	(0%)	4.2869	-	7.1383	(0%)	7.1383	-
1000	5.9314	(0%)	5.9314	-	7.6897	(0%)	7.6897	-

$K_0 = 4\beta^4 l^4$	λ_3				λ_4			
	Present study	% Difference between present results and GITM results	GITM (Ike 2023)	HPM (Coşkun 2003; Mutman and Coşkun 2013)	Present study	% Difference between present results and GITM results	GITM (Ike 2023)	HPM (Coşkun 2003; Mutman and Coşkun 2013)
1	10.2104	(0%)	10.2104	10.2104	13.3519	(0%)	13.3519	13.3519
10	10.2125	(0%)	10.2125	-	13.3529	(0%)	13.3529	-
100	10.2336	(0%)	10.2336	-	13.3623	(0%)	13.3623	-
1000	10.4374	(0%)	10.4374	-	13.4556	(0%)	13.4556	-

4.1.4. Cantilever EBBWF

The natural vibration of EBBWF clamped at the left and $x = 0$ and free at the right end ($x = l$) as shown in Fig. 6 is considered.

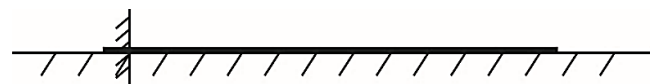


Fig. 6. Natural vibration of EBBWF with clamped end ($x = 0$) and free at $x = l$.

The BCs are:

At the clamped end, $x = 0$,

$$w(x = 0, t) = 0 \quad (127)$$

$$w'(x = 0, t) = 0 \Rightarrow W'(x) = 0 \quad (128)$$

At the free-end, $M(x = l, t) = -EIw''(x = l, t) = 0$

Hence,

$$W''(x = l) = 0$$

$$Q(x = l, t) = -EIw'''(x = l, t) = 0 \quad (129)$$

Hence,

$$W'''(x = l) = 0 \quad (130)$$

Hence,

$$\begin{pmatrix} \left(\frac{\cosh \alpha_n l + \cos \alpha_n l}{2} \right) & \left(\frac{\sinh \alpha_n l + \sin \alpha_n l}{2\alpha_n} \right) \\ \alpha_n \left(\frac{\sinh \alpha_n l - \sin \alpha_n l}{2} \right) & \left(\frac{\cosh \alpha_n l + \cos \alpha_n l}{2} \right) \end{pmatrix} \begin{pmatrix} W''(0) \\ W'''(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (131)$$

For nontrivial solutions,

$$\begin{pmatrix} \left(\frac{\cosh \alpha_n l + \cos \alpha_n l}{2} \right) & \left(\frac{\sinh \alpha_n l + \sin \alpha_n l}{2\alpha_n} \right) \\ \alpha_n \left(\frac{\sinh \alpha_n l - \sin \alpha_n l}{2} \right) & \left(\frac{\cosh \alpha_n l + \cos \alpha_n l}{2} \right) \end{pmatrix} = 0 \quad (132)$$

Expanding the determinant gives:

$$\frac{(\cosh \alpha_n l + \cos \alpha_n l)^2}{4} - \frac{(\sinh^2 \alpha_n l - \sin^2 \alpha_n l)}{4} = 0 \quad (133)$$

$$\frac{1}{4}(\cosh^2 \alpha_n l + 2 \cosh \alpha_n l \cos \alpha_n l + \cos^2 \alpha_n l - \sinh^2 \alpha_n l - \sin^2 \alpha_n l) = 0 \quad (134)$$

Using trigonometric identities in Eq. (101), we have:

$$\frac{1}{4}(2 + 2 \cosh \alpha_n l \cos \alpha_n l) = 0 \quad (135)$$

or,

$$\cosh \alpha_n l \cos \alpha_n l = -1 \quad (136)$$

Rewriting Eq. (130) as a standard function, $f(x) = \cosh x \cos x + 1 = 0$

The Newton-Raphson iteration formula for the root is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{\cosh x_n \cos x_n + 1}{\sinh x_n \cos x_n - \cosh x_n \sin x_n} \quad (137)$$

Since $f(x = 1) > 0$ and $f(x = 2) < 0$, a real root occurs within the internal $1 < x < 2$.

Let $x_0 = 2$

The Newton-Raphson iteration calculations are presented in Table 7.

Using different initial guesses of the roots yields the successive roots. Second root is obtained as $x = 4.694091$ using the initial guess $x_0 = 4.50$. The third root is obtained as $x = 7.854757$ using the initial guess $x_0 = 7.5$.

The eigenvalues extracted using mathematical software tools, Mathematica and Newton-Raphson's iterative methods for obtaining roots of transcendental equations are:

$$\alpha_1 l = 1.87510; \alpha_2 l = 4.69409; \alpha_3 l = 7.85476; \alpha_4 l = 10.99554; \alpha_n l = \frac{(2n-1)\pi}{2} \quad \text{for } n \geq 5 \quad (138)$$

The dimensionless frequency parameters for the first four modes and for $4\beta^4 l^4 = 1, 10, 100, 1000$, are presented in Table 8.

Table 7. Newton-Raphson iteration for $f(x) = \cosh x \cos x + 1 = 0$.

Iteration n	Approximate root x_n	Function value $f(x_n)$	Derivative value $f'(x_n)$	Error
0	2.000000	-0.565646	-4.930441	-0.565646
1	1.885275	-0.046399	-4.137837	-0.046399
2	1.874062	-0.003926	-4.062095	-0.003926
3	1.875104	0.0000	-4.069150	0.0000
4	1.875104	0.0000	-4.069150	0.0000

4.2. Discussion

This study has explored the Elzaki transform method for the closed form analytical solutions of free harmonic transverse vibration frequencies of prismatic Euler-Bernoulli beams resting on Winkler foundations for classical boundary conditions. The assumption of harmonic response simplified the governing partial differential equation to an ordinary differential equation (ODE) in the spatial variable. The Elzaki transformation of the ODE simplified it to an algebraic equation in the Elzaki transform space. It also considered the initial conditions. The inversion of the Elzaki transformed solution gave

the general solution in terms of the physical (problem) domain variable. The enforcement of boundary conditions gave the characteristic frequency equations for the various BCs cases considered. The expansion of the determinants resulted in transcendental equations whose eigenvalues were used to find the natural frequencies of vibration for each considered BC. The dimensionless eigenfrequencies λ_n for the first five vibratory modes $n = 1, 2, 3, 4, 5$ and for $K_0 = 4\beta^4 l^4 = 1, 10, 100, 1000, 10000$ are presented in Table 1 for EBBWF with simply supported ends. A comparison of present results and previous results shows that the present results are identical with previous results by Ike (2023, 2024b) who used

Generalized Integral Transform Method (GITM), and Kacer et al. (2011). Table 1 further shows that the present

results agree remarkably well with previous results by Rahbar-Ranji and Shahbazzabar (2017) and Zhou (1993).

Table 8. Dimensionless frequency parameters λ_n for the first four transverse vibratory modes of cantilever EBBowF.

$K_0 = 4\beta^4 l^4$	λ_1				λ_2			
	Present study	% Difference between present results and Ike (2023) and Kacer et al. (2011)	Ike (2023)	Kacer et al. (2011)	Present study	% Difference between present results and Ike (2023) and Kacer et al. (2011)	Ike (2023)	Kacer et al. (2011)
1	1.9119	(0%)	1.9119	1.9119	4.6965	(0%)	4.6965	4.6965
10	2.1746	(0%)	2.1746	2.1746	4.7181	(0%)	4.7181	4.7181
100	3.2558	(0%)	3.2558	3.2558	4.9191	(0%)	4.9191	4.9191
1000	5.6407	(0%)	5.6407	5.6407	6.2083	(0%)	6.2083	6.2083

$K_0 = 4\beta^4 l^4$	λ_3				λ_4			
	Present study	% Difference between present results and Ike (2023) and Kacer et al. (2011)	Ike (2023)	Kacer et al. (2011)	Present study	% Difference between present results and Ike (2023) and Kacer et al. (2011)	Ike (2023)	Kacer et al. (2011)
1	7.8553	(0%)	7.8553	7.8553	10.9957	(0%)	10.9957	10.9957
10	7.8599	(0%)	7.8599	7.8599	10.9974	(0%)	10.9974	10.9974
100	7.9058	(0%)	7.9058	7.9058	11.0143	(0%)	11.0143	11.0143
1000	8.3264	(0%)	8.3264	8.3264	11.1790	(0%)	11.1790	11.1790

The eigenfrequency λ_n results for EBBowF with clamped ends are presented in Table 3 for the first four vibratory modes $n = 1, 2, 3, 4$ and for foundation parameter values $K_0 = 4\beta^4 l^4 = 1, 10, 100, 1000, 10,000$. Table 2 shows that the present results for λ_n are identical with previous results that used GITM by Ike (2023), and remarkably identical to previous results by Kacer et al. (2011) and Balkaya et al. (2009). Table 4 also shows the present results for λ_n^2 for EBBowF with both ends clamped for $\rho = A = I = E = 1$ and $4\beta^4 l^4 = 1$. Table 4 confirms that present results for λ_n^2 are identical with previous results that used GITM by Ike (2023) and close to previous results by Mutman and Coşkun (2013), Balkaya et al. (2009) and Chen (2000).

The dimensionless natural frequencies λ_n for EBBowF simply supported at $x = 0$ and clamped at $x = l$ are shown in Table 6 for the first four vibratory modes $n = 1, 2, 3, 4$ and for the foundation parameters $4\beta^4 l^4 = 1, 10, 100, 1000$. Table 6 also compares the present results with previous results by Ike (2023), Coşkun (2003) and Mutman and Coşkun (2013). Table 6 illustrates that the present results by for λ_n are identical to previous results by Ike (2023) and close to previous results by Coşkun (2003) and Mutman and Coşkun (2013) which considered only the case for the value of foundation parameter $K_0 = 4\beta^4 l^4 = 1$.

The results for the dimensionless natural frequency parameters for cantilever EBBowF are presented for the first four vibratory modes $n = 1, 2, 3, 4$ and for foundation parameters $K_0 = 4\beta^4 l^4 = 1, 10, 100, 1000$ in Table 8. Table 8 also displays the previous results by Ike (2023) that used GITM and previous results by Kacer et al. (2011). Table 5 shows that the present results for λ_n are identi-

cal with previous results by Ike (2023) and Kacer et al. (2011). Table 5 shows that the cantilever BC intrinsically yields the lowest natural frequency. This is because the clamped free end does not have intermediate or end supports that constrains transverse deflection, rendering it inherently more flexible when compared to beams with other classical BCs.

The natural transverse vibration frequency results obtained in this work using ETM for all the BCs considered have greatly illustrated the mathematical simplicity, computational efficiency, ease of implementation and broader applicability of the ETM as compared to other previously used methods.

The percentage error between natural vibration frequency results and the results obtained using GITM is nil for all the BC considered. This is because like the GITM, the ETM gives exact analytical solutions to the free vibration problem of EBBowF considered, within the assumptions and theoretical framework of Euler-Bernoulli theory, Winkler foundation model, linear elastic behaviour, small displacement elasticity theory and classical boundary conditions.

When the Winkler foundation parameter tends to zero, the values of natural frequencies approach the values of natural frequencies of corresponding thin beam (without foundations).

Graphical depictions of the results presented in Tables 1, 2, 3, 4 and 5 are shown in Figs. 7, 8, 9 and 10. Fig. 7 is a composite graph that visually illustrates the variations of the first natural frequency λ_1 with the foundation parameter K_0 for the classical boundary conditions considered in this study. Fig. 7 illustrates that for given foundation parameters K_0 , the fundamental natural frequencies

increase in the order: clamped-free (C-free) BC, simply supported (SS) BC, simple-clamped (SC) BC and clamped-clamped (CC) BC. The CC BC gives the highest λ_1 , followed by SC, then SS and the lowest λ_1 is obtained for C-free BCs. Fig. 8 is a composite graph that presents the variation of the first four natural frequencies with foundation parameter K_0 for clamped-clamped EBBowF. Fig. 8 shows that the natural frequencies increase with increase in foundation parameter and increase in vibratory mode.

Fig. 9 presents a composite bar chart showing the variation of the first natural frequencies λ_1 of EBBowF with boundary conditions for foundation parameters $K_0 = 1, 10, 100, 1000$. Fig. 9 shows that the least value of λ_1 is

found for cantilever EBBowF for any given foundation parameter. It also shows that the greatest value of λ_1 is found for clamped-clamped EBBowF for any given foundation parameter. Similarly, Fig. 10 presents a composite bar chart showing the variation of the natural frequencies for the second vibration mode (λ_2) of EBBowF with boundary conditions for values of foundation parameter $K_0 = 1, 10, 100, 1000$. Fig. 10 shows that for the second vibration mode, least λ_2 occurs for cantilever EBBowF for any value of foundation parameter K_0 . Fig. 10 also shows that the greatest value of λ_2 occurs for EBBowF with both ends clamped for any given foundation parameter.

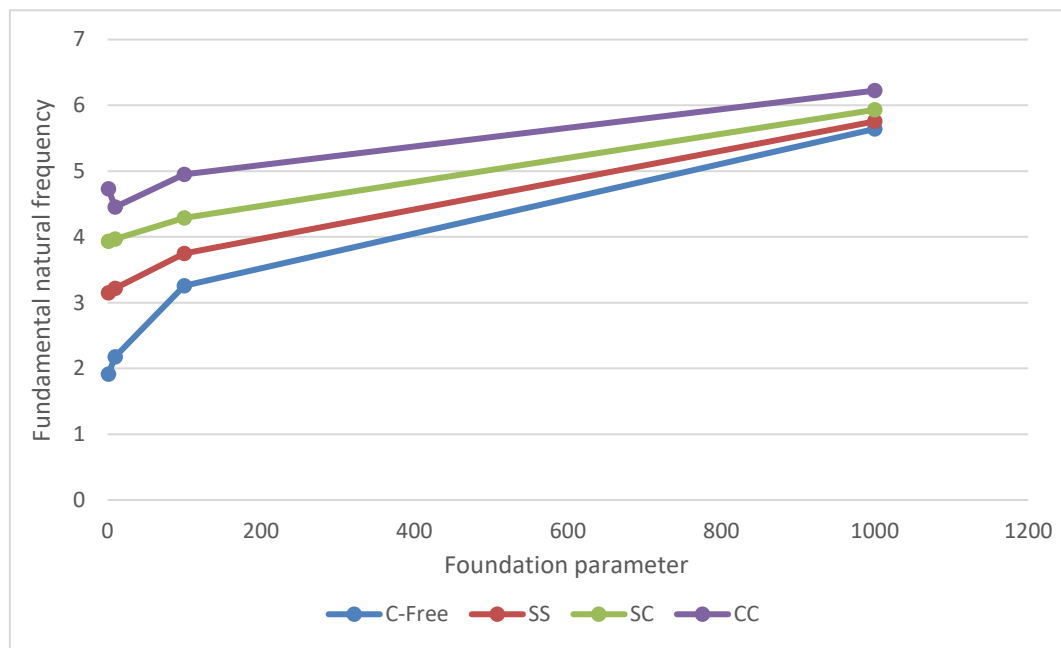


Fig. 7. Graph of variation of λ_1 with K_0 for various support conditions of EBBowF.

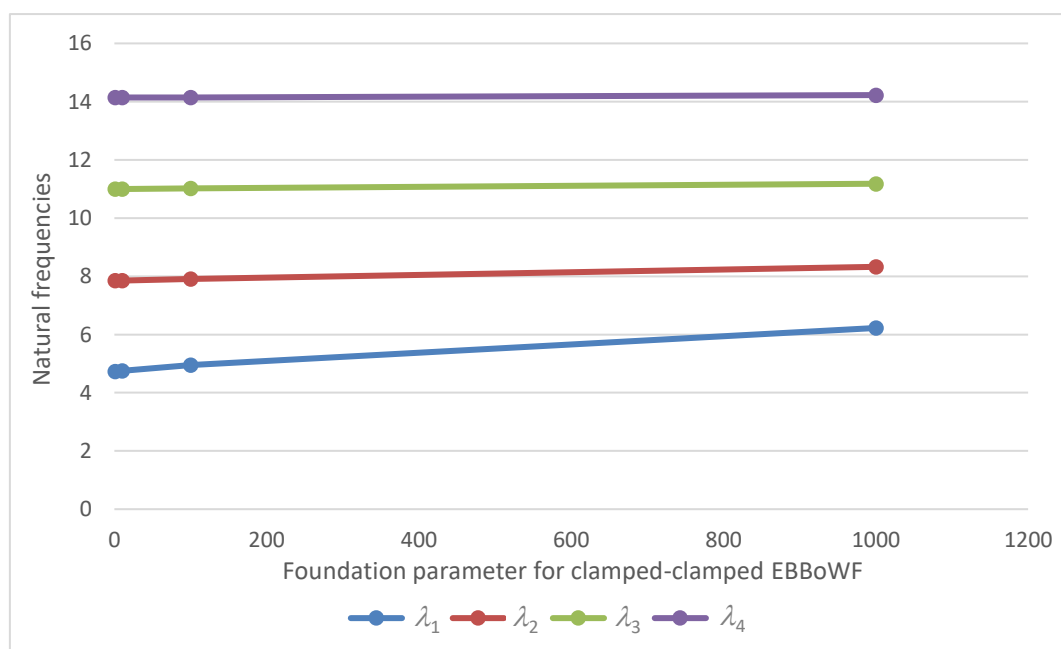


Fig. 8. Variation of first four natural frequencies with K_0 .

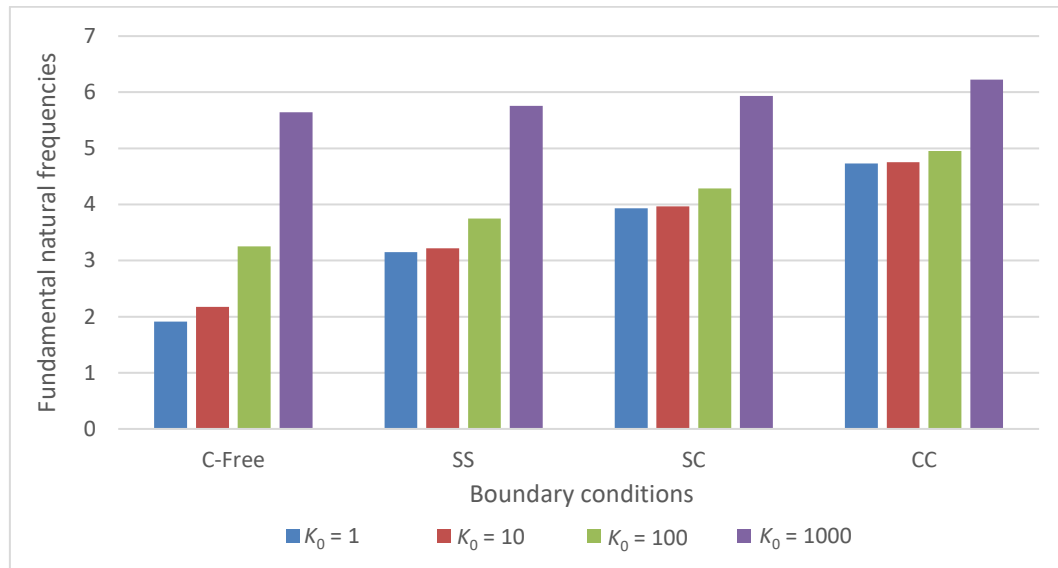


Fig. 9. Composite bar chart showing variation of first natural frequencies λ_1 of EBBowF with boundary conditions for foundation parameters; $K_0 = 1, 10, 100, 1000$.

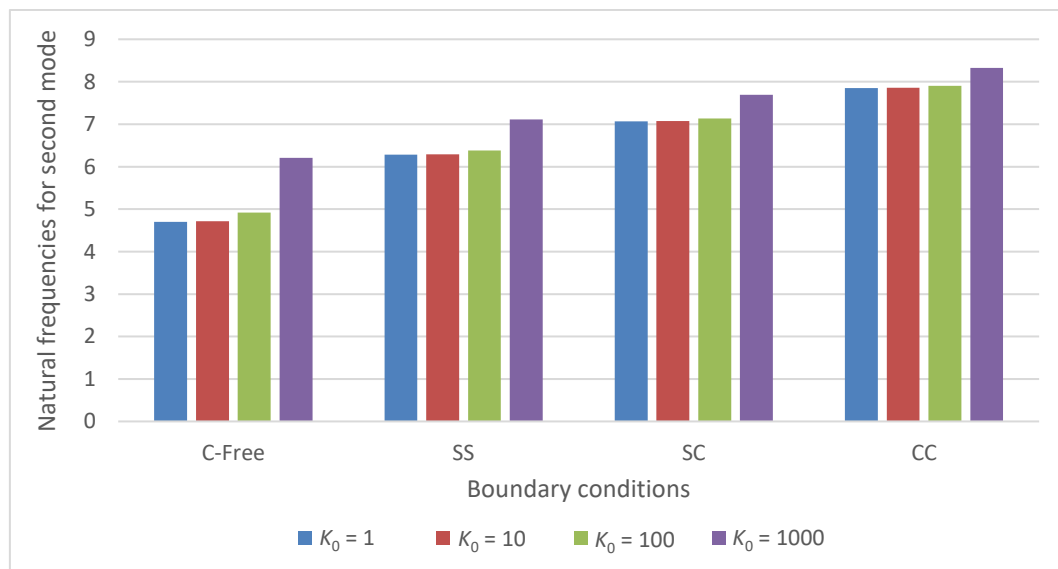


Fig. 10. Variation of natural frequencies for second mode of vibrating EBBowF with different boundary conditions for $K_0 = 1, 10, 100, 1000$.

4.2.1. Discussion on computational efficiency of the ETM and convergence of root extraction iteration schemes

Though the ETM does not require domain mesh refinements, prior selection of basis (trial) functions for BCs satisfaction, spatial discretization or perturbation procedures, numerical methods of root extraction are still required for the transcendental characteristic equations which result when BCs are considered. The ETM offers considerable simplification by converting the governing differential equation into more easily solvable algebraic equations in the frequency domain of Elzaki space. It also natively incorporates initial conditions, removes the necessity for prior general solutions and circumvents the intractable matrix exponentiation requirements of conventional techniques. The simplification is achieved through the core advantages of the integral transform method.

The basic advantage of the ETM and other integral transform methods is that it converts the governing differential equation from a problem of calculus to an algebraic problem which incorporates the initial conditions. Traditional methods of solving differential equations require the determination of the complete general solution by the superposition principle as the sum of the homogeneous and particular solutions, and subsequent applications of initial conditions in order to determine the arbitrary integration constants. The ETM on the other hand directly incorporates the given initial boundary values into the simplified algebraic equation from the initial steps of the formulation. This significantly improves comprehension.

The Elzaki transformation of the governing equation is applied on the entire spatial domain, and hence spatial discretization is not needed. The enforcements of BCs resulted in characteristic equations which are found as

transcendental equations because they contain both trigonometric and hyperbolic functions.

The characteristic eigenfrequency equation for simply supported EBBWF was found as Eq. (85). The characteristic eigenfrequency equation for EBBWF with both ends clamped was found as Eq. (103). The characteristic eigenfrequency equation of EBBWF clamped at the right end and simply supported at the left end was found as Eq. (124).

The characteristic natural frequency equation for cantilever EBBWF was found as Eq. (136). The characteristic natural frequency equations for all the BCs are transcendental equations that are solved for using numerical methods. Numerical methods seek to approximate the roots by starting with an interval or initial guess and repeatedly performing iterations using already established numerical root finding iteration schemes.

The commonly used numerical root extraction methods are (a) Bisection method (Bolzano method), (b) Regula-Falsi method and (c) Newton-Raphson method.

Software applications have been designed for fast implementation of the root extraction of transcendental functions. Some of them are Wolfram Alpha and Wolfram Mathematica, Python, Matlab and Maple. The transcendental equation solvers use Newton-Raphson's numerical iteration algorithms, or other established numerical iteration schemes.

The Newton-Raphson iteration method was used to manually determine the first natural frequency of EBBWF with both ends clamped. The iterations rapidly converged at the third iteration and the error was zero for the root computed to six decimal places. The first five non-trivial roots were determined using Online Wolfram Alpha Equation Solver as given by Eq. (109).

Similarly, the Newton-Raphson iteration method was used to manually compute the first natural frequency of EBBWF with simple support at $x = 0$ and clamped support at $x = l$. The Newton-Raphson's iteration calculations are presented in Table 5 for an initial guess at $x_0 = 3.927$. Table 5 shows a rapid convergence after two iterations with an error of zero and eigenvalue of $x = 3.92660$ to five decimal places. The first four natural frequencies were determined for the SC boundary conditions using Online Wolfram Alpha Equation Solver as Eq. (126).

The Newton-Raphson iteration solution of the characteristic frequency equation is presented in Table 7 for an initial guess of $x_0 = 2.0$. Table 7 relied on Eq. (137) to compute successive approximations of the root of the characteristic frequency equation. Table 7 shows that the root converged at the third iteration with zero error, illustrating the rapidly convergent nature of the Newton-Raphson iteration. The first four non-trivial eigenvalues were found using Online Wolfram Alpha Equation solver and other mathematical software tools as Eq. (138). The dimensionless forms of the natural frequencies for each considered BC were then obtained from α_n/l by using Eq. (115) and inputting the various values of the foundation parameters.

The application of ETM to EBBWF can be complicated by foundation discontinuities, variable Winkler foundation moduli and complex inverse transformations, all of which limit its scope and effectiveness in

the absence of supplementary numerical techniques. ETM inherently assume smooth continuous functions across the beam's longitudinal axis. Some practical beam foundation problems involve discontinuous loads, variable beam cross-sections and non-classical BCs. Discontinuous loads, variable cross-sections and non-classical BCs disrupt the transformation process and introduce piecewise functions or Dirac delta functions, thereby complicating the transformation to an algebraic problem. Also, when the Winkler foundation modulus varies along the longitudinal (x) axis, thus making K become dependent on x , the governing equation would contain variable parameters. It would be very difficult to apply the ETM to such variable parameter differential equations without converting them to complex differential forms, thus defeating the initial objective of a swift transform solution.

5. Conclusions

This work has explored the ETM for the exact analytical solutions of free harmonic transverse vibrations of EBBWF for the following BCs: simply supported at both ends, clamped at both ends, simply supported at left end and clamped at right end, and clamped at left end and free at the right. In conclusion:

- The assumption of harmonic response decouples the PDE to a fourth order ODE in terms of the space variable.
- Application of the ETM to the ODE gives an integral equation which further simplifies to an algebraic equation in the Elzaki transform parameter. Solving and inverting the solution in the transformed space gives the general solution.
- Applying the BCs for the four considered cases gave the characteristic frequency equation as a transcendental equation whose roots are extracted using mathematical software tools such as Wolfram Mathematica, Mathcad or iterative methods such as the Newton-Raphson iteration.
- The present eigenfrequency solutions for simply supported EBBWF are exact within the assumptions and theoretical framework of Euler-Bernoulli theory, Winkler foundation model, linear elastic behaviour, small displacement elasticity theory and classical boundary conditions (for all five modes and foundation parameters).
- The present eigenfrequency solutions are identical with previous solutions obtained using Generalized Integral Transform Method (GITM) and Stodola-Vianello iteration method (SVIM), and are close to previous solutions based on DTM and Rayleigh-Ritz method using Legendre polynomials as basis functions.
- For all the four vibration modes, and foundation parameters considered, the present eigenfrequency (natural frequency) solutions for the other classical BCs are exact within the assumptions and theoretical framework of Euler-Bernoulli theory, Winkler foundation model, linear elastic behaviour, small displacement elasticity theory and classical boundary conditions.

tions and are identical with previous solutions obtained using GITM and close to previous solutions using DTM.

- The natural frequencies increase as the foundation parameters $K_0 = 4\beta^4 l^4$ increase for all the vibration modes and for all the classical boundary conditions considered. This is because the elastic foundation increases the dynamic stiffness of the beam on elastic foundation system, thus raising the natural frequency.
- The key advantages of ETM include the following:
 - It does not require domain discretization.
 - The BC that produces the lowest natural frequencies is the cantilever BC. This is because it lacks intermediate or boundary support, making it more flexible than other BCs.
 - The BC that produces the highest natural frequency is the clamped-clamped BC. This is because the clamped-clamped ends restrict transverse deflection, making the structure inherently more rigid.
 - The effects of the elastic foundation on the first few natural frequencies are to increase the natural frequencies of vibration.
- The ETM has potential future extensions to other more complex applications, such as non-uniform beam, Timoshenko beam theory, non-linear foundations, damping effects and thick beams.

5.1. Limitations of the ETM

The ETM is a good mathematical tool for solving linear ordinary and partial differential equations. However, its main limitations include the difficulties in extending to nonlinear partial differential equations without hybrid modifications, restricted use to complex initial conditions, lack of widespread software support unlike the established Laplace or Fourier transforms.

5.2. Recommendations for future research

In order to overcome some of the limitations of ETM, further research could explore the use of recently developed generalized and modified variants of ETM, such as Fractional Elzaki transforms and Elzaki decomposition methods EDMs. Then future research could explore the applications to studies of nonhomogeneous EBBowF, discontinuous EBBowF, and EBBowF with nonclassical BCs.

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Appendix A. Elzaki transform table

$$T(x^n) = n! u^{n+2}$$

$$T e^{ax} = \frac{u^2}{1-au}$$

$$T \sin(ax) = \frac{au^3}{1+a^2u^2}$$

$$T \cos(ax) = \frac{au^2}{1+a^2u^2}$$

$$T \sinh(ax) = \frac{au^3}{1-a^2u^2}$$

$$T \cosh(ax) = \frac{au^2}{1-a^2u^2}$$

Hence,

$$T^{-1}u^n = \frac{x^{n-2}}{(n-2)!}; \quad n = 2, 3, 4, \dots$$

$$T^{-1}\left(\frac{u^2}{1-au}\right) = a^{ax}$$

$$T^{-1}\frac{u^3}{1+a^2u^2} = \frac{1}{a} \sin(ax)$$

$$T^{-1}\frac{u^2}{1+a^2u^2} = \frac{1}{a} \cos(ax)$$

$$T^{-1}\frac{u^3}{1-a^2u^2} = \frac{1}{a} \sinh(ax)$$

$$T^{-1}\frac{u^2}{1-a^2u^2} = \frac{1}{a} \cosh(ax)$$

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Conflict of Interest

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Data Availability

The datasets generated and/or analyzed during the current study are not publicly available but are available from the corresponding author upon reasonable request.

AI Assistance

No AI-based tools were used in the preparation of this manuscript.

Author Contributions

All authors made substantial contributions to the conception and design of the study, acquisition of data, analysis and interpretation of data; drafted or critically revised the manuscript for important intellectual content; and approved the final version to be published.

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Research Article

Seismic performance and strain localization in a historic masonry minaret: A comparative evaluation of internal steel strengthening strategies

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ABSTRACT

In this study, the seismic performance of a historic masonry minaret was investigated through nonlinear time-history analyses, and the relationship between global stiffness enhancement and local strain localization was evaluated. Four different internal steel strengthening scenarios, developed within a finite element model constructed by jointly considering experimental studies conducted on similar stone specimens and parameter ranges proposed in the literature, were comparatively analyzed under three distinct ground motion records. The modal analysis results indicated that the natural frequencies increased significantly in all strengthening scenarios, demonstrating a substantial improvement in overall structural stiffness. However, nonlinear dynamic analyses revealed that an increase in stiffness does not necessarily lead to an improvement in seismic performance. Displacement-time histories showed that the onset of instability was delayed in certain strengthening configurations; nevertheless, none of the interventions was able to completely prevent global collapse. The maximum principal strain distributions demonstrated that the collapse behavior was governed by tensile-dominated strain localization developing at the shaft-substructure transition region. In particular, strengthening configurations exhibiting partial continuity were found to induce stiffness discontinuities that could trigger early instability. The results indicate that the seismic performance of masonry minarets is closely related not only to global stiffness enhancement but also to the tensile strength, damage-softening characteristics, and energy dissipation capacity of the material. The study provides significant implications for performance-based strengthening design of masonry structures with cultural heritage value.

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1. Introduction

Türkiye is located within one of the world's most active seismic regions due to the interaction of the Anatolian plate with the Eurasian, African, and Arabian plates, which has generated major tectonic structures such as the North Anatolian Fault, the East Anatolian

Fault, and the Western Anatolian graben system (Bozkurt 2001). As a result, destructive earthquakes occur periodically across the country, causing significant loss of life and property as well as extensive damage to cultural heritage structures. The 6 February 2023 Kahramanmaraş earthquakes once again highlighted the seismic vulnerability of both the existing

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building stock and historic masonry structures in Türkiye (Kocaman 2023).

Anatolia hosts a rich inventory of historic structures dating from the Roman, Byzantine, Seljuk, and Ottoman periods, including mosques, madrasas, churches, bridges, towers, and monumental buildings. Among these structures, masonry minarets represent a distinctive architectural and structural typology. Their slender geometries, masonry construction with limited tensile capacity, and often partially independent structural interaction with the main mosque building make them particularly vulnerable to seismic actions (Yılmaz et al. 2025), consistent with broader risk assessments identifying tower-type historic masonry structures among the highest-priority seismic risk categories (Kahraman et al. 2026).

Historic masonry minarets are generally characterized by a cylindrical shaft supported on a square or polygonal base and terminated by a balcony and conical spire. Structural discontinuities such as transition zones,

door openings, and internal staircases introduce localized weaknesses in the load-bearing system. Under seismic loading, these features lead to high displacement demands, particularly in the upper shaft, balcony, and spire regions, where damage frequently concentrates (Usta 2021).

Recent earthquakes have repeatedly confirmed the seismic vulnerability of such structures. Field observations following the 6 February 2023 events showed that, although many mosque bodies remained largely intact, their minarets either collapsed or experienced severe structural damage. The observed damage patterns closely correspond to collapse mechanisms reported in the literature, with critical damage zones typically occurring at the base-shaft transition, beneath the balcony, along the upper shaft, and within the spire segment (Kocaman et al. 2025). Fig. 1 illustrates typical damage mechanisms observed in historic masonry minarets after earthquakes in Türkiye.



Fig. 1. Examples of historic masonry minarets damaged following earthquakes in Türkiye (Kocaman et al. 2025).

Various strengthening techniques have been proposed to improve the seismic performance of historic masonry minarets. These interventions generally aim to reduce displacement demands associated with their slender geometries and to control damage mechanisms developing in critical regions of the structure.

FRP- and CFRP-based wrapping systems represent one of the commonly investigated approaches. More broadly, fiber- and polymer-based composite systems have been investigated across structural applications for improving mechanical performance (Çelik et al. 2024; Mohd Nazri et al. 2025; Ünal et al. 2025; Khalbous et al.

2026). Jadallah et al. (2025) demonstrated through nonlinear time-history analyses of the Şirvani Minaret that FRP confinement effectively limited tensile crack propagation, particularly in the upper shaft and spire regions, and reduced displacement demands under repeated earthquake excitations. Although FRP strengthening provided only a limited increase in global stiffness, it proved effective in controlling damage progression.

Alternative strategies focus on improving axial continuity and global load transfer along the shaft. Bayraktar and Hökelekli (2021) reported that vertically oriented FRCM-based reinforcement elements acting as longitudinal strengthening components significantly reduced tensile stresses and displacement demands when soil-structure interaction was considered. Such approaches directly influence the global structural response of the minaret.

Circumferential strengthening techniques based on steel bands or tension rings aim to enhance sectional integrity in regions characterized by geometric discontinuities. Kocaman et al. (2025) showed through nonlinear dynamic analyses of the Gaziantep Kabasakal Mosque minaret that circumferential steel rings reduced displacement demands in the upper regions and prevented the development of global collapse mechanisms by improving circumferential stiffness.

More recently, mechanically anchored strengthening techniques have also been proposed. Cavuşlu and Ülger (2025) demonstrated that iron clamp and dowel systems applied to a nineteenth-century stone minaret improved the continuity between masonry units and reduced stress concentrations, particularly in transition regions. Their results highlighted the advantages of mechanically anchored interventions in terms of minimal intervention and reversibility.

Overall, previous studies show that different strengthening techniques improve seismic behavior through distinct structural mechanisms. However, most investigations focus on a single intervention applied to a specific minaret. Comparative evaluations of different strengthening strategies on the same structure and systematic assessments of their influence on damage mechanisms remain limited. Consequently, the selection of an optimal strengthening strategy depends on multiple factors including the geometric characteristics of the minaret, existing damage conditions, target performance levels, conservation constraints, and implementation feasibility.

Within this framework, the present study investigates the historic masonry minaret of Yakutiye Madrasa located in Erzurum, a region characterized by significant seismic hazard. A detailed finite element model was developed to represent the geometric and structural characteristics of the minaret, with material parameters defined through a combination of field observations, experimental data obtained from stone specimens, and information reported in the literature. Three representative ground motion records were scaled to match the target design spectrum, and nonlinear time-history analyses were conducted to evaluate the existing seismic response and potential damage mechanisms.

Based on the identified vulnerabilities, four strengthening scenarios were developed in accordance with the

conservation principle of minimum intervention and evaluated under identical seismic inputs. These scenarios include (i) a stepped internal plate system integrated along the spiral staircase, (ii) localized strengthening elements applied in critical transition regions, (iii) a distributed internal steel plate system along the shaft, and (iv) a hybrid system combining internal steel plates with longitudinal reinforcement elements. All strengthening configurations were analyzed using the same nonlinear dynamic framework to allow direct comparison of their structural performance.

Through this approach, the study systematically evaluates the relative effectiveness of different strengthening strategies on the seismic response of a historic masonry minaret under identical structural and seismic conditions. Rather than proposing a single strengthening solution, the objective is to provide a mechanism-based comparative framework that can support the selection of appropriate intervention strategies for similar historic masonry towers.

2. Yakutiye Madrasa and Minaret

Yakutiye Madrasa was constructed in the early 14th century during the Anatolian Seljuk–Ilkhanid period and represents one of the most significant monumental structures located in the city center of Erzurum. The building is considered an authentic example of the Anatolian madrasa tradition and reflects the architectural, artistic, and structural characteristics of Seljuk-era educational complexes. Due to their outstanding cultural and historical significance, Anatolian madrasas are recognized within the UNESCO World Heritage Tentative List framework, and Yakutiye Madrasa is considered one of the most prominent representatives of this architectural tradition (UNESCO 2014). Therefore, the structure possesses not only architectural importance but also significant cultural heritage value requiring careful preservation.

The minaret, constructed adjacent to the madrasa, constitutes one of the most visually distinctive and structurally sensitive components of the complex. It consists of a cylindrical shaft seated on a square base and terminated by a conical spire. According to the geometric dimensions illustrated in Fig. 2, the total height of the minaret is approximately 24.7 m, the external diameter of the shaft is about 3.18 m, and the base dimensions are approximately 3.18 m × 3.18 m. The internal spiral staircase core has a diameter of approximately 0.50 m. Abrupt cross-sectional transitions at the base–shaft and shaft–spire interfaces create geometric discontinuities that represent critical regions where damage may concentrate under seismic loading.

The load-bearing system of the minaret consists of irregularly arranged masonry stone units. Over time, the mortar layers between the stone blocks have partially lost their mechanical properties due to environmental effects and material aging, which is particularly relevant because mortar bond and physical properties strongly influence the integrity of masonry systems (Kara et al. 2025). The negligible tensile strength of masonry and the discontinuous nature of stone–mortar interfaces

make such systems highly susceptible to crack initiation and brittle behavior under seismic loading (Lourenço 2006). In addition, the internal spiral staircase extending

along the shaft introduces localized cross-sectional weaknesses at certain elevations, reducing the continuity of the load-bearing system.

General view of Yakutiye Madrasah

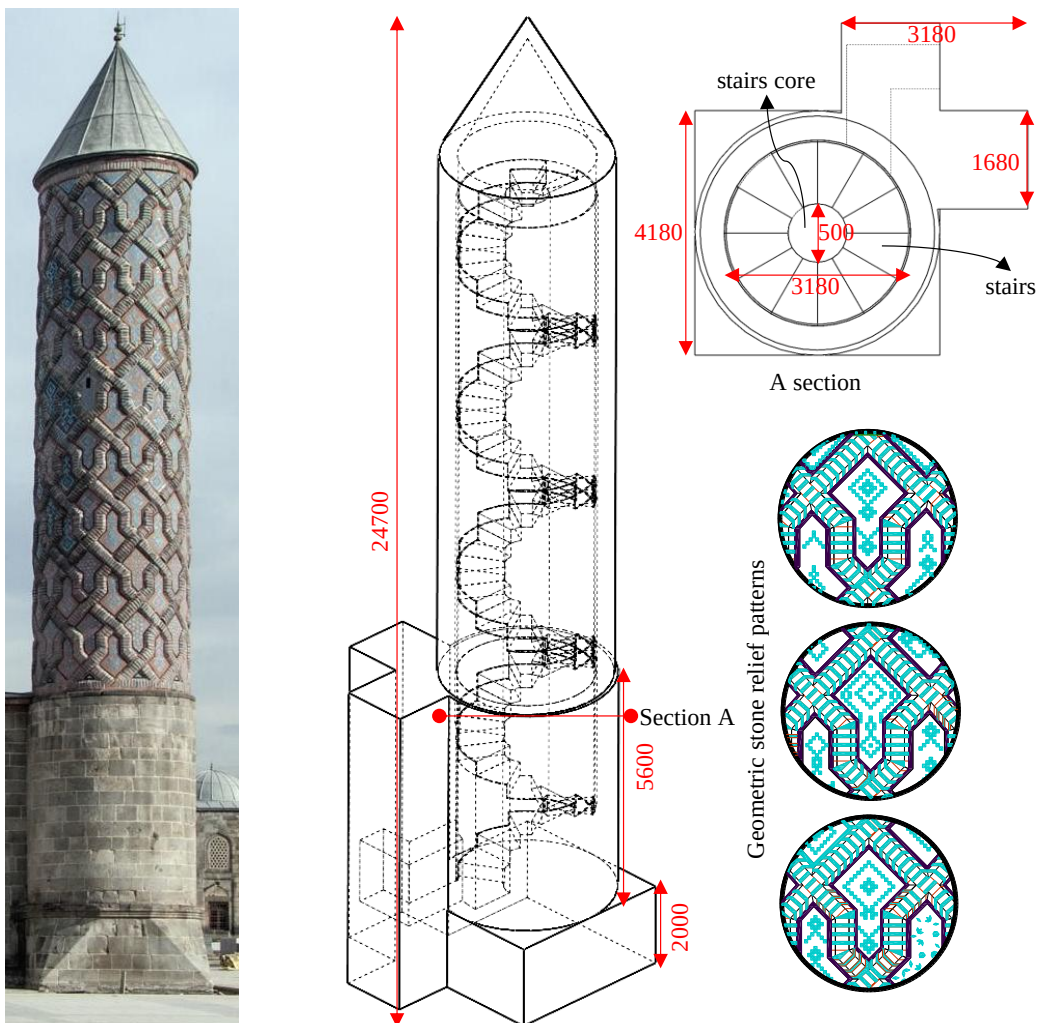


Fig. 2. Details of Erzurum Yakutiye Madrasa.

The exterior surface of the minaret is decorated with interlaced geometric stone reliefs, representing one of the most refined examples of Seljuk stone craftsmanship. Although these decorative elements enhance the cultural and artistic significance of the structure, they also create local protrusions and recesses on the surface, leading to geometric irregularities. Previous studies indicate that such relief-type geometries may influence local stiffness distribution and generate stress concentrations in certain regions (Binda et al. 2000). Therefore, these geometric features should be considered when evaluating the seismic behavior of the structure.

The Yakutiye Madrasa minaret has largely preserved its original materials and construction techniques and has not undergone any major reconstruction interventions. However, the harsh climatic conditions of Erzurum, particularly freeze–thaw cycles and large temperature fluctuations, have contributed to the gradual deterioration of both stone and mortar materials. Such environmental effects may lead to progressive weakening of material properties and consequently influence the seismic performance of the structure (ICOMOS 2003).

Considering these characteristics collectively, the Yakutiye Madrasa minaret represents a significant case study for seismic performance assessment and strengthening strategies. As an authentic Seljuk-period cultural heritage structure that has preserved its original construction features while exhibiting geometric and material discontinuities, the minaret provides a valuable opportunity to investigate the seismic behavior of historic masonry towers.

3. Numerical Modeling Approach and Finite Element Model

In recent years, the seismic resilience of historic structures such as churches, bridges, mosques, towers, and minarets has received increasing attention in earthquake engineering research (Kocaman et al 2025; Ferrante et al 2021; Orlando 2023; Colak Demirel 2024; Requena-Garcia-Cruz 2023). The preservation of these structures requires reliable prediction of their behavior under seismic loading and accurate identification of potential damage mechanisms. However, assessing historic masonry structures remains challenging due to their complex geometries, traditional construction techniques, heterogeneous material properties, and the limited availability of experimental data. Additional uncertainties related to boundary conditions, transition regions, connection details, and past repair interventions further increase the complexity of numerical analyses. Consequently, the reliable evaluation of their seismic performance requires the use of advanced numerical modeling approaches (Kocaman and Gürbüz 2023; Çelik et al. 2025; Kiral et al. 2024; Romero-Sanchez et al. 2024; Ergün and Tayfur 2024).

Two main numerical modeling strategies are commonly adopted in the analysis of historic masonry structures: micro-modeling and macro-modeling. In the micro-modeling approach, masonry constituents such as stone units and mortar layers are modeled separately, allowing detailed investigation of local damage

mechanisms. However, the high computational cost and long solution times limit its applicability in large-scale nonlinear dynamic analyses. In contrast, the macro-modeling approach represents masonry as an equivalent homogenized continuum, providing a computationally efficient framework for structural-scale simulations. Particularly in nonlinear time-history analyses under seismic loading, macro-modeling significantly reduces computational demand while maintaining reliable predictive capability. For this reason, nonlinear dynamic analyses based on macro-modeling are widely used in the seismic assessment of historic masonry structures (Valente 2021; Valente 2022; Kocaman et al. 2024).

Within the scope of this study, the three-dimensional geometric model of the Yakutiye Madrasa minaret was developed using SolidWorks software based on detailed architectural measurements. During the modeling process, superficial stone ornaments and decorative details with negligible influence on the structural behavior were excluded. The resulting three-dimensional geometric model was subsequently transferred to the ANSYS APDL environment, where the finite element analyses were conducted (Fig. 3). Rigid ground conditions were assumed at the foundation level, and the base was fully restrained in all degrees of freedom.

The finite element mesh was generated using the eight-node SOLID65 solid-element formulation available in ANSYS APDL (ANSYS 2024). Owing to the geometric complexity of the minaret, the tetrahedral degeneration available for SOLID65 was employed by assigning coincident or repeated nodal positions at the collapsed vertices. Accordingly, each tetrahedral-shaped mesh cell contains four distinct geometric vertices while retaining the eight-node SOLID65 connectivity convention. A nominal element size ranging from 15 to 20 cm was adopted. The final finite element model comprised approximately 258,209 elements and 54,563 nodes and provided sufficient geometric resolution for the global nonlinear analyses. However, the numerical convergence of the adopted discretization was not independently verified through a formal mesh-sensitivity analysis.

Consistent with the mesh definition provided above, SOLID65 is an eight-node, three-dimensional solid element with three translational degrees of freedom at each nodal position. It is widely used in the numerical analysis of historic masonry structures because of its capability to simulate nonlinear damage mechanisms, including tensile cracking and compressive crushing. The element formulation allows the representation of crack initiation under tensile stresses, crushing under compressive stresses, and limited plastic deformation. The standard element geometry, node-numbering convention, integration points, and tetrahedral degeneration adopted in the finite element mesh are illustrated in Fig. 4a.

Material behavior in SOLID65 is initially defined as isotropic; however, during the analysis independent crack formation can occur in three principal directions at each integration point. Once cracking initiates, the stiffness matrix is updated according to crack orientation, enabling the numerical representation of multiaxial cracking and separation behavior typical of masonry materials.

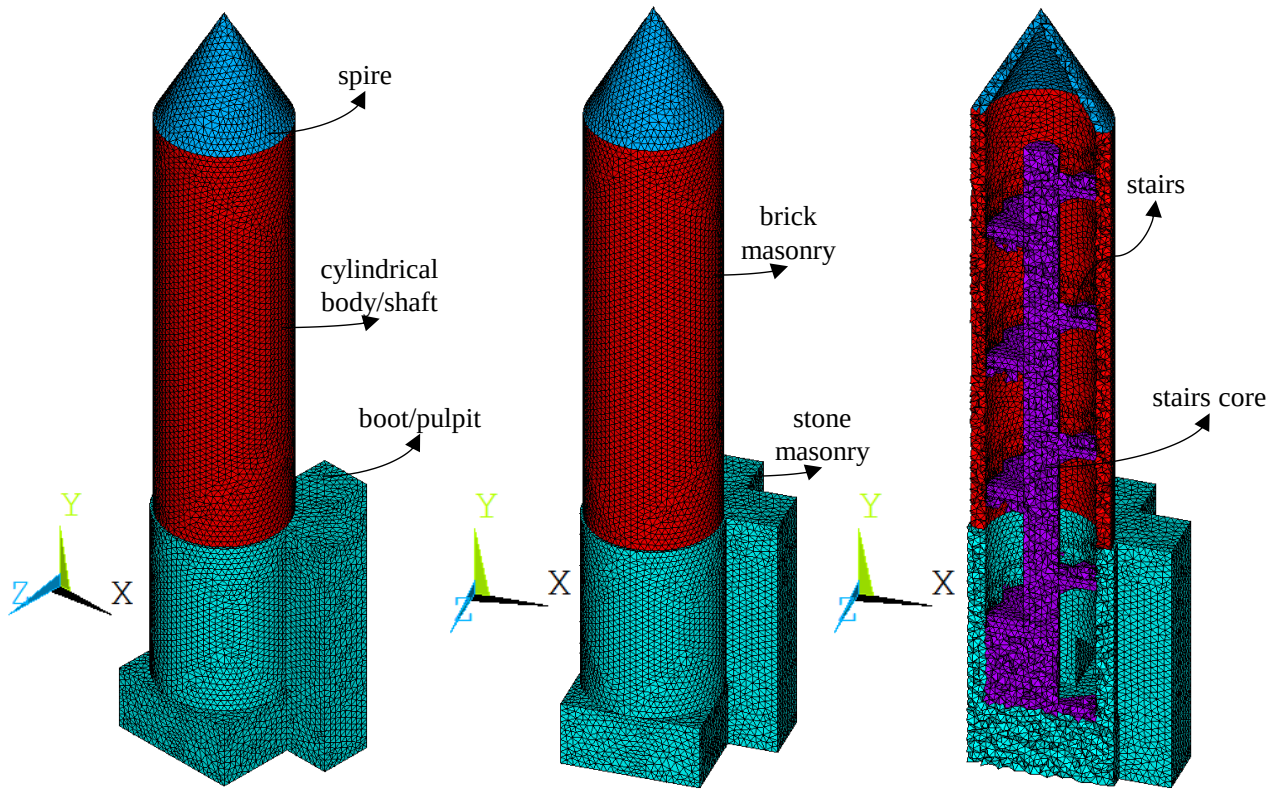


Fig. 3. Finite element model of the Yakutiye Madrasa minaret.

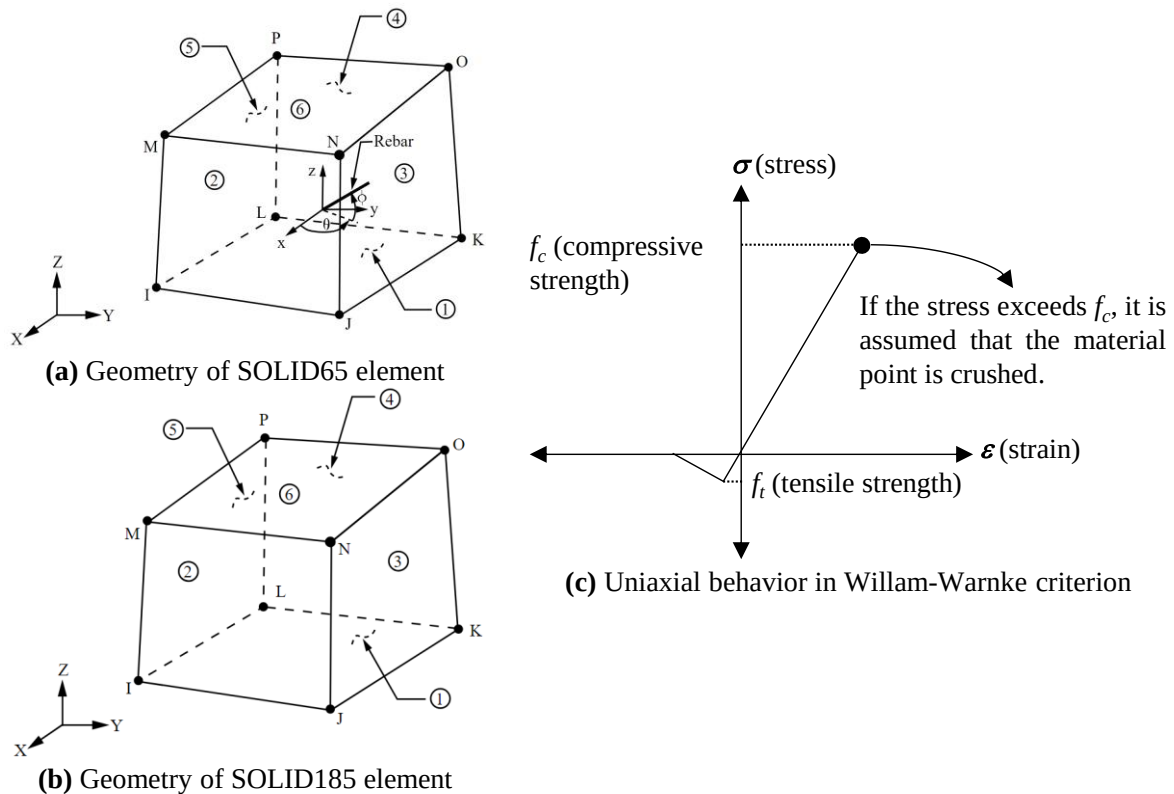


Fig. 4. Finite-element formulations and constitutive representation adopted in the numerical models: (a) Parent topology of the eight-node SOLID65 element used for masonry; (b) Parent topology of the eight-node SOLID185 element used for the S235 steel strengthening components; and (c) Schematic uniaxial tensile and compressive limits associated with the Willam–Warnke cracking–crushing criterion (Willam and Warnke 1975; ANSYS 2024).

In this study, the nonlinear behavior of the masonry material was modeled using the five-parameter Willam–

Warnke failure criterion (Karaton and Çanakçı 2020; Bartoli et al. 2015; Nebioğlu and Kocaman 2025). This

model defines a three-dimensional failure surface based on the uniaxial compressive strength (f_c) and tensile strength (f_t) of the material and accounts for crushing under uniaxial, biaxial, and triaxial stress states. When the crushing limit is exceeded, the stiffness contribution of the material at the corresponding integration points is eliminated, allowing the simulation of advanced damage and pre-collapse behavior. The corresponding failure surface and stress–strain relationships are presented in Fig. 4c.

Although the Willam–Warnke model was originally developed for concrete and geomaterials, it has been widely adopted in the numerical modeling of masonry systems. While the model does not explicitly represent post-peak softening, the collapse behavior of historic masonry minarets is largely governed by cracking, separation, and block-type mechanisms associated with low tensile strength. Therefore, the standalone use of the Willam–Warnke model was considered sufficient for the purposes of this study (Kocaman 2024).

The key parameters defining the damage surface include the shear transfer coefficients for open and closed cracks (β_t and β_c), as well as the compressive and tensile strengths of the material. In accordance with literature recommendations, $\beta_t=0.05$ was adopted for open cracks and $\beta_c=0.8$ for closed cracks (Kocaman and Gürbüz 2023). While SOLID65 was used to model the nonlinear behavior of masonry, the steel strengthening components were represented using the SOLID185 element, which is more suitable for modeling elastic–plastic material behavior and does not require crack–crushing capabilities. The geometric configuration and main characteristics of the SOLID185 element are illustrated in Fig. 4b.

All steel plate and bar components used in the strengthening scenarios were modeled using SOLID185 elements and assigned the MAT-5 material definition. MAT-5 represents isotropic S235 structural steel with an elastic modulus of 200,000 MPa, a Poisson's ratio of 0.30, a nominal yield strength of 235 MPa, and a mass density of 7850 kg/m³. The post-yield response was represented by a rate-independent elastic-perfectly plastic von Mises model; therefore, no strain hardening was considered and the post-yield tangent modulus was taken as zero.

The interfaces between the steel plates and masonry, between the longitudinal steel bars and masonry, and between connected steel components were idealized as fully bonded. Numerically, displacement compatibility was imposed in all three translational directions at the corresponding interface nodes. Consequently, relative normal separation and tangential slip were prevented throughout the analyses. No discrete anchor bolts, adhesive layers, frictional contact elements, or cohesive interface elements were explicitly modeled. This idealization represents the collective action of sufficient distributed anchorage and provides an upper-bound representation of composite load transfer between the steel strengthening system and the existing masonry.

4. Material Properties

In the numerical modeling of historic masonry structures, the definition of material properties represents

one of the most critical and uncertain stages of the analysis process. In masonry systems composed of stone and brick units, such as the Yakutiye Madrasa minaret, a single representative material set cannot be precisely defined due to the heterogeneity of masonry units, aging of mortar layers, and variability of stone–mortar interfaces. Therefore, homogenized (macro-scale) material parameters were adopted based on ranges reported in the literature for similar masonry systems, together with field observations and the existing structural characteristics. The material model was defined consistently with the cracking–crushing-based nonlinear modeling approach using compressive strength (f_c), tensile strength (f_t), modulus of elasticity (E), and density (ρ).

Five different material definitions were used in the finite element model. MAT-1 represents the primary load-bearing stone masonry walls, MAT-2 corresponds to the core and stair system, MAT-3 represents brick wall components, MAT-4 corresponds to the dome element, and MAT-5 represents the steel elements used in the strengthening scenarios. The strengthening components were modeled as S235 structural steel (MAT-5).

Due to the preserved condition and cultural heritage status of the structure, direct sampling from the Yakutiye Madrasa minaret was not possible. Based on field observations, material texture, and the geological characteristics of the Erzurum region, the stone used in the structure was identified as andesitic volcanic stone. To represent its mechanical behavior, sixteen cubic specimens (5 × 5 × 5 cm) were obtained from a local quarry supplying similar building stones for the region. Uniaxial compression tests were performed on these specimens, and the modulus of elasticity values were derived from the resulting stress–strain relationships.

The experimental results indicated that the compressive strengths of the stone specimens ranged between 33.8 MPa and 89.8 MPa, with an average value of approximately 72 MPa. Elastic modulus values were calculated from the stress–strain curves at 30% and 60% stress levels, yielding E_{30} values between 1893 MPa and 7832 MPa and E_{60} values between 2356 MPa and 9123 MPa. The observed failure modes and representative stress–strain curves of the tested specimens are presented in Fig. 5.

The experimental results indicate that the andesitic stones exhibit relatively high compressive strength. However, in historic masonry structures the effective mechanical properties at the structural scale are typically much lower than those of individual stone units due to weak mortar layers, discontinuities at stone–mortar interfaces, internal infill materials, and irregularities associated with traditional construction techniques. Therefore, the values obtained directly from stone unit tests were not used in the finite element model. Instead, homogenized masonry parameters were adopted to represent the composite behavior of the stone–mortar–infill system.

In defining these parameters, the methodology reported in a previous investigation on the structural behavior of the Yakutiye Madrasa was taken as a reference (Kocaman et al. 2018). That study demonstrated that, although individual stone blocks possess relatively high compressive strength, the effective strength of masonry walls decreases considerably due to mortar layers, inter-

nal infill materials, and multi-leaf wall configurations. Based on empirical relationships reported in the literature, the compressive strength of cut stone masonry walls was estimated to have an upper bound of approximately 13.45 MPa (Kocaman et al. 2018).

However, these empirical relationships were primarily developed for planar masonry walls with relatively

limited discontinuities. Masonry minarets exhibit a more complex structural behavior due to their cylindrical geometry, cross-sectional transitions, internal cavities, and spiral staircase systems. Consequently, directly adopting compressive strength values derived for planar walls may lead to overly optimistic estimations of the seismic performance of the minaret shaft.

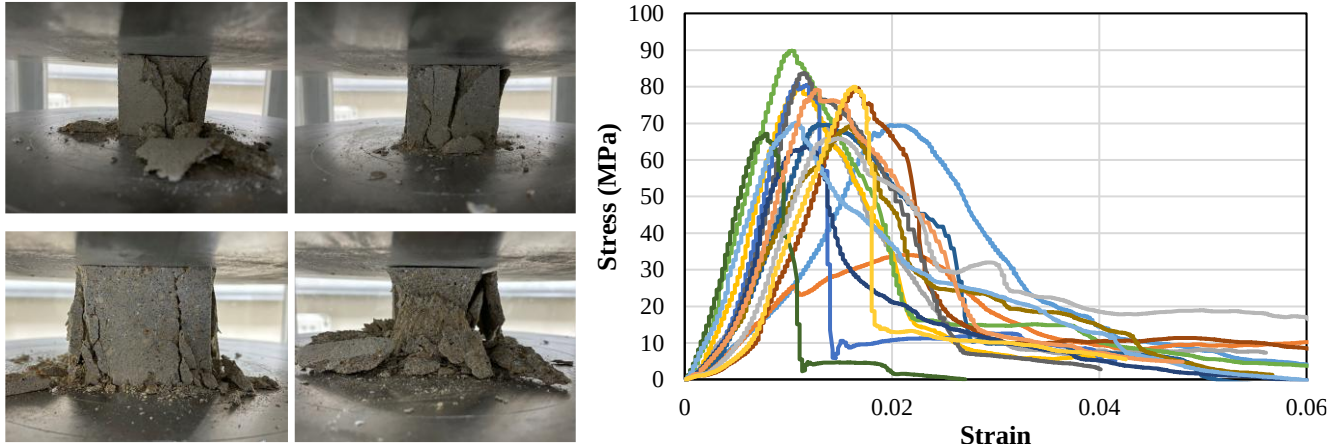


Fig. 5. Observed failure modes during compression tests of stone unit specimens.

For this reason, the homogenized masonry parameters adopted in this study were determined by considering both empirical findings and recommendations from national and international guidelines. According to the Guideline for the Management of Seismic Risks of Historic Structures in Türkiye, the compressive strength of well-cut stone masonry walls ranges between 3.0 MPa and 4.0 MPa (VGM 2017). Similarly, Eurocode 6 recommends defining the modulus of elasticity proportionally to the compressive strength using the relationship $E \approx (800-1000) \cdot f_c$ (CEN 2005).

Accordingly, a compressive strength of $f_c=3.0$ MPa and an elastic modulus of $E=3000$ MPa were adopted for the primary load-bearing stone walls of the minaret (MAT-1). The core region and spiral staircase system were assumed to exhibit a more integrated structural response; therefore, $f_c=7.0$ MPa and $E=7000$ MPa were assigned to these regions (MAT-2). This approach provides a conservative yet realistic representation of the structural behavior of the minaret.

Direct sampling of brick walls and dome elements was not feasible; therefore, their mechanical properties were defined based on experimental results reported in the literature. Long-term environmental exposure and aging may also lead historic brick units to exhibit distinct physical and mechanical characteristics (Niroula and Bhatt 2026). A study examining Ottoman-era masonry structures in the Erzincan region reported an average compressive strength of approximately 6.32 MPa for brick units obtained from $50 \times 50 \times 50$ mm specimens (Çelik et al. 2025).

Since the mechanical behavior of brick masonry depends on both brick and mortar properties, the empirical relationship proposed by Kaushik et al. (2007) was used to estimate the compressive strength of brick masonry elements:

$$f_m = 0.63 \times f_b^{0.49} \times f_j^{0.32} \quad (1)$$

where f_m is the compressive strength of brick masonry, f_b represents the compressive strength of the brick unit, and f_j denotes the mortar compressive strength. Using $f_b=6.32$ MPa and $f_j=3.1$ MPa for weak mortar conditions, the compressive strength of brick masonry elements was calculated as approximately 2.2 MPa, which is consistent with the ranges reported for historic masonry walls.

In the same study, the modulus of elasticity of brick masonry was suggested to be estimated using the relationship $E=550 \cdot f_m$ (Kaushik 2007). Based on this relationship, the elastic modulus was determined to be approximately 1225 MPa for brick walls (MAT-3) and 1500 MPa for dome elements (MAT-4). For all masonry components, the tensile strength was assumed to be approximately 10% of the compressive strength, consistent with common engineering practice. MAT-4, representing the dome masonry, was retained as a separate homogenized material definition because of the component-specific masonry arrangement. A compressive strength of $f_c=2.72$ MPa was adopted for this region. This value falls within the compressive-strength range of 2.4–4.0 MPa reported for historic solid-brick masonry constructed with lime mortar (Corradi et al. 2018). Applying the relationship $E=550 \cdot f_m$ gives $E=1496$ MPa; therefore, the elastic modulus of MAT-4 was taken as $E=1500$ MPa.

Through this approach, experimental findings, empirical relationships, and code recommendations were jointly considered to define a conservative and representative material set for the finite element model (Table 1). Despite the relatively high compressive strength of individual stone units, the adopted homogenized parameters account for the reduced effective mechanical properties at the wall and minaret scale and allow realistic simulation of cracking, separation, and collapse mechanisms in the nonlinear analyses.

Table 1. Homogenized material properties adopted in the finite element model.

Material	Structural component	f_c (MPa)	f_t (MPa)	f_y (MPa)	E (MPa)	ρ (kg/m ³)
MAT-1	Stone masonry walls	3.00	0.30	–	3000	2500
MAT-2	Core and stair system	7.00	0.70	–	7000	2800
MAT-3	Brick masonry walls	2.20	0.22	–	1225	2350
MAT-4	Dome element	2.72	0.15	–	1500	2350
MAT-5	Steel plates and longitudinal bars	–	–	235	200000	7850

5. Existing Condition of Reference Model

Although numerous strengthening techniques have been proposed to improve the seismic behavior of historic masonry minarets, the effectiveness of such interventions strongly depends on the geometry, material properties, and governing damage mechanisms of the specific structure. Therefore, a reliable assessment of the existing structural condition is required before proposing any strengthening strategy.

In this section, the current seismic response of the Yakutiye Madrasa minaret is evaluated using the Reference Model (RM), representing the unstrengthened condition of the structure. The objective is to establish a baseline for comparison with the strengthening scenarios examined in the following sections and to determine the initial seismic vulnerability of the minaret.

First, modal analyses were performed to identify the dominant dynamic characteristics and deformation patterns of the structure. Subsequently, nonlinear time-history analyses were conducted using the selected ground motion records to evaluate displacement demands, damage initiation, and stress-strain accumulation in critical regions. Based on these results, the governing damage concentration zones and potential collapse mechanisms of the minaret were identified, providing the basis for the strengthening strategies investigated in the subsequent sections.

5.1. Modal characteristics of the reference model

Linear elastic modal analyses were performed for the Reference Model (RM) representing the existing condition of the Yakutiye Madrasa minaret. The aim of these analyses was to identify the dynamic characteristics of the structure and determine the dominant vibration modes that may influence its seismic behavior. Accordingly, natural frequencies, vibration periods, and modal mass participation ratios were evaluated.

The first three natural frequencies of the reference model were calculated as 1.90 Hz, 2.02 Hz, and 8.15 Hz, respectively. The close proximity of the first two frequencies indicates similar stiffness characteristics in the two principal horizontal directions and confirms that the dynamic behavior of the minaret is dominated by flexural response under horizontal seismic excitation. Such behavior is typical for slender masonry minarets with cylindrical shafts. The third mode, characterized by a higher frequency, reflects localized deformations primarily concentrated in the upper shaft and transition regions (Fig. 6).

The effective modal mass participation ratios indicate that the first two modes represent the principal global lateral-bending modes but do not, by themselves, dominate the overall dynamic response. In the X direction, the cumulative effective mass participation ratios of the first two and first three modes are approximately 27.36% and 30.17%, respectively. The corresponding values in the Z direction are 27.55% and 30.92%. Mode-shape assessment further indicates that the third mode is characterized predominantly by torsional deformation. The remaining effective mass is therefore distributed among the higher modes, including modes associated with higher-order global deformation and localized response mechanisms. Consequently, the nonlinear time-history response should be interpreted as the combined contribution of lower and higher vibration modes, although the first two modes remain important in describing the principal global lateral behavior of the minaret.

In addition, a notable participation in the rotational (ROTY) direction is observed starting from the first mode. This suggests that bending-torsion interaction cannot be neglected despite the approximate axial symmetry of the structure, mainly due to stiffness and mass discontinuities caused by the stair core and transition regions. Such characteristics indicate that damage accumulation and crack concentration may develop in the upper shaft and transition zones during seismic excitation.

Overall, the modal analysis results reveal that the Yakutiye Madrasa minaret exhibits a low-frequency dynamic character dominated by horizontal flexural behavior with noticeable torsional interaction. This dynamic behavior implies significant displacement demands under seismic loading and indicates potential damage concentration in the upper shaft and transition regions, highlighting the structural vulnerability of the minaret in its current condition.

5.2. Nonlinear dynamic response of the reference model

Nonlinear time-history analyses were performed to evaluate the seismic behavior of the Yakutiye Madrasa minaret. In order to represent realistic seismic conditions, three recorded earthquake ground motions characteristic of strong earthquakes in Türkiye were selected: the 1992 Erzincan earthquake (Mw=6.8), the 1999 Gölcük–İzmit earthquake (Mw=7.6), and the 1999 Düzce–Bolu earthquake (Mw=7.1). These events represent the seismic characteristics of both the East Anatolian and North Anatolian fault systems and allow the structural response of the minaret to be examined under different frequency contents and acceleration levels. The acceleration records were obtained from the AFAD TADAS database (AFAD 2026).

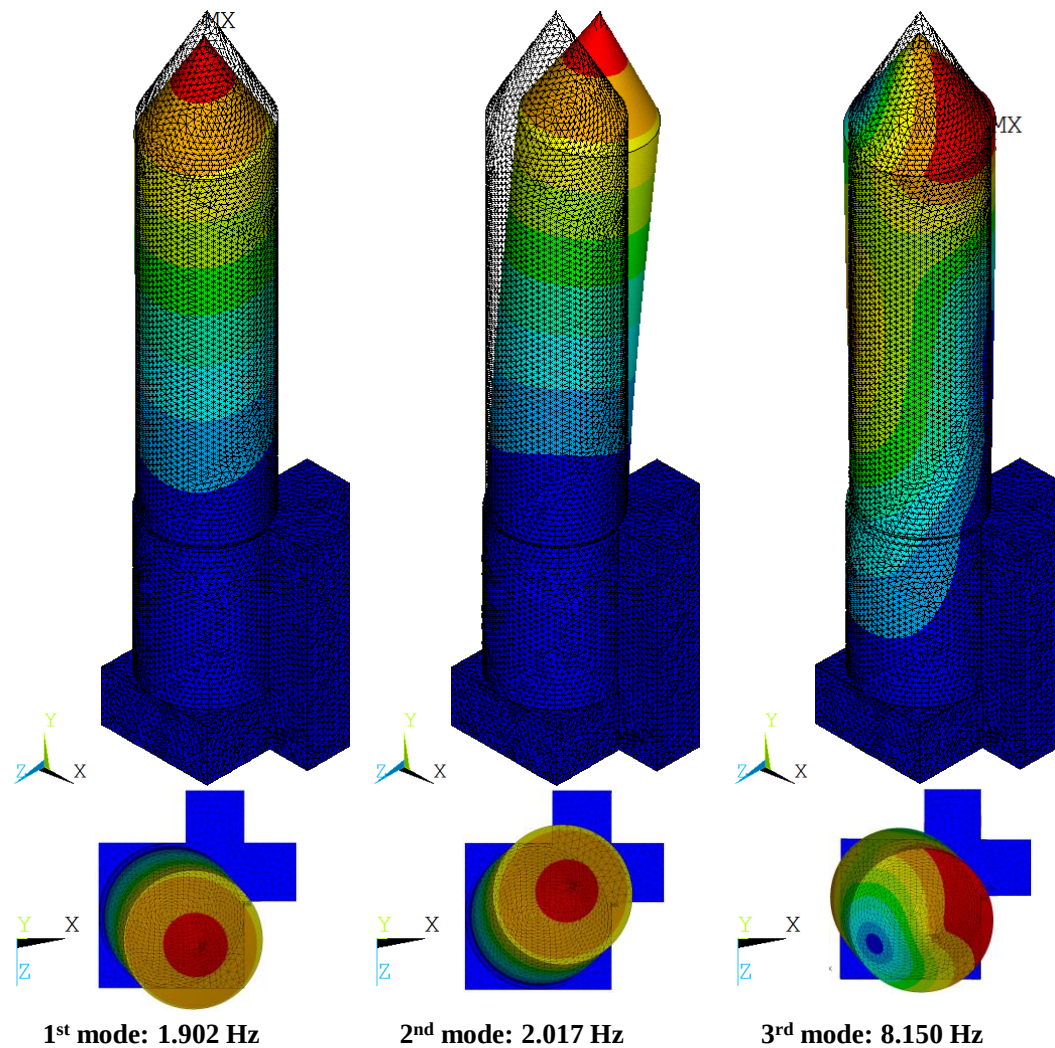


Fig. 6. Mode shapes corresponding to the first three natural vibration modes obtained for RM.

For each earthquake record, the two horizontal components (N-S and E-W) were scaled according to the provisions of the Turkish Building Earthquake Code (TBEC-2018). The scaling procedure was based on the dominant vibration period obtained from the modal analysis of the reference model. Accordingly, the average of the scaled response spectra was ensured to be not less than 90% of the target elastic design spectrum within the period range of $0.2T_p$ – $1.5T_p$. The same scaling factor was applied simultaneously to both horizontal components of each record.

Modal analyses were performed not only for the Reference Model (RM) but also for all strengthening scenarios (RET-1, RET-2, RET-3, and RET-4). Since strengthen-

ing interventions modify the dynamic properties of the structure, scaling coefficients were recalculated separately for each model. This procedure ensured that all nonlinear analyses were conducted within a code-compliant and mutually comparable analytical framework.

The main seismic parameters of the selected ground motions, including earthquake name, year, magnitude, station code, peak ground acceleration values, and the scaling factors used for each model, are presented in Table 2.

The acceleration–time histories of the selected ground motions are presented in Fig. 7, and these records were used as input for the nonlinear dynamic analyses.

Table 2. Characteristics of the selected ground motions and corresponding scaling coefficients.

ID	Event	Year	Mw	Station Code	PGA (gal)		Scale Factor				
					N-S component	E-W component	RM	RET-1	RET-2	RET-3	RET-4
GM-1	Merkez Erzincan	13.03.1992	6.8	Center	405.007	479.543	0.85	0.84	0.99	0.99	0.97
GM-2	Gölcük İzmit	17.08.1999	7.6	8101	314.340	365.930	0.98	0.98	1.04	1.04	1.02
GM-3	Düzce Bolu	12.11.1999	7.1	1401	719.330	803.830	0.55	0.54	0.57	0.57	0.58

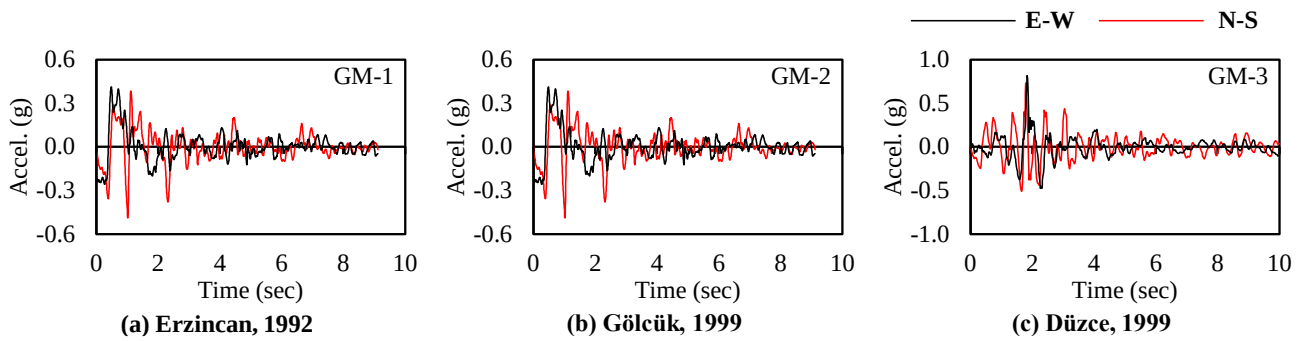


Fig. 7. Acceleration–time histories of the selected ground motions.

Considering the high computational demand of nonlinear time-history analyses, the duration of the earthquake records was limited to the time interval representing the significant portion of seismic energy. For this purpose, the records were truncated according to the Arias Intensity criterion proposed by Trifunac and Brady (1975), and only the interval corresponding to 5%–95% of the cumulative Arias intensity was used in the analyses. This approach ensures that the most energy-relevant portion of the ground motion is considered while reducing computational cost.

The nonlinear dynamic response of the reference model was evaluated using displacement–time histories

obtained from the three selected ground motions (GM-1, GM-2, and GM-3). Horizontal displacements were monitored at the top of the minaret, which represents the most dynamically sensitive region due to the slender geometry and flexure-dominated structural behavior.

The displacement–time histories shown in Fig. 8 indicate that the reference model exhibits significant displacement demands under all ground motions. For GM-1, horizontal displacements at the minaret top reached approximately ± 80 – 100 mm, and the response did not return to its initial position during the later stages of the record, indicating the development of permanent deformations and stiffness degradation.

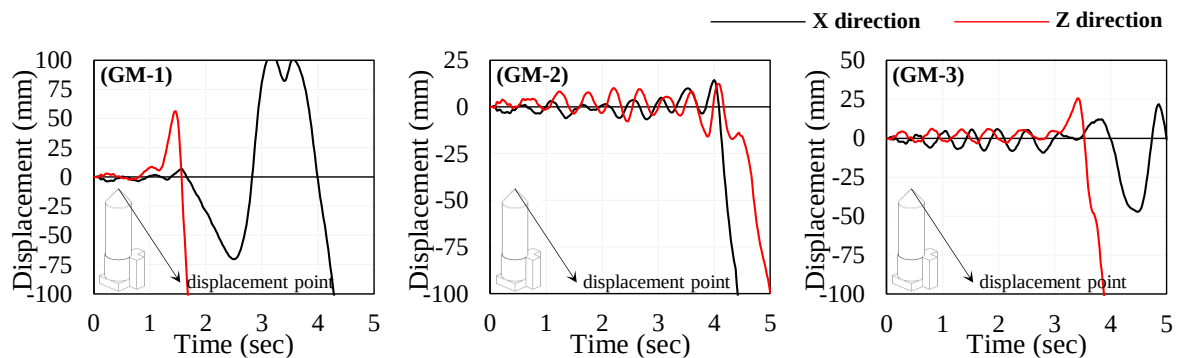


Fig. 8. Displacement–time histories obtained from dynamic analyses (for the RM model).

Under GM-2, the displacement response was slightly lower but still critical, reaching approximately ± 60 – 80 mm. The relatively long duration and low-frequency content of this ground motion interacted with the dominant vibration periods of the minaret, resulting in progressive amplification of displacement demands.

In contrast, GM-3 produced a different response pattern due to its impulsive characteristics and high peak accelerations. Although the displacement range remained within approximately ± 50 – 70 mm, sudden displacement jumps occurred over short time intervals. Such abrupt increases are particularly unfavorable for masonry structures with limited ductility.

Although specific displacement limits for minarets are not defined in the literature, comparison with performance limits proposed for historic masonry mosques and domed structures indicates that the displacement demands obtained in the reference model exceed commonly accepted thresholds (Kocaman 2024). The observed non-recoverable displacement increases suggest

that the structure cannot effectively dissipate seismic energy and that nonlinear behavior develops at an early stage.

Overall, the displacement histories demonstrate that the Yakutiye Madrasa minaret exhibits significant seismic vulnerability in its current condition. These results also provide the baseline for evaluating the effectiveness of the strengthening scenarios presented in the following sections and will be further interpreted together with the strain and damage distributions discussed subsequently.

The first principal strain (ε_1) distributions obtained from the nonlinear dynamic analyses of the reference model clearly reveal the dominant damage mechanism of the minaret. The ε_1 contours presented in Fig. 9 show that, under all ground motions (GM-1, GM-2, and GM-3), damage develops in a similar spatial pattern. This consistency indicates that damage evolution is governed primarily by the geometric and structural characteristics of the minaret rather than by the specific characteristics of the earthquake records.

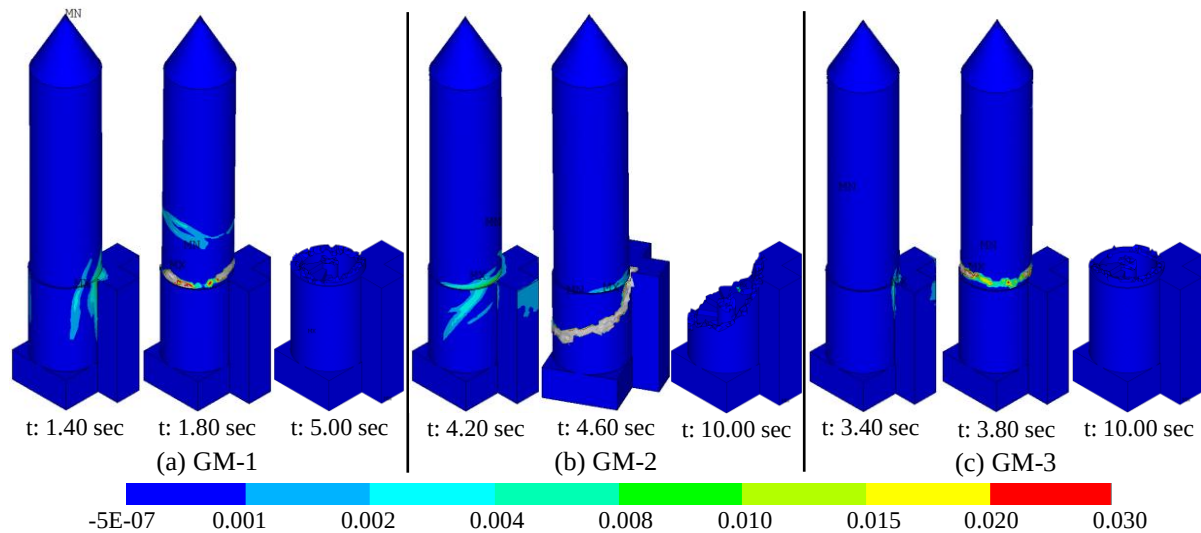


Fig. 9. Damage mechanisms obtained from the nonlinear dynamic analyses (for the RM model).

In all analyses, strain concentrations first emerge in the transition region between the minaret shaft and the base/substructure. In this zone, elevated ε_1 values form a discontinuous yet circumferential band, indicating the development of a tension-dominated strain field. Given the low tensile capacity of masonry materials, these strain levels correspond to the initiation and propagation of cracking, consistent with numerical evidence that historic stone and brick elements are particularly vulnerable in tension under seismic loading (Zardari et al. 2024).

As seismic excitation continues, the strain concentration in the transition region becomes progressively more continuous around the shaft circumference, weakening the effective load-carrying capacity of the cross-section. The circumferential distribution of elevated ε_1 values indicates that the damage mechanism involves not only localized cracking but also progressive degradation of the structural integrity of the entire cross-section. This behavior highlights the transition region—characterized by abrupt stiffness changes and slender geometry—as the most seismically vulnerable part of the minaret.

The consistent damage pattern observed under all ground motions suggests that the governing collapse mechanism of the reference model involves circumferential tensile cracking in the transition region followed by progressive weakening of the shaft–substructure interaction. These strain-based observations provide direct insight into the location and evolution of damage and form the basis for the strengthening strategies proposed in the subsequent sections.

Collapse was not defined solely by the termination of the nonlinear solution or by the loss of numerical convergence. Instead, the occurrence and time of collapse were determined through a combined and mutually corroborating evaluation of the global displacement response, literature-based drift limits, first-principal-strain distributions, crack-line continuity, and the development of physically plausible regional failure mechanisms.

The global drift ratio was calculated as the relative lateral displacement between the top and reference base levels divided by the corresponding structural height.

Because specific and universally accepted drift limits are not available for historic masonry minarets, the calculated drift demands were evaluated with reference to the ranges reported for comparable historic masonry structures. This structure-specific interpretation is consistent with evidence that damage-state thresholds can significantly affect masonry vulnerability estimates and should not be transferred directly between different structures (Yesilyurt 2025). Kocaman and Kazaz (2023), based on nonlinear analyses of eleven historic masonry mosques, reported global drift-ratio ranges of 0.01–0.07%, 0.06–0.25%, and 0.08–0.45% for the Damage Limitation, Significant Damage, and Near Collapse states, respectively. These ranges were used as comparative performance benchmarks rather than as a single deterministic collapse threshold, since drift capacity is strongly affected by structural geometry, material stiffness, boundary conditions, and the governing damage mechanism.

Accordingly, the displacement-time histories and force-displacement response were first examined for rapid displacement growth, accumulation of non-recoverable deformation, loss of stable cyclic behavior, and pronounced stiffness or strength degradation. These global-response indicators were then evaluated together with the first-principal-strain distributions. Isolated peak strain values occurring in a limited number of finite elements were not interpreted as collapse. Instead, attention was given to the magnitude, spatial extent, propagation, and continuity of tensile-strain localization bands and to whether these bands formed a mechanically meaningful crack pattern.

For the reference model, the governing collapse mechanism was identified by the formation and propagation of an approximately continuous circumferential tensile-damage band in the shaft–substructure transition region. This regional damage pattern indicated progressive loss of sectional integrity and weakening of load transfer between the minaret shaft and the supporting substructure. Therefore, collapse was assigned only when the literature-based global drift demand, the deterioration observed in the global response, and the strain-based regional damage mechanism provided consistent and simultaneous evidence of structural instability.

The reported collapse time, (t_c), corresponds to the earliest analysis time step at which this combined collapse condition became established. It does not necessarily correspond to the final converged time step. In some analyses, the numerical solution continued after the structural response had already satisfied the adopted collapse criteria. Conversely, numerical non-convergence was not accepted as evidence of collapse unless it was supported by the displacement, drift, strain-localization, crack-propagation, and regional damage findings described above.

6. Strengthening Scenarios

The nonlinear dynamic analyses of the reference model (RM) clearly indicated that the Yakutiye Madrasa minaret exhibits significant seismic vulnerability in its current condition. The combined evaluation of displacement demands, first principal strain distributions, and circumferential cracking patterns revealed that seismic demands concentrate primarily in the shaft-substructure transition region. These findings demonstrate that strengthening intervention is necessary to improve the seismic performance of the structure.

However, strengthening strategies for historic masonry structures must not rely solely on structural capacity enhancement. Interventions must comply with international conservation principles defined by ICOMOS, including minimum intervention, reversibility, respect for original materials, and preservation of historical authenticity. For the Yakutiye Madrasa minaret, this requirement is particularly important due to the highly refined Seljuk stone ornamentation on the exterior surface. Consequently, all strengthening strategies considered in this study were designed to be implemented entirely within the internal volume of the minaret without altering the original exterior stone texture.

The analyses of the reference model further indicated that the seismic response of the minaret is governed by a global damage mechanism initiated by strain concentrations in the shaft-substructure transition region. Therefore, strengthening strategies limited to localized repair would be insufficient; instead, interventions capable of redistributing stresses and reorganizing load transfer mechanisms were required.

Within this framework, four strengthening scenarios (RET-1, RET-2, RET-3, and RET-4) were developed. Rather than representing incremental variations of a single solution, these scenarios were formulated as alternative engineering strategies addressing the same structural problem with different levels of stiffness continuity and structural influence. The layouts of the strengthening systems are illustrated schematically in Fig. 10.

The same geometric and material assumptions were maintained for the steel components in all strengthening scenarios. All internal steel plates used in RET-1, RET-2, RET-3, and the plate component of RET-4 were assigned a uniform thickness of 30 mm. The plates and the 30 mm diameter longitudinal bars used in RET-4 were modeled using SOLID185 elements and the MAT-5 steel properties reported in Table 1.

The strengthening systems investigated herein represent comparative numerical configurations rather than fully detailed construction designs. Mechanical anchors were therefore not represented as discrete finite elements. Instead, adequate anchorage was idealized through the fully bonded steel-masonry interface condition described in Section 3. Accordingly, the analyses do not simulate local anchor failure, plate debonding, pull-out, or relative slip. The type, diameter, spacing, embedment length, corrosion protection, and installation sequence of the mechanical anchors should be determined through a separate site-specific application design if any of the investigated systems is considered for implementation.

6.1. RET-1 strengthening scenario: Discontinuous steel plate system along the internal spiral staircase

In RET-1, the discontinuous modules consist of 30 mm thick steel plates arranged along the internal spiral staircase from the first to the last step. Each module is approximately 750 mm high and follows the internal circumference of the shaft at the corresponding stair level. The modules remain structurally discontinuous from one another, while each module is fully bonded to the adjacent masonry surface through the adopted interface constraint.

RET-1 represents the first strengthening strategy developed to mitigate tensile strain concentrations while maintaining minimal intervention. In this scenario, a discontinuous internal banding system composed of steel plate modules was introduced along the spiral staircase within the interior of the minaret.

The steel plates are arranged along the stair levels, extending from the first to the last step, with each module approximately 750 mm in height. The modules are positioned against the inner surface of the shaft and follow the staircase geometry. This configuration allows the strengthening elements to remain concealed while providing additional confinement through the interaction between the internal stair core and the shaft.

The engineering concept of RET-1 is to partially restrain circumferential opening of the masonry walls by introducing localized internal banding. These steel modules aim to slow crack propagation, preserve sectional integrity, and reduce displacement demands by distributing tensile stresses along the shaft.

6.2. RET-2 strengthening scenario: Concentrated internal steel plate at the transition region

In RET-2, a continuous cylindrical steel plate with a uniform thickness of 30 mm is placed along the internal circumference of the shaft between approximately +5.00 m and +12.00 m elevation. The entire plate-masonry contact surface is modeled as fully bonded. Continuity is maintained within the strengthened region, whereas no steel lining is provided above or below these elevations.

RET-2 was developed based on the results obtained from RET-1. Although RET-1 redistributed stresses along the shaft, the nonlinear analyses indicated that circumferential cracking persisted in the shaft-substructure transition region.

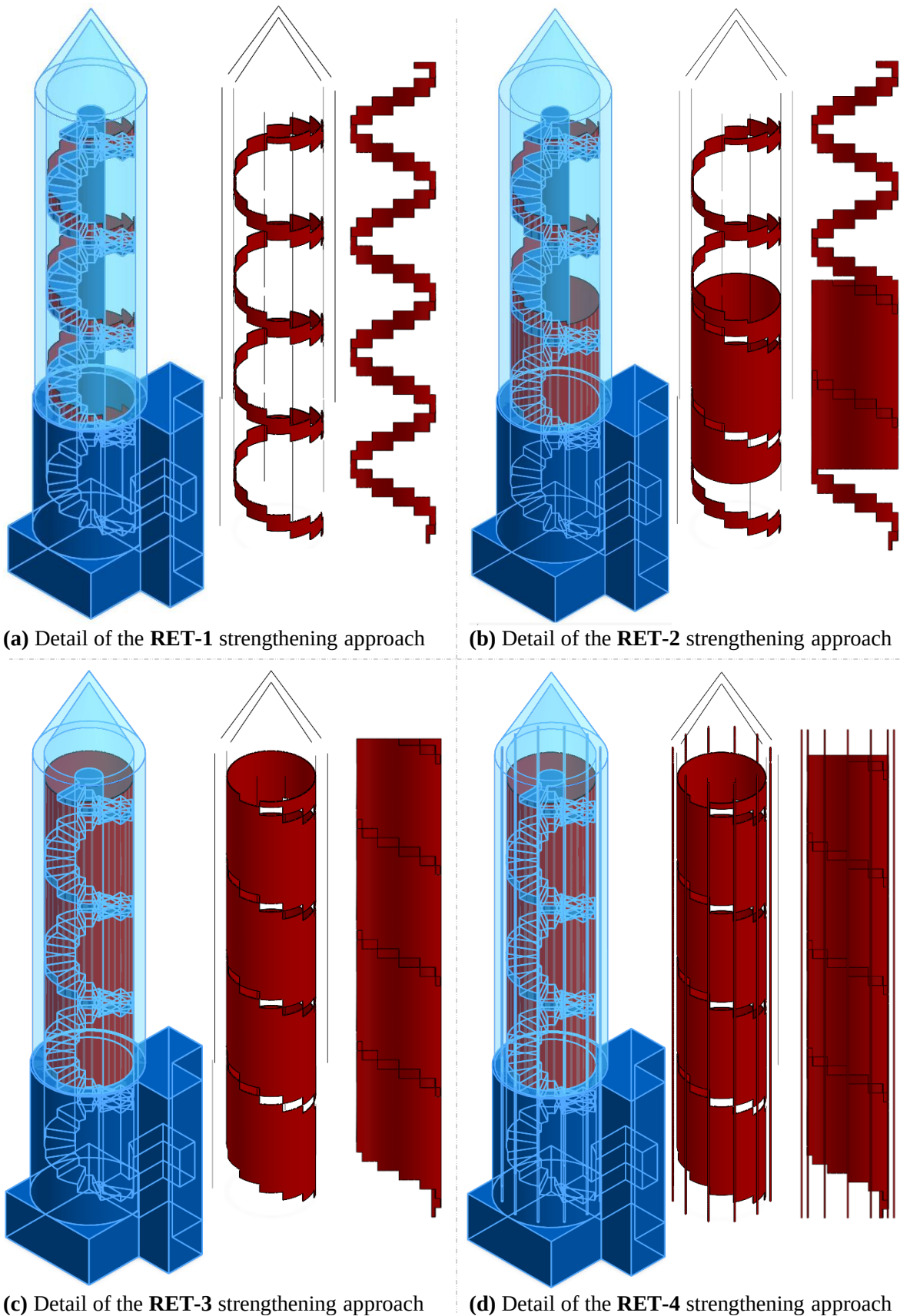


Fig. 10. Geometric and numerical definitions of the proposed internal strengthening configurations:
 (a) RET-1, discontinuous 30 mm thick steel plate modules with an approximate module height of 750 mm;
 (b) RET-2, continuous 30 mm thick internal steel plate between +5.00 m and +12.00 m elevation;
 (c) RET-3, continuous 30 mm thick internal steel lining along the entire shaft;
 (d) RET-4, RET-3 lining supplemented with twelve 30 mm diameter longitudinal steel bars anchored to the foundation. Steel-masonry and connected steel-steel interfaces were assumed to be fully bonded.

To address this limitation, RET-2 introduces a continuous cylindrical steel plate located specifically in the transition region. The plate extends approximately from +5.0 m to +12.0 m elevation and is placed along the inner surface of the shaft.

The objective of this strengthening system is to increase circumferential stiffness at the sectional level and limit crack propagation within the transition region. By providing continuous confinement, the steel plate promotes more integrated structural behavior and delays stiffness degradation under seismic loading. The strengthening element remains confined within the internal volume and therefore does not affect the exterior stone surface, maintaining compatibility with conservation principles.

6.3. RET-3 strengthening scenario: Continuous internal steel plate along the entire shaft

In RET-3, a 30 mm thick continuous steel lining is extended along the entire internal shaft surface, from the substructure level to the base of the spire. All adjoining steel regions are modeled with full displacement compatibility, and the lining is fully bonded to the masonry along its complete contact surface.

The RET-3 scenario was developed after examining the nonlinear response of RET-2. Although the transition region strengthening reduced damage concentrations locally, new strain concentrations were observed at the boundaries of the strengthened zone due to abrupt stiffness changes.

To eliminate these secondary weakness planes, RET-3 extends the internal steel plate along the entire height of the shaft. The continuous steel lining covers the inner surface from the base to the base of the spire. This configuration aims to homogenize stiffness distribution along the shaft and prevent localized strain concentrations. By acting as an internal steel shell, the system enables more uniform distribution of bending, shear, and torsional demands. Despite being a more extensive intervention than previous scenarios, RET-3 remains compatible with conservation requirements because all elements are located inside the minaret and do not interfere with the exterior architectural features.

6.4. RET-4 strengthening scenario: Hybrid system with longitudinal steel reinforcements

RET-4 retains the 30 mm thick continuous internal lining defined for RET-3 and supplements it with twelve longitudinal steel bars, each 30 mm in diameter. The bars are uniformly distributed around the shaft to preserve sectional symmetry, extend continuously along the full height of the minaret, and are anchored to the foundation in the numerical model. The bar-masonry and bar-plate connections are assumed to be fully bonded. The foundation anchorage is represented by displacement compatibility rather than by explicitly modeled anchor plates, bolts, or embedment zones.

RET-4 represents the most comprehensive strengthening strategy considered in this study. Although RET-3 significantly improved stiffness distribution, nonlinear analyses indicated that increasing stiffness alone was in-

sufficient to fully prevent damage mechanisms governed by the low tensile capacity of masonry.

Therefore, RET-4 introduces a hybrid strengthening system combining the continuous internal steel lining of RET-3 with additional longitudinal reinforcement elements that directly increase tensile capacity.

In this configuration, twelve steel bars with a diameter of 30 mm are installed along the full height of the shaft and anchored to the foundation. The reinforcement layout preserves sectional symmetry and avoids amplification of torsional effects while remaining compatible with the internal staircase geometry.

The resulting system allows the minaret to behave as a composite structure in which the internal steel shell regulates stiffness distribution and the longitudinal reinforcements enhance tensile resistance and limit crack development. RET-4 therefore provides simultaneous control of stiffness distribution and tensile-dominated masonry failure mechanisms.

7. Comparative Analysis Results

In this section, the seismic performance of the reference model (RM) and the four strengthening scenarios (RET-1–RET-4) is comparatively evaluated. The analyses include both modal characteristics and nonlinear time-history responses obtained under the three selected ground motions (GM-1, GM-2, and GM-3). First, the natural vibration frequencies and mode shapes of the models are compared to examine the influence of strengthening interventions on global stiffness. Subsequently, displacement–time histories and first principal strain distributions are evaluated to investigate the evolution of collapse time, maximum displacement demands, and damage mechanisms. This comparison provides a quantitative basis for assessing the effectiveness of the proposed strengthening strategies.

7.1. Modal analysis results

The modal analysis results of the reference model (RM) and the strengthening scenarios (RET-1–RET-4) are comparatively presented in Table 3, while the deformation patterns corresponding to the first mode are illustrated in Fig. 11.

Examination of Table 3 indicates that the strengthening interventions lead to gradual and significant increases in natural frequencies. Compared to the RM model, the increase in the first natural frequency was calculated as 3.6% for RET-1, 45.9% for RET-2, 74.3% for RET-3, and 82.5% for RET-4. A similar increasing trend is observed in the second and third modes, with the increase in the second mode frequency reaching approximately 96% in the RET-4 model. These results demonstrate that, particularly following RET-2, the interventions result in a substantial increase in overall system stiffness.

The modal analyses were performed by extracting 100 vibration modes. The cumulative effective modal mass participation ratios reached approximately 78% in the X direction and 82% in the Z direction, indicating that the structural mass contribution is distributed over a rel-

atively broad range of modes. The first three modes, representing the fundamental lateral-bending and torsional characteristics of the minaret, provide cumulative horizontal mass participation ratios of approximately 30.17% and 30.92% in the X and Z directions, respectively. Therefore, although these modes are important for describing the principal global deformation characteristics, higher-order modes also contribute substantially to the overall dynamic behavior. It should be em-

phasized that the nonlinear time-history analyses were performed using direct time integration rather than modal superposition; consequently, the nonlinear response was not restricted to the modes extracted in the modal analysis. The third mode was identified as a torsion-dominated mode based on its deformation pattern. A numerical ROTY participation ratio was not reported because a consistently normalized rotational effective-mass ratio was not available for all models.

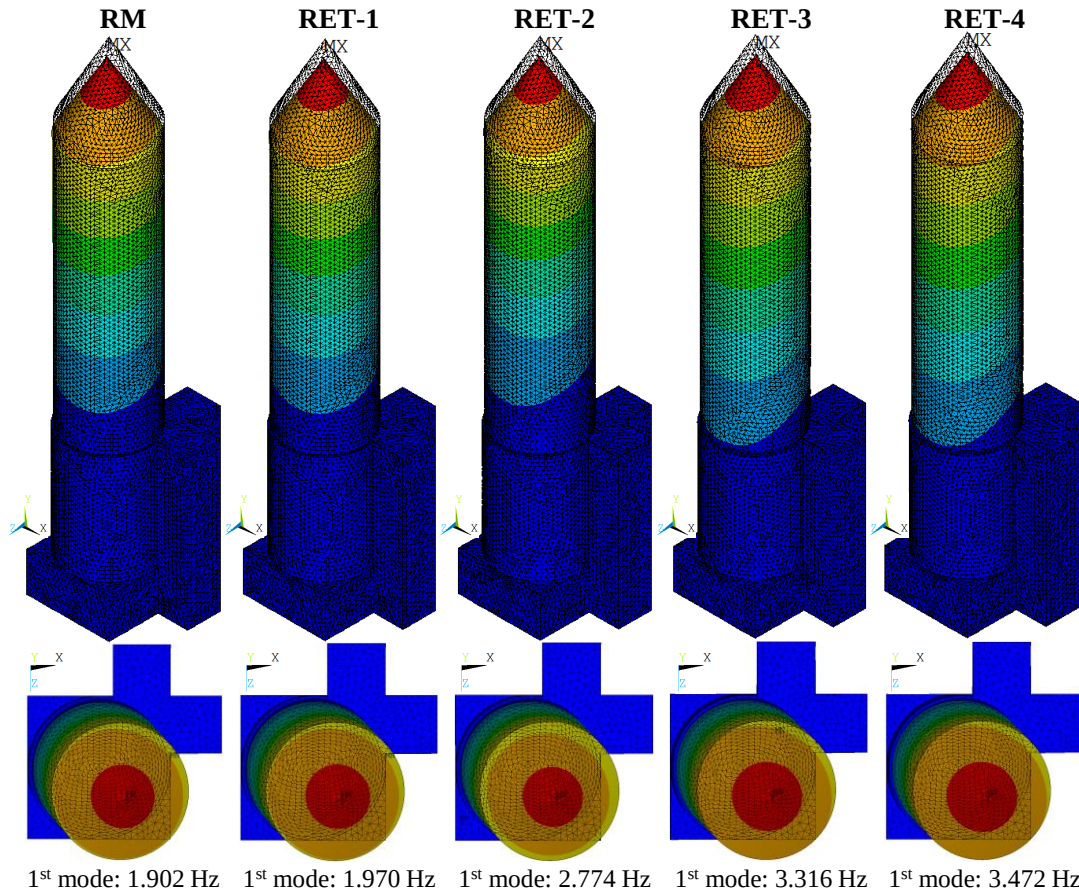


Fig. 11. Comparison of the first mode shapes of the reference model (RM) and the strengthening scenarios.

Table 3. Comparison of natural frequencies and modal mass participation ratios (MPR) of the RM and RET models.

Model	f_1 (Hz)	MPR (%)	Δf_1 (%)	f_2 (Hz)	MPR (%)	Δf_2 (%)	f_3 (Hz)	Δf_3 (%)
RM	1.902	15.84 (Z)	–	2.017	15.00 (X)	–	8.150	–
RET-1	1.970	15.46(Z)	+3.6	2.091	14.61 (X)	+3.7	8.571	+5.2
RET-2	2.774	15.26 (Z)	+45.9	3.052	13.52 (X)	+51.3	10.082	+23.7
RET-3	3.316	17.01 (X)	+74.3	3.772	15.46 (Z)	+87.0	13.759	+68.8
RET-4	3.471	17.35 (Z)	+82.5	3.957	15.736 (X)	+96.2	14.255	+74.9

In contrast, examination of the first mode shapes presented in Fig. 11 indicates that the dominant behavior remains globally flexural in all models, and the modal deformation patterns are nearly identical. No significant change in torsional dominance or modal shape distribution is observed following the strengthening interventions. The maximum modal displacements continue to concentrate in the upper regions of the minaret shaft across all models.

This observation demonstrates that, although the strengthening scenarios increase the global stiffness of the system, they do not alter the fundamental dynamic behavior of the structure. Accordingly, the collapse mechanisms observed in the nonlinear dynamic analyses are associated not with changes in modal characteristics but rather with localized tensile cracking and compressive crushing developing within the masonry units. This finding clearly indicates that an increase in natural

frequencies alone is not sufficient to ensure improvement in seismic performance.

7.2. Nonlinear time-history analyses

The five different models (RM, RET-1, RET-2, RET-3, and RET-4) were subjected to nonlinear time-history analyses under three different ground motion records (GM-1, GM-2, and GM-3). The analysis results were comparatively evaluated to reveal the influence of the strengthening scenarios on the global dynamic response and the evolution of local damage within the masonry units. In this context, the global response was first exam-

ined through displacement-time histories, followed by an assessment of strain distributions and the progression of damage mechanisms.

7.2.1. Displacement-time histories

Fig. 12 presents the displacement-time histories of the RM and RET-1–RET-4 models in the X and Z directions under the three different ground motion records. The results indicate that the strengthening interventions influence the dynamic response in different ways depending on the characteristics of the ground motion record.

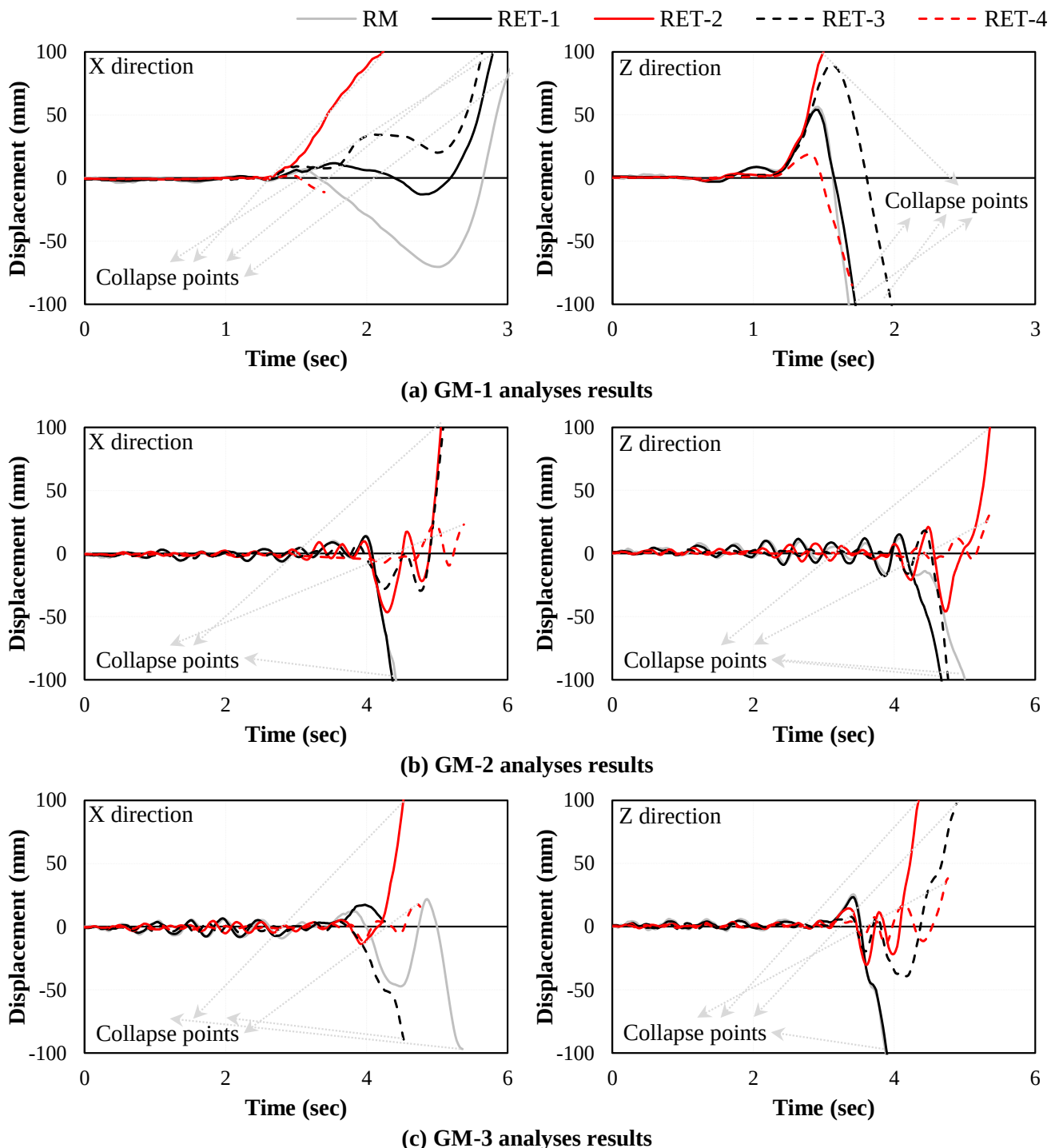


Fig. 12. Comparison of displacement-time histories of the reference model (RM) and the strengthening scenarios.

Under GM-1, the RET-1 model reduced the maximum displacement amplitude in the X direction compared to the RM; however, this improvement was insufficient to prevent global collapse. In the Z direction, the response of RET-1 remained largely similar to that of the RM. This indicates that the discontinuous internal steel plate system defined along the spiral staircase (RET-1) partially redistributed local strain demands but did not eliminate the dominant damage mechanism concentrated in the transition region.

For the RET-2 model under GM-1, a more critical outcome was observed. Although the continuous internal steel plate in the transition region increased global stiffness, earlier divergence was detected in the displacement histories. This suggests that stiffness enhancement may, in certain cases, adversely affect stress redistribution and trigger premature instability. In other words, a targeted yet partially continuous steel plate application may introduce secondary weakness planes at elevations where stiffness discontinuities occur.

The RET-3 model, owing to the continuous internal steel plate system along the shaft, distributed displacement demands more uniformly; however, it was unable to fully eliminate the collapse mechanism. For RET-4, although complete data in the X direction were not available, a certain reduction in post-elastic displacement amplitudes was observed in the Z direction. Nevertheless, global instability developed as the loading cycles progressed, resulting in collapse. This outcome indicates that, despite the increase in tensile capacity provided by the longitudinal reinforcements, the system-level collapse behavior could not be entirely prevented.

Under GM-2, the RET-1 model exhibited a dynamic response in both X and Z directions that was very similar to the RM. This finding indicates that the discontinuous internal banding approach does not significantly modify global behavior under certain ground motion characteristics. RET-2 and RET-3 produced comparable displacement histories under GM-2, demonstrating that increasing the extent of strengthening elements does not necessarily result in proportional improvement in nonlinear dynamic performance. In particular, stiffness enhancement concentrated in the transition region did not fundamentally alter system behavior for some ground motions.

Although complete collapse-time data for RET-4 under GM-2 could not be fully tracked, it is evident that lower displacement amplitudes were achieved compared to the other models. Nevertheless, examination of strain distributions indicates that the structure ultimately collapsed. This result suggests that RET-4 is relatively more effective in limiting displacement demands; however, it does not entirely eliminate global instability.

Under GM-3, none of the RET models exhibited earlier collapse than the RM in the X direction. This indicates that the tendency for early instability observed in GM-1 is record-dependent. In the Z direction, a different pattern emerged. The RET-1 model displayed a collapse behavior similar to that of the RM, with no significant delay. In contrast, RET-2 and RET-3 delayed the time of collapse to a certain extent. Particularly, strengthening systems providing greater continuity along the shaft (RET-3) managed displacement growth in a more controlled

manner. The RET-4 scenario also delayed collapse; however, ultimate instability could not be prevented.

These results demonstrate that the effectiveness of strengthening strategies is sensitive to the frequency content and duration characteristics of the ground motion. Discontinuous or regionally concentrated stiffness increases may lead to early instability under certain records, whereas more comprehensive strengthening approaches can delay collapse. However, as long as the tensile-dominated damage mechanism developing within masonry units is not fully controlled, ultimate collapse cannot be entirely prevented.

When the displacement–time histories are evaluated collectively, it is evident that global instability persists in all strengthening scenarios despite the significant increases observed in natural frequencies. This finding indicates that stiffness enhancement alone is insufficient in historic masonry minarets and that collapse behavior is primarily governed by nonlinear material response and strain localization.

7.2.2. Strain distributions and damage mechanism

The maximum principal strain distributions obtained under the three ground motions are summarized in Fig. 13. The results indicate that the instability behavior observed in the displacement–time histories is associated with tensile-dominated strain localization occurring in specific regions of the structure.

For the reference model (RM), the dominant damage mechanism under all ground motions appears as a circumferential strain band concentrated in the shaft–substructure transition region. Tensile strain accumulation develops on one side of the cross-section due to bending effects and eventually leads to loss of sectional integrity. The consistency of this damage location across different ground motions indicates that the mechanism is primarily controlled by structural geometry and stiffness distribution.

In the RET-1 scenario, the discontinuous steel plates installed along the spiral staircase partially redistributed the strain field, but the circumferential damage band remained. Although peak strain values were reduced, the governing damage mechanism was not eliminated, indicating limited effectiveness of the discontinuous internal banding system.

In the RET-2 model, the continuous steel plate located in the transition region reduced strain intensity locally; however, new strain concentrations developed at the boundary of the strengthened zone due to stiffness discontinuities. This behavior explains the early instability observed under GM-1.

The RET-3 configuration, consisting of a continuous internal steel lining along the shaft, resulted in a more uniform strain distribution and reduced local peaks. Nevertheless, the circumferential tensile band mechanism persisted, indicating that improved stiffness continuity alone cannot fully prevent damage governed by the low tensile capacity of masonry.

In the RET-4 hybrid system, the addition of longitudinal steel reinforcements further reduced circumferential strain concentrations under some ground motions. However, new strain patterns associated with longitudinal

cracking developed in certain cases and collapse still occurred.

Overall, while all strengthening scenarios modified the strain distribution and reduced strain magnitudes to varying degrees, none completely eliminated the tensile-

dominated damage mechanism. The results indicate that the ultimate seismic performance of the minaret is largely controlled by the limited tensile strength and damage-softening characteristics of the masonry material.

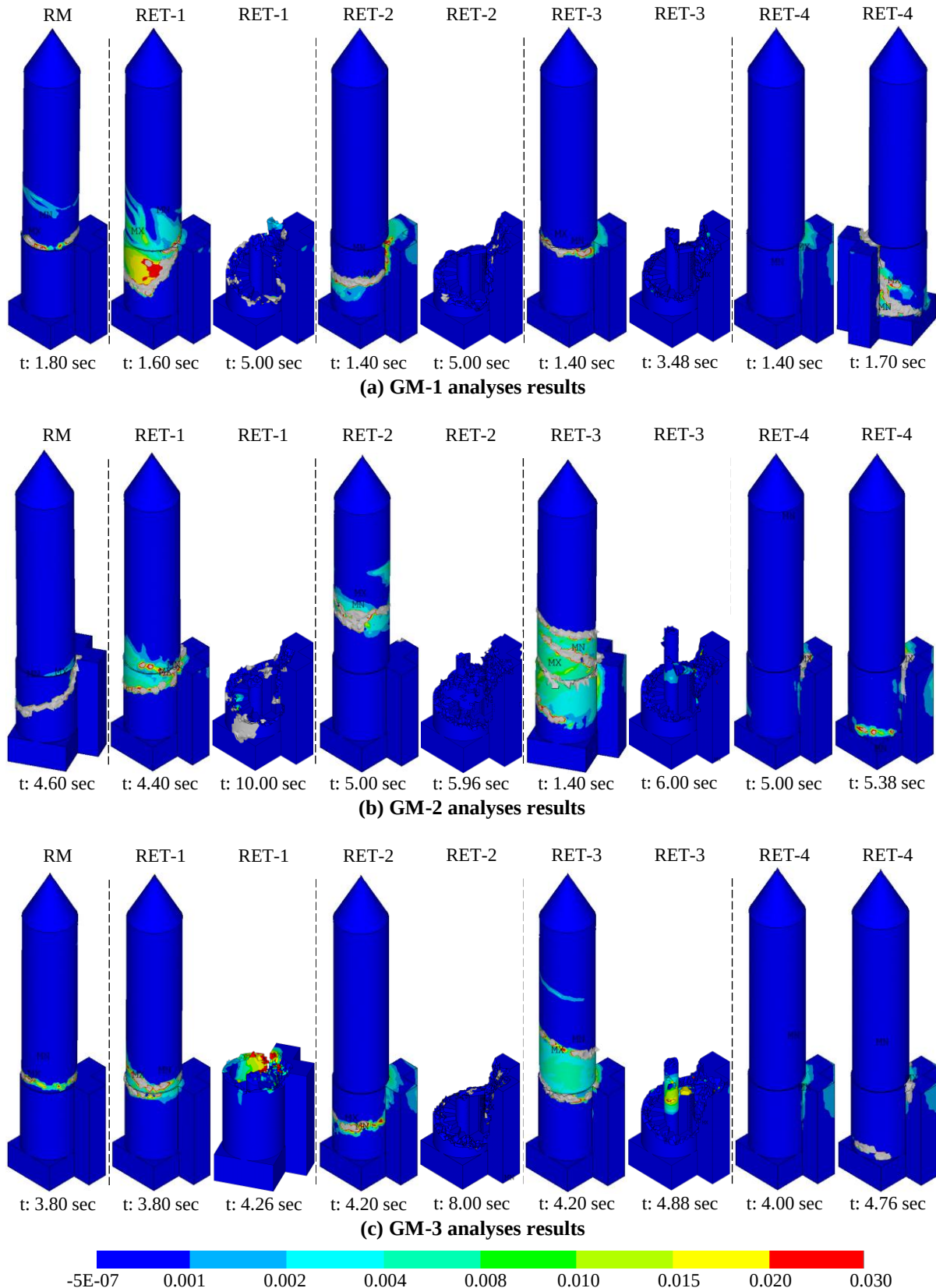


Fig. 13. Comparison of damage distributions of the reference model (RM) and the strengthening scenarios.

7.2.3. Integrated quantitative comparison and availability of response measures

For a consistent comparison, the response of each model was evaluated at two distinct times: the collapse time, (t_c), defined according to the combined criterion introduced in Section 5.2, and the final solved time, (t_{end}). These quantities should not be interpreted interchangeably because several analyses continued after the adopted structural collapse condition had already been reached.

Under GM-1, the (t_c/t_{end}) values were 1.68/5.00 s for RM, 1.72/5.00 s for RET-1, 1.50/5.00 s for RET-2, 1.98/3.48 s for RET-3, and 1.70/1.70 s for RET-4, with the latter determined from the Z-direction displacement response. Relative to RM, RET-3 delayed the collapse condition by approximately 17.9%, whereas RET-2 reduced the collapse time by approximately 10.7%. The changes obtained for RET-1 and RET-4 were limited to approximately 2.4% and 1.2%, respectively.

Under GM-2, the corresponding (t_c/t_{end}) values were 4.98/10.00 s, 4.66/10.00 s, 5.36/5.96 s, 4.76/6.00 s, and 5.34/5.38 s for RM and RET-1–RET-4, respectively. Compared with RM, RET-2 and RET-4 delayed the collapse condition by approximately 7.6% and 7.2%, whereas RET-1 and RET-3 reduced the collapse time by approximately 6.4% and 4.4%, respectively.

Under GM-3, the (t_c/t_{end}) values were 3.88/10.00 s for RM, 3.88/4.26 s for RET-1, 4.34/8.00 s for RET-2, 4.86/4.88 s for RET-3, and 4.76/4.76 s for RET-4. Accordingly, RET-2, RET-3, and RET-4 delayed the collapse condition by approximately 11.9%, 25.3%, and 22.7%, respectively, whereas RET-1 produced no change in the identified collapse time. These comparisons quantitatively demonstrate that the influence of strengthening on collapse evolution was strongly ground-motion dependent and was not proportional to the increase in natural frequency.

Maximum top displacement was evaluated only over the interval ($t-t_c$), because displacement growth occurring after the formation of a global collapse mechanism does not represent stable structural capacity. The absolute pre-collapse peak top displacement reached approximately 100 mm, corresponding to a response envelope extending to approximately (± 100) mm. Residual displacement was not defined as a comparable response quantity because none of the model–ground-motion combinations reached a stable post-earthquake equilibrium state after satisfying the adopted collapse criterion. Therefore, the displacement at (t_{end}) should not be interpreted as a conventional residual displacement.

The first-principal-strain distributions were plotted using the same contour interval, ranging from $-5E-07$ to 0.030, for all models and ground motions. However, a model-wide scalar maximum tensile strain was not used as an independent performance measure. In a smeared-crack formulation, isolated integration-point peaks are highly sensitive to mesh discretization and may not represent a physically meaningful structural damage state. The strain assessment was therefore based on the magnitude, spatial extent, propagation, and continuity of the localized tensile-strain bands rather than on an isolated numerical maximum.

A damaged-element ratio was not calculated because SOLID65 identifies cracking and crushing at individual integration points and in different principal directions. Consequently, classification of an entire element as either damaged or undamaged would require an additional arbitrary threshold that was not included in the original analysis framework. In the strengthened configurations, the calculated equivalent stress demand in the steel components exceeded the adopted yield strength, indicating that the yielding threshold was reached. Exact peak steel-stress values were not introduced as an additional ranking parameter because stresses beyond the onset of yielding are strongly governed by the adopted constitutive idealization and do not independently characterize the effectiveness of the strengthening system.

Base-shear histories were not retained among the original time-history output requests and would require reconstruction from the time-dependent foundation reactions. Dissipated energy was also not reported as a scalar response quantity because the adopted Willam–Warnke smeared-crack model was not formulated using calibrated fracture-energy regularization. Consequently, a numerical energy-dissipation comparison would be mesh dependent and could imply a level of physical accuracy not supported by the adopted constitutive model.

Collectively, the available quantitative results show that the strengthening systems substantially modified stiffness, displacement development, and the timing of damage progression, but their effectiveness was record dependent. The decisive performance indicator was not the stiffness increase alone, but whether the intervention prevented or delayed the development of a continuous tensile-damage mechanism at the shaft–substructure transition.

8. Conclusions

The four strengthening scenarios (RET-1–RET-4) developed in this study significantly increased the global stiffness of the minaret and resulted in noticeable increases in natural frequencies. However, the nonlinear time-history analyses demonstrated that stiffness enhancement alone does not necessarily improve seismic performance. Although the onset of instability varied among the strengthening scenarios, none of the interventions completely eliminated the ultimate collapse mechanism. This indicates that global stiffness primarily modifies the dynamic response but does not control the governing damage evolution.

Strain distribution analyses revealed that collapse behavior is mainly governed by tensile-dominated strain localization concentrated in the shaft–substructure transition region. Under all ground motions, the dominant damage mechanism developed as a circumferential strain band in this zone. While the strengthening scenarios altered the magnitude and spatial distribution of strain concentrations, they did not eliminate the mechanism itself. These findings highlight the decisive role of sectional tensile capacity and damage-softening behavior in the seismic response of masonry minarets.

The early instability observed in the RET-2 scenario also emphasizes the importance of stiffness continuity in

strengthening design. Although the localized steel plate reduced strain intensity in the transition region, it introduced a stiffness discontinuity that triggered secondary damage planes. This outcome demonstrates that localized stiffness enhancement may cause unintended stress redistribution within the structural system.

The analyses further indicate that structural response is highly sensitive to masonry material parameters, including compressive strength (f_c), tensile strength (f_t), damage-softening characteristics, and energy dissipation capacity. The survival of the structure through historical earthquakes suggests that the adopted parameters may represent conservative assumptions. Future studies should therefore incorporate material models calibrated through field observations and experimental data, together with systematic parameter sensitivity analyses.

Overall, the findings indicate that strengthening strategies for historic masonry structures should not focus solely on increasing global stiffness. Instead, effective interventions should enhance tensile resistance, ensure stiffness continuity, and mitigate local strain concentrations to improve seismic performance.

This study is subject to several limitations. The steel-masonry interfaces were idealized as fully bonded, the material parameters were defined primarily based on literature data rather than structure-specific experimental characterization, and soil-structure interaction was not considered. Moreover, the strengthening alternatives examined in this study were limited to internal intervention strategies. A formal mesh-sensitivity or convergence analysis was not performed; therefore, the numerical results may retain some dependence on the adopted nominal element size of 15–20 cm. This dependence is expected to be particularly relevant to localized response quantities, including first principal strain concentrations and the predicted spatial distribution of cracking, which are generally more sensitive to mesh discretization than global parameters such as natural frequencies and overall displacements. Accordingly, the strain contours should be interpreted as comparative indicators of potential damage localization rather than as mesh-objective predictions of crack width or highly lo-

calized failure. Future studies should incorporate in situ dynamic measurements, structure-specific material characterization, and experimental testing of representative masonry components and steel-masonry connection details to calibrate and validate the numerical model. Systematic comparisons of coarse, intermediate, and refined meshes should also be conducted to evaluate the convergence of natural frequencies, global displacements, collapse times, and strain-localization patterns. Further research should additionally investigate material and interface sensitivity, soil-structure interaction, and alternative strengthening configurations within a performance-based conservation framework.

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Conflict of Interest

The authors declare no potential conflicts of interest with respect to the research, authorship, and/or publication of this manuscript.

Data Availability

The datasets generated and/or analyzed during the current study are not publicly available but are available from the corresponding author upon reasonable request.

AI Assistance

During the preparation of this manuscript, ChatGPT-5 (OpenAI) was used to support various aspects of the writing process, including improving clarity, grammar, spelling, and academic tone. All content generated with the assistance of this tool was carefully reviewed and edited by the authors, who take full responsibility for the final content of the manuscript.

Author Contributions

All authors made substantial contributions to the conception and design of the study, acquisition of data, analysis and interpretation of data; drafted or critically revised the manuscript for important intellectual content; and approved the final version to be published.

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Research Article

Optimal design of reinforced concrete cantilever retaining walls via simulated annealing

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ABSTRACT

This study investigates the optimal design of reinforced concrete cantilever (RCC) retaining walls considering both environmental and economic objectives under three concrete alternatives: green, sustainable, and traditional concrete. The design problem is formulated as two independent single-objective optimization models aimed at minimizing CO₂ emissions and material cost while satisfying geotechnical, structural, and material constraints. A simulated annealing-based approach is employed to efficiently explore and obtain the optimal design. The results demonstrate a strong interdependence between cost and emissions for all material types. Green concrete consistently provides the lowest CO₂ emissions and cost values, followed by sustainable concrete, while traditional concrete yields the highest values for both objectives. Furthermore, the traditional concrete configurations require greater material consumption compared to green and sustainable alternatives, whereas green concrete consistently achieves the most efficient overall performance. Green concrete consistently achieved the best performance, reducing the material cost by up to 16.9% and carbon emissions by up to 20.3% compared with traditional concrete while satisfying all structural and geotechnical design requirements. Overall, the findings confirm that the proposed simulated annealing approach is a robust and effective tool for the two independent single-objective optimization of RCC retaining walls, enabling efficient decision-making between environmental and economic criteria.

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1. Introduction

Geotechnical retaining structures are crucial in civil engineering for ensuring slope stability and safeguarding nearby infrastructure as well as natural ecosystems. Furthermore, these structures are essential for preventing soil erosion, managing water runoff, and mitigating the risk of landslides and flooding. Among the various types of retaining walls, Reinforced Concrete Cantilever (RCC) retaining walls have long been a cornerstone of modern engineering, offering strength, durability, and versatility. However, the growing demand for environmentally responsible construction practices has prompted the need for more sustainable

design approaches in the construction of RCC retaining walls.

In recent decades, the importance of sustainability in civil engineering has grown significantly, with an increasing emphasis on designing structures that minimize emissions while maintaining functional integrity. Reinforced Concrete Cantilever (RCC) retaining walls are widely preferred in numerous engineering projects due to their ability to withstand complex loads, high soil pressures, and seismic forces (Kaveh et al. 2013). Their structural superiority lies in the synergy of concrete's compressive strength and steel's tensile capacity, allowing for versatile geometric configurations such as cantilever, counterfort, and anchored designs. This adaptabil-

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ity ensures efficient integration into diverse urban and natural environments (Shakeel et al. 2022). However, as the construction industry seeks to reduce its massive carbon footprint, the challenge has shifted from purely functional design to achieving a balance between safety, cost-efficiency, and environmental preservation. The evolution of optimization research for RCC retaining walls can be categorized into three primary frameworks. The first category focuses on Geometric and Cost-Centric Optimization, where metaheuristic algorithms are used to minimize material weight or construction costs. Studies by Kashani et al. (2022) utilized genetic algorithms (GA), while Gandomi et al. (2017a) and Tayfur and Kamiloğlu (2024) employed evolutionary and improved optimization methods to achieve significant savings. Similarly, Kumar and Suribabu (2017) demonstrated that differential evolution could reduce wall weight by 15% compared to manual designs. While these studies, including the works of Kaveh et al. (2013, 2020a) and Kayabekir et al. (2021) using Teaching-Learning-Based Optimization (TLBO) and Collision Bodies Optimization (CBO), established the efficiency of metaheuristics, they primarily treated concrete as a conventional material with fixed environmental properties. The second category involves the Integration of Environmental Criteria, where CO₂ emissions are introduced as a secondary objective. Khajehzadeh et al. (2014) applied a hybrid gravitational search algorithm to minimize both cost and emissions, while Aydoğdu (2017) highlighted the benefits of lower-grade concrete in reducing the carbon footprint through Social Spider and Biogeography-Based Optimization. Despite these advancements, most existing literature (Akin et al. 2016; Das et al. 2016; Sheikholeslami et al. 2016; Ukritchon et al. 2017; Kaveh et al. 2022) relies on generic emission factors and a limited range of standard concrete mixes, often overlooking the complex performance characteristics of modern sustainable alternatives. The third and most recent framework, which defines the present study, addresses the gap between material science and structural optimization.

This research distinguishes itself by integrating a comprehensive mixture database (Naseri et al. 2022)—comprising green, sustainable, and traditional concrete types—directly into the optimization loop. Unlike traditional models that use simplified material inputs, our approach evaluates the specific binder intensities and cost-performance ratios of eco-friendly mixtures incorporating recycled aggregates and low-carbon additives. By utilizing a specifically tailored Simulated Annealing (SA) algorithm to navigate the non-linear constraints, this study provides a more nuanced understanding of how advanced material selection dictates the optimal geometry of low-carbon infrastructure. Accordingly, a significant gap exists between the use of theoretical concrete grades in optimization models and the practical application of real-world sustainable mixtures. Moreover, the present study fundamentally diverges from existing literature utilizing SA, Jaya, GA, TLBO, BBO, CBO, and other metaheuristic applications for RCC retaining walls in its systemic integration of material-stiffness-emission variables. While previous research has primarily focused on the mathematical convergence and computational speed of these algorithms (e.g., Kaveh et al. 2020b; Gandomi et

al. 2017b; Temur and Bekdaş 2016), they often treat the material input as a secondary, static constant. In contrast, our approach reconfigures the Simulated Annealing (SA) framework to act as a bridge between structural geometry and a mixture database. The distinctive contribution of this study is rooted in several key methodological advancements that separate it from traditional metaheuristic applications. Firstly, through algorithmic coupling, our modified Simulated Annealing (SA) framework simultaneously navigates a discrete set of green and sustainable concrete mixtures—each characterized by unique binder intensities and cost-performance ratios—rather than optimizing for a single, static concrete grade as seen in standard TLBO or Jaya implementations. Secondly, the study emphasizes constraint rigidity by maintaining high-fidelity compliance with rigorous safety factors. This ensures that the resulting optimal designs are technically reproducible in real-world engineering contexts, unlike many GA and BBO studies that simplify geotechnical and structural constraints to achieve faster convergence. Finally, by addressing the sensitivity to material volatility, this research explores the non-linear relationship between waste-based cementitious materials and structural volume. This critical analysis demonstrates that the efficacy of an optimization algorithm is fundamentally tied to the quality of the material database it operates upon, effectively shifting the research focus from purely numerical performance toward material-aware structural optimization. Unlike previous studies (Table 1), which generally focused on a single concrete type or solely on geometric optimization, this study investigates the optimal design of RCC retaining walls by simultaneously considering environmental and economic objectives for three different concrete alternatives. This unified framework enables a systematic comparison of the influence of different concrete materials on the optimal design, material cost, and environmental performance of RCC retaining walls.

Consequently, sustainable design in retaining walls involves not only considering geotechnical evaluation, structural stability, energy efficiency, and resource conservation. This includes optimizing material usage, reducing carbon footprints, and ensuring that materials and construction processes are in harmony with the surrounding ecosystem. Thus, incorporating sustainable design principles into RCC retaining walls requires a careful balance between geotechnical design, structural integrity, environmental responsibility, and cost-effectiveness. This study highlights the significance of sustainable RCC retaining wall designs in promoting infrastructure safety and longevity through the optimization of cost and CO₂ emissions by utilizing green and sustainable concrete compared to traditional mixtures. This paper is organized as follows. Section 2 describes the methodology, including the RCC retaining wall design variables, material properties, design constraints, objective functions, and the implementation of the Simulated Annealing (SA) algorithm. Section 3 presents and discusses the optimization results for the cost and CO₂ emission objectives, together with the sensitivity analysis, regression analysis, and comparison of the optimized designs. Finally, Section 4 summarizes the main findings, contributions, and limitations of the study.

Table 1. Summary of recent studies on RCC retaining wall optimization problem.

Study	Objective function(s)	Design variables	Constraints	Optimization technique
Shakeel et al. (2022)	Review of cost, weight, CO ₂ emission, and reliability-based optimization approaches	Various wall geometry, reinforcement, backfill, and foundation	Geotechnical stability, structural strength, and serviceability requirements	Complex method (CM); GA; PSO; SA
Villalba et al. (2010)	Minimize CO ₂ emissions	Wall dimensions, footing geometry, reinforcement quantities	Sliding, overturning, bearing capacity, and structural design requirements	Mathematical/parametric optimization GA; PSO; SA
Öztürk and Türkel (2019)	Minimize construction cost	Stem, toe, heel, base, key dimensions, and reinforcement details	Sliding, overturning, bearing capacity, flexural, and shear design constraints	Jaya Algorithm
Schmied and Karlsson (2021)	Simultaneous minimization of environmental impact and investment cost	Wall geometry, reinforcement ratios, and material selection	Eurocode structural design requirements and geotechnical stability constraints	Multi-objective optimization framework; PSO
Khajehzadeh et al. (2014)	Minimize construction cost and maximize stability	Geometric dimensions of retaining wall	Sliding, overturning, bearing capacity, and structural safety constraints	Hybrid Adaptive Gravitational Search Algorithm (HAGSA)
Kayabekir et al. (2020)	Simultaneous minimization of construction cost and CO ₂ emissions	Wall dimensions and reinforcement quantities	Sliding, overturning, bearing capacity, flexural, and shear design constraints	Multi-objective Harmony Search Algorithm
Khajehzadeh et al. (2013)	Minimize construction cost and environmental impact	Retaining wall geometry and reinforcement variables	Sliding, overturning, bearing capacity, and structural design constraints	Opposition-Based Gravitational Search Algorithm (OBGSA)
Kaveh and Behnam (2013)	Minimize construction cost	Wall geometry and reinforcement details	Geotechnical stability and reinforced concrete design constraints	Charged System Search (CSS)
Kaveh and Khayatizad (2014)	Minimize construction cost	Geometric dimensions and reinforcement variables	Sliding, overturning, bearing capacity, and structural requirements	Ray Optimization (RO)
Kaveh and Soilemani (2015)	Minimize construction cost	Wall geometry and reinforcement parameters	Geotechnical and structural design constraints	Colliding Bodies Optimization (CBO) and Democratic Particle Swarm Optimization (DPSO)
Kalemci et al. (2020)	Minimize construction cost	Wall dimensions and reinforcement variables	Sliding, overturning, bearing capacity, flexural, and shear constraints	Grey Wolf Optimizer (GWO)
Yücel et al. (2021)	Minimize construction cost with improved convergence performance	Wall geometry and reinforcement parameters	Strict geotechnical and structural design constraints	Adaptive hybrid optimization algorithm
Kaveh et al. (2022)	Minimize construction cost	Geometric dimensions and reinforcement variables	Geotechnical stability and reinforced concrete design constraints	Plasma Generation Optimization (PGO)
Keivanian et al. (2023)	Multi-objective optimization considering sustainability and cost	Wall geometry, reinforcement, and material parameters	Geotechnical stability, structural safety, and sustainability constraints	Hybrid metaheuristic optimization approach
Gandomi et al. (2017a)	Minimize construction cost	Wall geometry and reinforcement variables	Sliding, overturning, bearing capacity, and reinforced concrete design constraints	Evolutionary Algorithms
Gandomi et al. (2017b)	Minimize construction cost	Geometric dimensions and reinforcement details	Sliding, overturning, bearing capacity, and structural design constraints	Interior Search Algorithm (ISA) with different bound constraint handling strategies
Sheikholeslami et al. (2016)	Minimize construction cost	Wall dimensions and reinforcement quantities	Sliding, overturning, bearing capacity, flexural, and shear design constraints	Hybrid Firefly Algorithm with Upper Bound Strategy
Kashani et al. (2022)	Simultaneous minimization of construction cost and CO ₂ emissions (multi-objective)	Wall geometry, reinforcement quantities, and material parameters	Geotechnical stability and reinforced concrete design requirements	Comparative multi-objective metaheuristic optimization algorithms
Tayfur and Kamiloğlu (2024)	Minimize construction cost	Geometric dimensions and reinforcement variables	Sliding, overturning, bearing capacity, and structural safety constraints	Improved Teaching–Learning–Based Optimization (ITLBO) Algorithms

2. Methodology

The design process involves two groups of design parameters. The first group consists of the geometric dimensions of the RCC retaining wall, represented by variables X_1 to X_5 and illustrated in Fig. 1, which include the width of the toe slab (X_1), the width of the base slab (X_2),

the thickness of the stem at the top (X_3), the thickness of the base slab (X_4), and the thickness of the stem at the bottom (X_5). The second group involves the required steel reinforcement quantities, denoted as R_1 to R_3 , comprising the area of vertical reinforcement in the stem (R_1), the area of horizontal reinforcement in the toe (R_2), and the area of horizontal reinforcement in the heel (R_3).

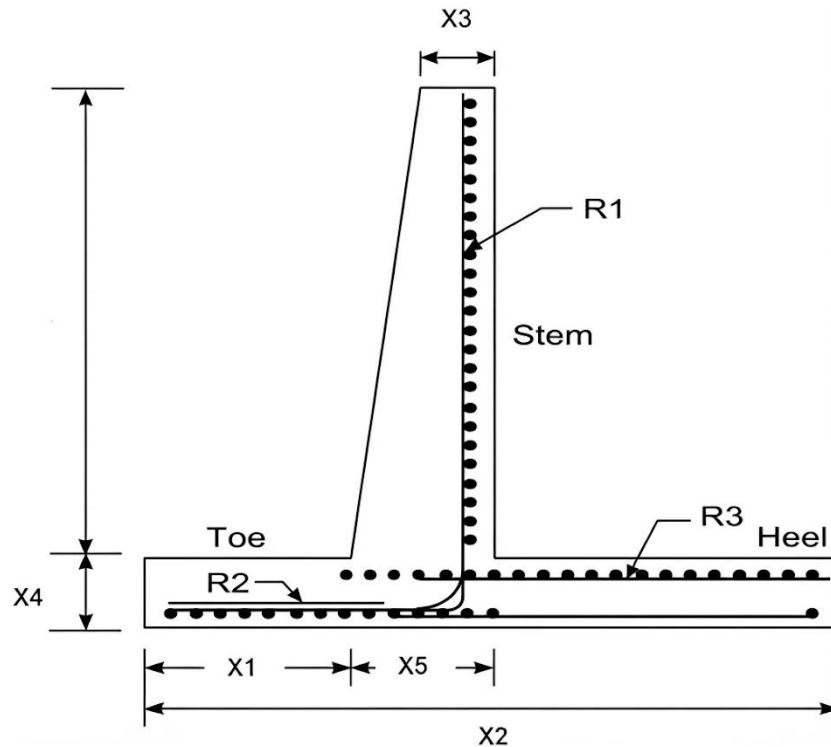


Fig. 1. Design variables of the RCC retaining wall.

The areas of steel reinforcement are represented by a combination of bar diameters and their spacings. Various reinforcement configurations were used, combining 3–30 evenly spaced bars with diameters ranging from 10 to 30 mm. These configurations were chosen to ensure that the reinforcement area per meter of wall length met the allowable minimum and maximum limits (ACI 318-14 2014) for the spacing between longitudinal bars, which is defined by the diameter of the maximum diameter of aggregate in the concrete and the diameter of the longitudinal reinforcements. For a given bar count (Bars #) and bar diameter (bar size), the corresponding total reinforcement area (A_s in cm^2) is calculated directly; for example, 3 bars of size 10 yield an A_s of 2.356 cm^2 ($\eta = 1$), 4 bars of size 10 yield 3.141 cm^2 ($\eta = 2$), and higher index combinations scale accordingly up to larger configurations such as 18 bars of size 30 yielding 127.234 cm^2 . To ensure practical constructability, lower and upper bounds for the geometric design variables were established based on commonly accepted reinforced concrete retaining wall design recommendations (ACI 318-05 2005). Specifically, the minimum stem top thickness was limited to 0.25 m, the minimum base slab thickness to 0.30 m, the toe projection was constrained within 0.10–0.15H, the base width was maintained 0.40–0.7H, and the stem bottom thickness was constrained to at least 0.08–0.10H. Simulated annealing was implemented to obtain optimum retaining wall designs with

respect to cost and CO_2 emissions over wall heights (H) of 4–10 m under constant soil conditions. The design variables considered in the optimization process and their corresponding lower and upper bounds are presented in Table 2.

The input parameters are defined with a reinforcing steel yield strength (f_y) of 400 MPa, and a concrete compressive strength (f'_c) of 21 MPa. The concrete cover (C_c) is 7 cm, and the shrinkage and temperature reinforcement percentage (ρ_{st}) is 0.002. The surcharge load (q) is set to 0 with a backfill slope (β) of 0° . For the soil properties, the internal friction angle of the retained soil (ϕ) is 36° , the internal friction angle of the base soil (ϕ') is 34° , the unit weight of the retained soil (γ_s) is 17.5 kN/m^3 , the unit weight of the base soil (γ'_s) is 18.5 kN/m^3 , the unit weight of concrete (γ_c) is 23.5 kN/m^3 , and the cohesion of the base soil (c) is 0 kPa.

2.1. Design of retaining wall

The Mononobe-Okabe (M-O) method, a commonly used pseudo-static approach developed by Mononobe and Okabe in the 1920s, was employed to determine the passive and active forces acting on a unit length of the retaining wall (Okabe 1924; Mononobe and Matsuo 1929). As an essential tool in modern geotechnical engineering, the Mononobe-Okabe (M-O) method offers a

more accurate representation of seismic effects on retaining walls than static methods, enabling the design of structures that maintain stability during seismic events. It is especially useful in seismic regions, where it is com-

monly applied to design earthquake-resistant structures such as highways, dams, and retaining walls. Fig. 2 illustrates the key forces involved in the Mononobe-Okabe (M-O) method.

Table 2. Boundary constraints of the design variables.

Design variable	Unit	Lower bound	Upper bound
X_1 (toe slab width)	m	0.480	1.300
X_2 (base slab width)	m	1.600	4.700
X_3 (stem top thickness)	m	0.270	1.350
X_4 (base slab thickness)	m	0.310	1.330
X_5 (stem bottom thickness)	m	0.550	2.400
R_1 (vertical reinforcement in stem)	(cm ²)	45.000	565.000
R_2 (horizontal reinforcement in toe)	(cm ²)	40.000	480.000
R_3 (horizontal reinforcement in heel)	(cm ²)	38.000	445.000

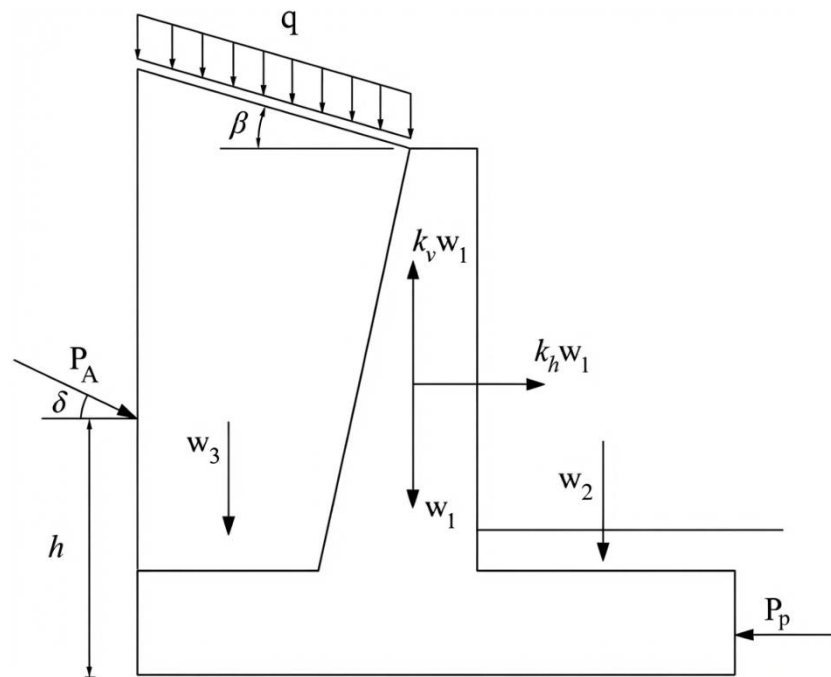


Fig. 2. Forces acting on an RCC retaining wall based on the M–O method.

These forces include: the weight of the concrete wall (W_1), the weight of the soil on the toe (W_2), the weight of the backfill material (W_3), the surcharge load (q), the inertial forces in both the horizontal ($k_h W_1$) and vertical ($k_v W_1$) directions, and the seismic passive and active earth pressures (P_p and P_a). The movement results in two forces ($k_v W_1$ and $k_h W_1$) that act in the vertical and horizontal directions, respectively. Here, k_v and k_h represent the vertical and horizontal coefficients that are specific to the local geographic conditions. In this study, the horizontal seismic coefficient was set at $k_h = 0.30$, while the vertical seismic coefficient was taken as $k_v = 0.15$. The total maximum active earth pressure acting on the wall (P_a) is given by the M-O theory as:

$$P_a = \frac{1}{2} k_a \gamma_{\text{soil}} H^2 (1 - k_v) \quad (1)$$

Here, H indicates the height of the wall, γ_{soil} stands for the unit weight of the soil, k_v is the coefficient of vertical acceleration, and k_a represents the dynamic active earth pressure coefficient, which is determined as:

$$k_a = \frac{\cos^2(\phi - \theta - \beta)}{\cos \theta \cos^2 \beta \cos(\delta + \theta + \beta)} \left[1 + \sqrt{\frac{\sin(\delta + \phi) \sin(\phi - \theta - i)}{\cos(\delta + \beta + \theta) \cos(i - \beta)}} \right]^2 \quad (2)$$

where, i represents the inclination of the backfill, θ defines the seismic inertia angle, ϕ is the soil's internal friction angle, β indicates the vertical inclination of the back wall, and δ denotes the angle of wall friction between the soil and the rear face of the stem. The passive earth pressure (P_p) is calculated similarly to the active earth pressure, as follows:

$$P_p = \frac{1}{2} k_{pe} \gamma_{\text{soil}} h^2 (1 - k_v) \quad (3)$$

where, the dynamic passive earth pressure coefficient (k_p), given by:

$$k_p = \frac{\cos^2(\phi + \theta - \beta)}{\cos^2 \theta \cos(\delta - \theta + \beta)} \left[1 + \sqrt{\frac{\sin(\phi - \delta) \sin(\phi + i - \theta)}{\cos(\delta - \beta + \theta) \cos(i - \beta)}} \right]^2 \quad (4)$$

where, the inclination of the backfill is represented by i , while β denotes the back wall angle. The internal friction angle of the soil is referred to as ϕ , and δ represents the friction angle between the stem and the soil. The seismic inertia angle θ is defined by the following equation:

$$\theta = \tan^{-1} \left(\frac{k_h}{1 - k_v} \right) \quad (5)$$

In this study, k_h represents the horizontal seismic acceleration coefficient, and combinations of horizontal and vertical (k_v) coefficients are applied, ranging from 0 to 0.3, with a step size of 0.15. The constraints for the optimization of RCC walls are analyzed using three core evaluation methods: (1) geotechnical requirement, (2) structural requirement, and (3) geometrical analysis.

2.1.1. Geotechnical requirements

Geotechnical stability checks include overturning, sliding, and bearing capacity analyses. Overturning stability is checked by calculating the factor of safety for overturning (FSO):

$$FSO = \frac{\sum MR}{\sum MO} \quad (6)$$

To ensure safety, the factor of safety must be at least 1.5 ($FSO \geq 1.5$), where $\sum MO$ denotes the sum of overturning moments and $\sum MR$ represents the sum of resisting moments. To check the sliding stability, the factor of safety for sliding (FSS) is evaluated as follows:

$$FSS = \frac{\sum FR}{\sum FD} = \frac{(\sum W_{wall} \cdot \tan(\frac{2\phi_{base}}{3})) + 2Bc_{base} + P_p + P_k}{P_a \cdot \cos \beta} \quad (7)$$

The sliding factor of safety must satisfy the condition $FSS \geq 1.5$, where, the base soil's internal friction angle is denoted by ϕ_{base} , while the adhesion between the base slab and soil is represented by c_{base} . P_a represents the active force, and P_p is the passive force located in front of the base. The sum of the horizontal driving forces is given by $\sum FD$, and $\sum FR$ denotes the sum of the horizontal resisting forces. The total weight of the wall is represented by $\sum W_{wall}$, and B indicates the width of the base slab.

The bearing capacity of the soil is assessed by the bearing factor of safety (FSB):

$$FSB = \frac{q_{ult}}{q_{max}} \quad (8)$$

where q_{ult} is the ultimate bearing capacity of the soil, and q_{max} is the maximum stress at the base slab. The ultimate bearing capacity is calculated using the following general expression:

$$q_{ult} = cN_c s_c + qN_q s_q + 0.5\gamma B' N_\gamma s_\gamma \quad (9)$$

where; c is Cohesion of the soil, q is Overburden pressure at the footing base level, B' is Effective footing width (B-2e) to account for eccentricity. N_c , N_q , N_γ are dimensionless bearing capacity factors based on the friction angle (ϕ). s_c , s_q and s_γ are the shape factors (Fredlund and Rahardjo 1993).

The limit stresses along the base slab are calculated using the equations below:

$$q_{max/min} = \frac{\sum V}{B} \left(1 \mp \frac{6e}{B} \right) \quad \text{and} \quad e = \frac{B}{2} - \frac{\sum M_R - \sum M_O}{\sum V} \quad (10)$$

where $\sum V$ is the sum of all of the vertical forces, and e represents the eccentricity of the corresponding distribution. The bearing strength criterion requires $FSB \geq 3$ (Gandomi et al. 2017a).

2.1.2. Structural requirements

The structural analysis includes verifying that the shear and moment capacities of all sections of the wall, such as the stem, heel, and toe meet the following conditions (ACI 318-05 2005):

$$\frac{M_n}{M_u} \geq 1 \quad \text{and} \quad \frac{V_n}{V_u} \geq 1 \quad (11)$$

The moment and shear capacities, M_n and V_n , were calculated for the toe, stem, and heel using following equations.

$$V_n = \phi_n 0.17 \sqrt{f_c} b d \quad \text{and} \quad M_n = \phi_m A_s f_y \left(d - \frac{a}{2} \right) \quad (12)$$

The shear strength is determined by ACI 318-05 (2005), where b is the width of the cross-section; f_c is the specified compressive strength of concrete; and ϕ_n is the nominal strength coefficient (equal to 0.75). In this context, a is the depth of the compressive stress block; d is the effective depth of the cross-section; f_y is the yield strength of steel; A_s is the cross-sectional area of the steel reinforcement; and ϕ_m is the nominal strength coefficient (equal to 0.9). As illustrated in Fig. 3, the pressure distribution from the base onto the foundation soil is non-uniform and is calculated using the following formulas for heel and toe. The ultimate shear and moment forces V_u and M_u are then used to compute the shear forces and design moment for the heel and toe slabs. The effective design forces acting on toe, heel, and stem are depicted in Fig. 3.

At the heel section of the base slab, the shear force and bending moment act on the critical section located a distance dh from the back face of the stem's base, defined as $dh = X4 - CC$. Similarly, the toe slab experiences its critical flexural moment at a section positioned dt away from the front face of the stem, where $dt = X4 - CC$. The shear force and bending moment requirement at the critical section of the toe slab are then determined using the following equations:

$$V_h = \left[1.6q + 1.2\gamma_c X_4 + 1.2\gamma_{rs} H + 1.2 \frac{W_{bs} + W_{bsdh}}{2} - 0.9 \frac{q_{dh} + q_{min}}{2} \right] (l_{heel} - d_h) \quad (13)$$

$$M_h = \left[\left(\frac{1.6q + 1.2\gamma_c X_4 + 1.2\gamma_{rs} H}{2} \right) + \frac{1.2W_{bs}}{3} - \left(\frac{q_1 + 2q_{min}}{6} \right) \right] l_{heel}^2 \quad (14)$$

$$V_t = \left[1.6 \left(\frac{q_{dt} + q_{\max}}{2} \right) - 0.9 (\gamma_c X_4 + \gamma_{bs} (D - X_4)) \right] (l_{\text{toe}} - dt) \quad (15)$$

$$M_t = \left[1.6 \left(\frac{q_2}{6} + \frac{q_{\max}}{3} \right) - 0.9 (\gamma_c X_4 + \gamma_{bs} (D - X_4)) \right] l_{\text{toe}}^2 \quad (16)$$

where, CC represents the concrete cover, while γ_c denotes the concrete's unit weight. The load resulting from the triangular distribution of the backfill soil at a height

dh behind the wall is represented as W_{bsdh} , with the corresponding soil pressure expressed as q_{dh} . The symbol W_{bs} identifies the peak load due to backfill acting at the heel section. At the bottom of the stem on the rear side, the soil pressure is denoted by q_1 , whereas q_2 corresponds to the pressure applied at the toe. The terms l_{heel} and l_{toe} describe the lengths of the heel and toe slabs, respectively.

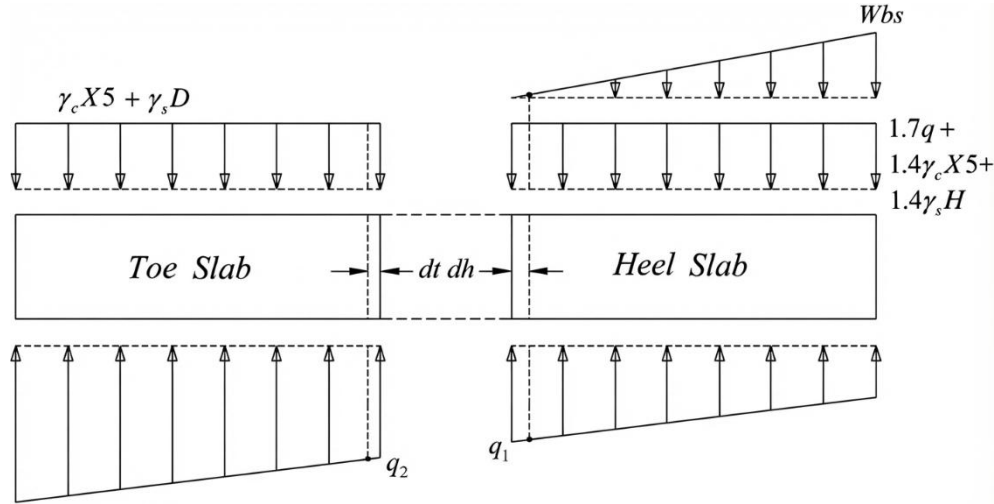


Fig. 3. The effective design forces on toe slab and heel slab.

Furthermore, the shear force and bending moment along the height of the stem, with the critical section for shear force located at a distance ds from the point where the stem intersects with the base slab ($ds = X_5 - CC$), can be represented as follows:

$$V_s = 1.6 \left(k_a q (H - ds) + \frac{1}{2} k_a \gamma_{rs} (H - ds)^2 \right) \cos \beta \quad (17)$$

$$M_s = 1.6 \left[\frac{1}{2} (k_a q H^2) + \left(\frac{1}{3} k_a \gamma_{rs} H^3 \right) \right] \cos \beta \quad (18)$$

In the final retaining wall design, all structural strength and geotechnical stability requirements must be satisfied. The optimization algorithm enforces the design constraints through a penalty-based approach. Designs satisfying all constraint conditions remain unpenalized, while infeasible solutions are assigned penalty values proportional to the extent of their constraint violations.

2.1.3. Geotechnical constraints

The design optimization of the retaining wall is governed by a set of inequality constraints that ensure structural integrity and geotechnical stability. The first constraint, $g_1(x) = (FS_{O\text{design}}/FS_O) - 1 \leq 0$ is defined to maintain overturning stability by ensuring the calculated factor of safety does not fall below the required design limit. Similarly, the sliding resistance is controlled by $g_2(x) = (FS_{S\text{design}}/FS_S) - 1 \leq 0$ which guarantees that the resisting forces are sufficient against lateral earth pressures. To prevent bearing capacity failure of the foundation soil, the constraint $g_3(x) = (FS_{B\text{design}}/FS_B) - 1 \leq 0$ is employed to keep the applied pressure within safe limits. Finally, to ensure the entire base of the wall remains in compression and

maintains full contact with the soil, the condition $g_4(x) = q_{\min} \geq 0$ is imposed, effectively preventing any tensile stress at the heel or toe.

2.1.4. Structural constraints

In addition to geotechnical stability, the structural integrity of the retaining wall components (stem, toe, and heel slabs) is maintained through a set of reinforcement-related inequality constraints. The primary structural safety is governed by the moment and shear capacity ratios; specifically, the constraints $g_{[5-7]}(x) = (M_u/M_n) - 1 \leq 0$ and $g_{[8-10]}(x) = (V_u/V_n) - 1 \leq 0$ ensure that the factored design moment and shear force do not exceed the nominal resistance capacities of the sections. Furthermore, to ensure ductile behavior and prevent sudden failure, the amount of reinforcement steel must stay within standardized limits. This is controlled by the minimum reinforcement constraint, $g_{[11-13]}(x) = (A_{s\min}/A_s) - 1 \leq 0$ which ensures sufficient steel area is provided, and the maximum reinforcement constraint, $g_{[14-16]}(x) = (A_s/A_{s\max}) - 1 \leq 0$ which prevents over-reinforcement and ensures the concrete reaches its crushing strain only after the steel yields. $g_{[5-7]}(x)$ are the moment capacity constraints for each section (stem, toe slab, and heel slab), $g_{[8-10]}(x)$ are the shear capacity constraints for each section, $g_{[11-13]}(x)$ are the minimum reinforcement area constraints, $g_{[14-16]}(x)$ are the maximum reinforcement area constraints

2.1.5. Geometric constraints

To ensure the constructability and structural stability of the retaining wall, several geometric and reinforcement development length constraints are implemented. The geometric constraint, $g_{17}(x) = (X_1 + X_3)/(X_2) -$

$1 \leq 0$ is used to maintain proportional dimensions between different parts of the wall structure. Furthermore, to ensure that the reinforcement bars can fully develop their design strength without slipping, the development length must be sufficiently accommodated within the concrete sections. These requirements are controlled by the following constraints: The stem reinforcement development within the footing is governed by $g_{18}(x) = (l_{db \text{ stem}})/(X_5 - \text{cover}) - 1 \leq 0$. The toe slab reinforcement development is ensured by $g_{19}(x) = (l_{db \text{ toe}})/(X_2 - X_1 - \text{cover}) - 1 \leq 0$. The heel slab reinforcement anchorage is checked using $g_{20}(x) = (l_{db \text{ heel}})/(X_2 + X_3 - \text{cover}) - 1 \leq 0$.

2.1.6. Objective functions

This study introduces an innovative automated optimization technique designed to improve geotechnical design and structural efficiency, applying a metaheuristic method for the first time to automate the dynamic analysis of sustainable and material cost-effective RCC walls. By optimizing 8 design variables, including 5 geometric and 3 steel reinforcement factors, the method aims to minimize material cost and CO₂ emissions using the simulated annealing technique:

$$\text{Cost} = C_s W_s + C_c V_c \quad (19)$$

$$\text{CO}_2 = e_s W_s + e_c V_c \quad (20)$$

The unit costs of steel and concrete are denoted by C_s and C_c , respectively, while W_s represents the weight of the reinforcing steel and V_c is the volume of concrete. The unit weight of concrete, scaled by a factor of 100, is represented by γ_c . The CO₂ emissions resulting from the RCC wall materials are calculated as shown in Eq. (20). The unit CO₂ emissions of reinforcement and concrete are referred to CO_{2s} and CO_{2c}, respectively, and are taken from Naseri et al. (2022). CO₂ emission for steel reinforcement is 2.82 kg/kg for traditional concrete (ITeC 2009). Concrete, the most widely used man-made material in construction, faces a growing demand worldwide. As a result, the use of eco-friendly concrete has become increasingly

important on geotechnical infrastructures. In this study, the environmental objective function accounts exclusively for the emissions associated with concrete and reinforcing steel. Accordingly, the system boundary and functional unit are clearly defined to reflect only the material-related embodied CO₂ emissions within the adopted database and system boundary.

The mixture proportions of eco-friendly concrete (green mixture and sustainable mixture) were evaluated in RCC retaining walls as a key environmental design element. Green concrete is defined as concrete with an optimal balance of cost-effectiveness, environmental footprint, workability, and strength. Sustainable concrete, considered a durable form of green concrete, provides the best balance in terms of these factors (Naseri et al. 2022). The methodology simultaneously considers various critical environmental factors in concrete mixture proportioning, including hazardous waste disposal (HWD), non-hazardous waste disposed (NHWD), and global warming potential (GWP) is considered in this study. The HWD represents the amount of hazardous waste generated throughout a material's life cycle and is expressed in kilograms (kg). Lower HWD values associated with hazardous waste management. The GWP quantifies the contribution of greenhouse gas emissions to climate change and is expressed as kilograms of carbon dioxide equivalent (kg CO₂-eq). Lower GWP values indicate lower greenhouse gas emissions and better environmental performance. The use of fly ash (FA) in mixture design helps mitigate the environmental impact of cement and improve concrete properties. Subsequently, the optimal mixture proportions for sustainable and green concrete are introduced in Table 3. Green concrete is designed to minimize environmental impacts while maintaining adequate strength, workability, and cost. Sustainable concrete further incorporates durability as an additional performance criterion. Tables 3 and 4 present the source material data adopted directly from Naseri et al. (2022). No recalculation or modification was performed, and the reported material properties were used without alteration. These literature-derived data were incorporated into the optimization model as the material properties of the reinforced concrete cantilever (RCC) retaining wall.

Table 3. Optimal mixture proportion of green and sustainable concrete with different strengths (Naseri et al. 2022).

Compressive strength (MPa)	Concrete type	Mixture ID	Cement (kg/m ³)	Water (kg/m ³)	Fine aggregate (kg/m ³)	Coarse aggregate (kg/m ³)	Fly ash (kg/m ³)
30	Green	G30	145	177.40	812.62	1020.40	174.13
40	Green	G40	145.02	173.25	892	939	183.17
50	Green	G50	177.59	170.80	804.74	982.21	205.24
60	Green	G60	209.09	165.08	709.08	1049	223.07
30	Sustainable	S30	158.39	179.79	892	942.28	154.23
40	Sustainable	S40	173.89	169.98	830.78	996.07	174.23
50	Sustainable	S50	207.81	160	785.30	1045.60	169.85
60	Sustainable	S60	239.31	160	737.95	1049	185.61
30	Traditional	T30	176.92	188.60	954.45	1015.86	171.18
40	Traditional	T40	194.45	178.48	889.94	1074.31	193.40
50	Traditional	T50	232.75	168.10	840.27	1129.25	188.80
60	Traditional	T60	269.10	176	789.61	1132.92	206.52

Furthermore, the costs of green (G), traditional (T) and sustainable (S) concrete with a mixture ID of G30, G40, G50, T30, T40, T50, S30, S40, and S50 are detailed in Table 4, with GWP (kg CO₂-eq), NHWD (kg), Carbona-

tion (mm), and HWD (kg) including material cost (Euros/m³). Table 4 summarizes the environmental impact indicators and unit material costs for the concrete types used in this study.

Table 4. The environmental impact indicators and unit material costs for green, sustainable, and traditional concrete (Naseri et al. 2022).

Mixture ID	GWP (kg CO ₂ -eq)	HWD (kg)	Cost (Euros/m ³)	NHWD (kg)	Carbonation (mm)
G30	139.13	0.021	59.66	0.335	26.77
S30	151.09	0.023	61.18	0.347	26.77
T30	265.99	0.057	91.36	0.572	26.51
G40	139.28	0.022	61.14	0.329	26.62
S40	165.36	0.025	62.91	0.378	19
T40	300.46	0.099	93.87	0.625	21.65
G50	169	0.026	64.86	0.386	26.53
S50	196.06	0.030	66.63	0.484	6.23
T50	392.41	0.067	98.77	0.777	10.36

2.1.7. Simulated annealing algorithm

The heuristic optimization approach adopted in this study is Simulated Annealing (SA), first introduced by Kirkpatrick et al. (1983). The method is inspired by the annealing process, where a material is heated and then gradually cooled to achieve a stable structure. At high temperatures, the algorithm may accept worse solutions, but this probability decreases as the temperature is lowered. In SA, a new solution is accepted based on the probability $\exp(-\Delta E/T)$, where ΔE is the change in the objective function and T is the current temperature. The temperature is reduced geometrically using a cooling coefficient, and a fixed number of iterations (Markov chain) is performed at each temperature level. This enables the algorithm to escape local optima at early stages and gradually converge as the temperature approaches zero. The performance of SA depends on parameters such as the initial temperature, cooling rate, and Markov chain length. In this study, the initial solution is generated randomly within the variable bounds and repeated until a feasible solution is obtained.

To ensure the rigorous technical reproducibility of the optimization procedure, the Simulated Annealing (SA) algorithm is implemented with precise control mechanisms and a robust thermal schedule. The optimization process initiates with the definition of an eight-dimensional design vector, $Y = \{Y_1, Y_2, \dots, Y_8\}$, representing the geometric dimensions ($X_1, \dots, X_5$) and reinforcement areas ($R_1, \dots, R_3$). The search space is bounded by practical engineering limits, and an initial feasible solution is generated through a stochastic sampling process. To maintain structural and geotechnical validity from the onset, the algorithm utilizes a rejection sampling technique, repeatedly generating initial candidate vectors until all inequality constraints—governing overturning, sliding, bearing capacity, and structural requirements—are strictly satisfied. The thermal schedule, which governs the exploration-exploitation balance, is

calibrated based on the initial sensitivity of the objective functions (Cost and CO₂).

The initial temperature, T_0 , is established at 100. This specific value is selected through pre-computational trials to ensure an initial acceptance probability, P , of approximately 0.8 for worsening moves, as defined by the Metropolis criterion: $P = \exp(-\Delta E/T)$. This high initial energy state allows the algorithm to perform extensive global exploration, effectively bypassing local minimum that frequently occurs in non-convex structural landscapes. The transition from exploration to fine-tuning (exploitation) is controlled by a geometric cooling law, $T_{k+1} = \alpha \cdot T_k$, where the cooling rate, α , is set to 0.95. This relatively slow cooling rate ensures a thorough search of the design space. At each discrete temperature level, a Markov chain length, L , of 100 iterations is maintained. This ensures that the system reaches a state of quasi-equilibrium, allowing the design variables to stabilize before the next temperature reduction. During each iteration, a neighborhood move is generated by applying a random perturbation to the current design vector, scaled by the variable's permissible range.

Constraint handling is managed through a sophisticated static penalty function approach. Given the highly constrained nature of RCC retaining wall design, a penalized objective function, $f_{\text{aug}}(x)$, is formulated to guide the search toward the feasible region:

$$f_{\text{aug}}(x) = f(x) + R \sum_{i=1}^{20} \max(0, g_i(x))^2 \quad (21)$$

where $f(x)$ is the raw cost or CO₂ emission, $g_i(x)$ represents the i -th inequality constraint, and R is a robust penalty multiplier set to 10^6 . This high penalty coefficient ensures that any violation of safety factors results in a significant increase in the energy state, making such solutions highly unlikely to be accepted in the later stages of annealing. The optimization procedure concludes based on dual-termination criterion. The algorithm stops when either the system temperature reaches

a final threshold of $T_{\text{final}} = 0.001$ (thermal convergence) or the best-found solution remains unchanged for 50 consecutive temperature steps (stagnation convergence). By integrating these specific parameters and penalty mechanisms, the optimization framework provides a reproducible and technically sound method for the design of sustainable geotechnical structures.

3. Results and Discussion

Table 5 summarizes the CO₂ emission results and computational times obtained from nine independent runs for wall heights ranging from 4 m to 10 m, considering green, sustainable, and traditional concrete alter-

natives. Across all wall heights, CO₂ emissions increase as expected with structural size. In terms of material performance, green concrete consistently provides the lowest emissions, followed by sustainable concrete, while traditional concrete yields the highest emission. The gap between minimum and average emission values remains small at lower heights but becomes more pronounced as the wall height increases, indicating higher variability for larger design cases. The standard deviation also tends to grow with height, particularly for traditional concrete, reflecting greater dispersion in solutions. CPU times show a slight increasing trend with wall height, while remaining largely similar across all three concrete types, suggesting that environmental performance differences do not significantly affect computational effort.

Table 5. Comparative CO₂ emissions of retaining walls for different wall heights and material alternatives.

H (m)	Concrete type	Min. emissions (kg CO ₂ -eq)	Avg. emissions (kg CO ₂ -eq)	Std. deviation	Avg. CPU time (s)
4	Traditional	1285.420	1290.115	1.250	92.450
4	Sustainable	1156.880	1158.310	0.840	95.120
4	Green	1024.350	1025.040	0.520	98.610
6	Traditional	2715.300	2724.850	6.420	96.180
6	Sustainable	2443.770	2448.320	4.150	98.340
6	Green	2165.210	2166.590	2.880	102.150
8	Traditional	4982.150	5012.440	84.120	100.240
8	Sustainable	4483.930	4505.210	52.340	103.550
8	Green	3968.450	3975.120	35.180	108.420
10	Traditional	8125.680	8182.350	105.400	105.820
10	Sustainable	7313.110	7345.920	92.150	109.140
10	Green	6475.240	6488.770	68.220	114.350

Table 6 presents the cost optimization results and corresponding CPU times for nine runs over wall heights from 4 m to 10 m, evaluated for green, sustainable, and traditional concrete alternatives. In all cases, costs increase progressively with wall height due to higher material and structural demands. Green concrete consistently achieves the lowest cost values, followed by sustainable concrete, whereas traditional concrete results in the high-

est costs among the three options. The differences between minimum and average costs remain relatively limited, indicating stable optimization performance across all scenarios. Although standard deviation increases slightly at higher wall heights, the overall variability remains moderate. CPU time exhibits a gradual increase with wall height, and similar to the emissions case, it is not significantly influenced by the type of concrete used.

Table 6. Comparative cost analysis of retaining walls for different wall heights and material alternatives.

H (m)	Concrete type	Minimum cost (euros)	Average cost (euros)	Standard deviation	Average CPU time (s)
4	Traditional	815.420	816.050	0.450	92.450
4	Sustainable	742.180	742.850	0.380	95.120
4	Green	676.806	677.187	0.344	96.734
6	Traditional	1618.300	1620.450	2.850	96.180
6	Sustainable	1475.250	1476.900	2.420	98.340
6	Green	1343.588	1344.762	2.166	100.619
8	Traditional	2758.120	2768.440	19.450	100.240
8	Sustainable	2515.650	2524.320	17.800	103.550
8	Green	2290.699	2298.642	16.300	103.894
10	Traditional	4228.950	4235.120	9.500	105.820
10	Sustainable	3852.400	3858.750	8.450	109.140
10	Green	3513.353	3518.385	7.724	107.788

A sensitivity analysis was conducted to evaluate the influence of the Simulated Annealing cooling rate on the optimization performance. Five cooling rate values ranging from 0.90 to 0.99 were examined while keeping all other algorithmic parameters constant. The cooling rate has a noticeable influence on both the solution quality and the computational performance of the Simulated Annealing algorithm. When the cooling rate is set to 0.90, the algorithm converges rapidly, resulting in the shortest CPU time (71.4 s); however, it produces the highest best and mean cost values (€681.52 and €683.41, respectively) together with the largest standard deviation (€1.74), indicating reduced solution quality and stability with 12,500 iterations. As the cooling rate increases, the performance gradually deteriorates: at 0.92, the metrics shift to a best result of €688.20, mean of €685.10, standard deviation of 1.05, 112.0 s CPU time, and 17,000 iter-

ations; at 0.95, the best, mean, and worst costs rise to €698.80, €690.50 with a standard deviation of 1.25, taking 155.0 s and 25,400 iterations; at 0.97, the values further increase to €705.30 (best), €702.15 (mean) with a standard deviation of 1.52, 195.4 s CPU time, and 36,100 iterations; and finally, at 0.99, the algorithm yields its highest costs with a best result of €707.52, mean of €704.41, standard deviation of 1.74, requiring the maximum computational effort of 245.6 s CPU time and 48,200 iterations. All tests were executed in Python 3.10 using an Intel Core i7 processor and 16 GB of RAM.

Fig. 4 illustrates the relationship between the minimum values of CO₂ emissions and costs (Cost) with respect to the total wall height. The results indicate that both responses follow a clear quadratic trend. The average CO₂ emissions can be approximated by a second-order polynomial of the form:

- $CO_{2(\text{traditional})}(\text{kg}) = 108.4484H^2 - 370.0634H + 1036.6668$, with a coefficient of determination $R^2 = 0.9998$ for traditional concrete.
- $CO_{2(\text{sustainable})}(\text{kg}) = 96.9188H^2 - 325.8765H + 911.9630$, with a coefficient of determination $R^2 = 0.9999$ for sustainable concrete.
- $CO_{2(\text{green})}(\text{kg}) = 85.7563H^2 - 290.6015H + 817.2530$, with a coefficient of determination $R^2 = 0.9996$ for green concrete.
- Similarly, the average cost (Cost) variation is well represented by a function given as:
- $\text{Cost}_{(\text{traditional})}(\text{euros}) = 41.3925H^2 - 9.2350H + 189.4650$, with $R^2 = 0.9995$ for traditional concrete.
- $\text{Cost}_{(\text{sustainable})}(\text{euros}) = 37.52375H^2 - 5.5765H + 163.4580$, with $R^2 = 0.9994$ for sustainable concrete.
- $\text{Cost}_{(\text{green})}(\text{euros}) = 34.5105H^2 - 9.2733H + 161.0901$, with $R^2 = 0.9995$ for green concrete.

These regression models confirm a strong nonlinear dependency between wall height and both objective functions, with excellent agreement between the fitted curves and the observed data. The results are evaluated independently for each objective. The comparative assessment of the traditional, sustainable, and green concrete alternatives is therefore based on their individual performances with respect to each optimization objective. Moreover, it should be noted that the proposed regression equations are empirical relationships derived from the optimization results obtained for the investigated retaining wall heights (4–10 m). Therefore, extrapolation beyond the investigated wall height interval requires additional optimization analyses and further validation.

The results indicate that both the minimum cost and minimum carbon emissions increase with retaining wall height due to the higher material demand. For all investigated wall heights, green concrete consistently achieved the lowest cost and carbon emission values, followed by sustainable concrete, whereas traditional concrete resulted in the highest values. Compared with traditional concrete, the use of green concrete reduced the minimum cost by approximately 17% and the minimum carbon emissions by approximately 20%, while maintaining the required structural and geotechnical performance. These findings demonstrate the economic and environmental advantages of adopting green concrete in optimized retaining wall design (Fig. 4).

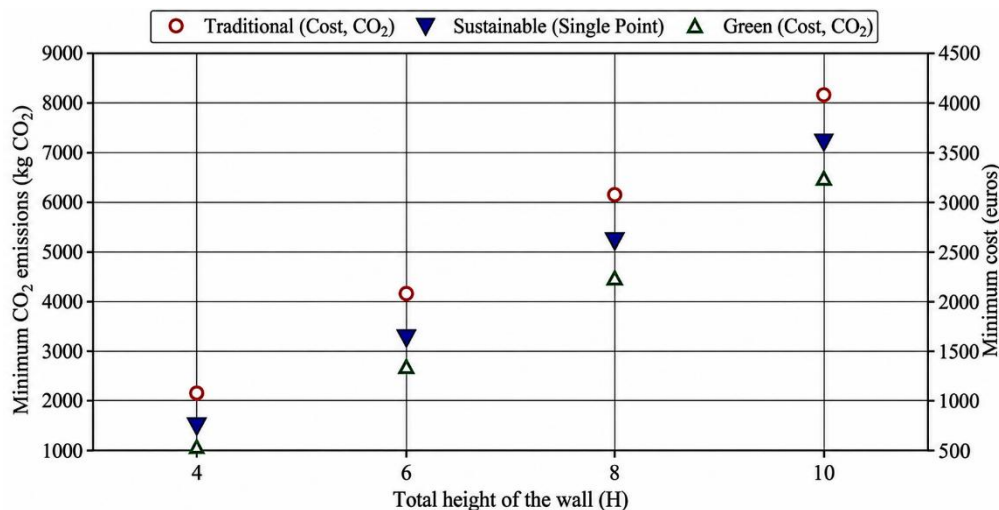


Fig. 4. Comparative variation of minimum cost and minimum emissions for traditional, sustainable, and green concrete types across the wall height.

Fig 5. illustrates the cost-versus-emission objective space for traditional, sustainable, and green concrete. As shown, green concrete consistently occupies the lower emission region, providing a clear visual representation of how variations in concrete type affect the balance between sustainability goals and budgetary

requirements.

Finally, it is necessary to analyze if the economic optimized walls and the emissions optimized walls present similar characteristics. The characteristics for these walls are compiled in Tables 7 and Table 8 for optimized emission and cost, respectively.

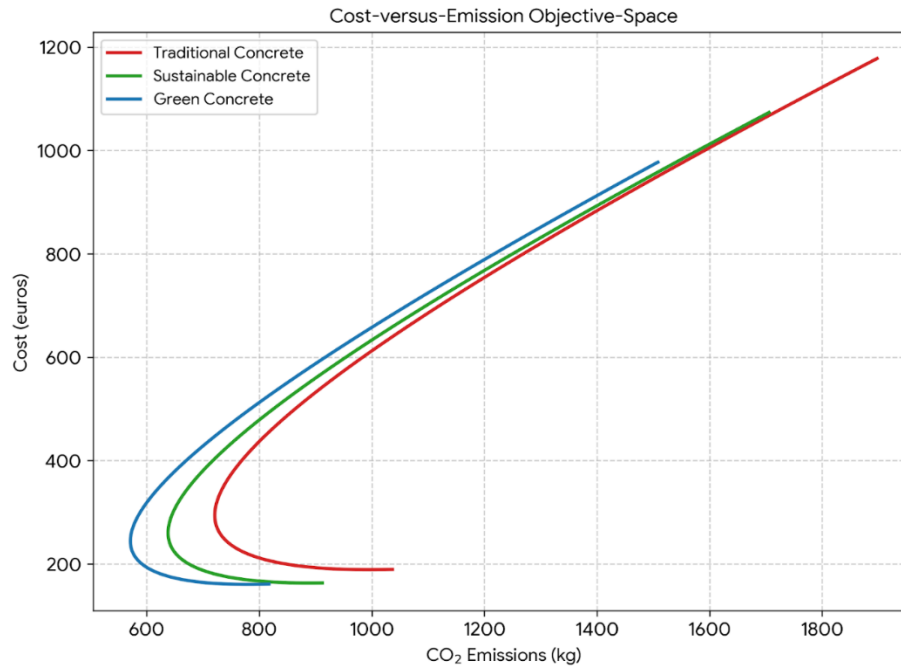


Fig. 5. The cost-versus-emission objective space for traditional, sustainable, and green concrete.

Table 7. Characteristics of RCC retaining walls with traditional, sustainable, and green concrete for emission optimization.

Design variables	H (m)	4	6	8	10
X1 (toe slab width) (m)	Traditional	0.640	0.960	1.150	1.260
	Sustainable	0.612	0.905	1.062	1.092
	Green	0.485	0.725	0.820	0.950
X2 (base slab width) (m)	Traditional	1.780	2.960	3.950	4.600
	Sustainable	1.685	2.748	3.781	4.286
	Green	1.650	2.460	3.280	4.040
X3 (stem top thickness) (m)	Traditional	0.360	0.600	0.900	1.220
	Sustainable	0.318	0.548	0.823	1.118
	Green	0.270	0.470	0.720	1.000
X4 (base slab thickness) (m)	Traditional	0.410	0.650	0.820	0.990
	Sustainable	0.362	0.588	0.742	0.903
	Green	0.310	0.500	0.650	0.800
X5 (stem bottom thickness) (m)	Traditional	0.700	1.150	1.650	2.200
	Sustainable	0.620	1.050	1.500	2.050
	Green	0.550	0.900	1.300	1.800
R1 (vertical reinforcement in stem) (cm ²)	Traditional	65.200	165.000	330.000	565.000
	Sustainable	56.870	142.880	287.940	492.330
	Green	45.000	115.000	230.000	390.000
R2 (horizontal reinforcement in toe) (cm ²)	Traditional	58.000	140.000	285.000	480.000
	Sustainable	50.120	121.760	252.880	420.780
	Green	40.000	100.000	200.000	330.000
R3 (horizontal reinforcement in heel) (cm ²)	Traditional	52.000	128.000	265.000	445.000
	Sustainable	45.500	112.000	235.000	395.000
	Green	38.000	92.000	190.000	320.000

The optimization results demonstrate a profound shift in the structural and environmental performance of RCC retaining walls when traditional design paradigms are replaced with metaheuristic-driven approaches. The primary finding of this study is the consistent superiority

of green concrete across all height parameters. At a structural height of 10 m, green concrete achieved a minimum CO₂ emission of 6475.24 kg, representing a substantial reduction of approximately 20% compared to the 8125.68 kg generated by traditional mixtures (Fig. 6).

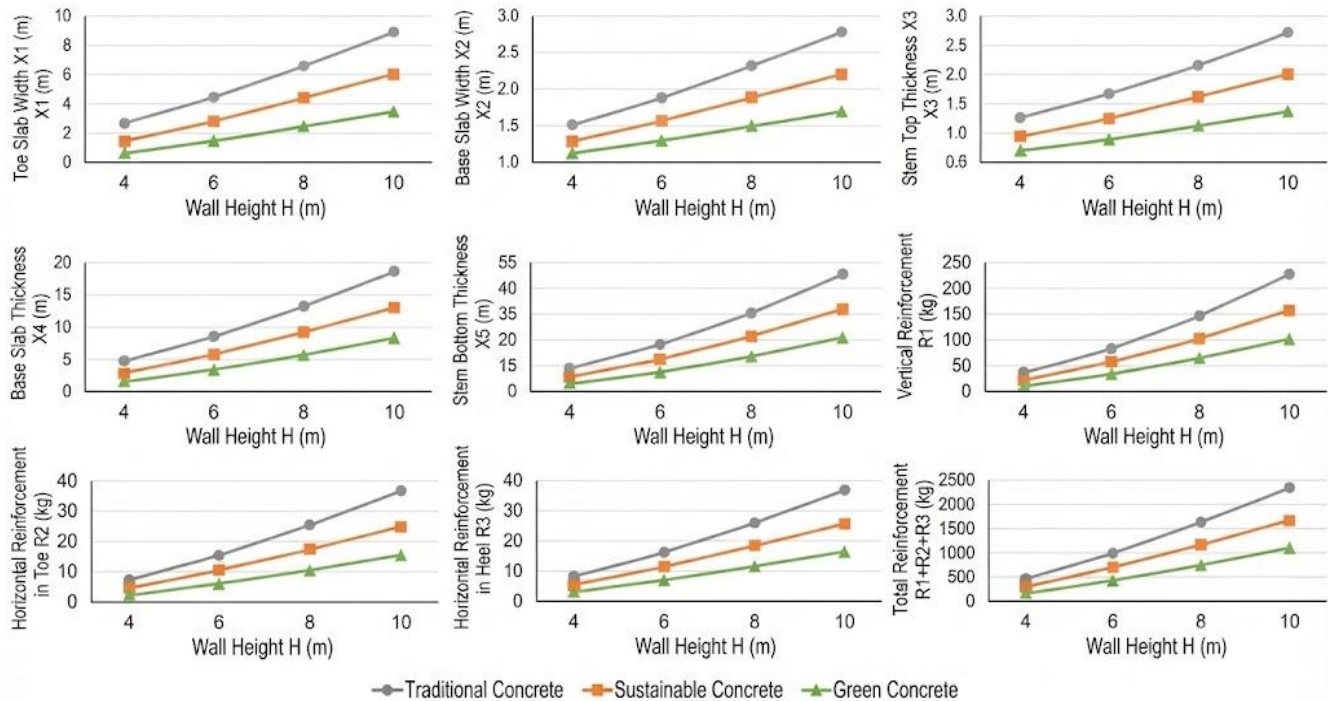


Fig. 6. Variation of design variables with wall height for optimized emissions.

This environmental efficiency is largely attributed to the optimized replacement of cement with fly ash, which significantly lowers the Global Warming Potential (GWP), where G30 concrete exhibits a GWP of only 139.13 kg-CO₂-eq compared to 265.99 kg-CO₂-eq for T30. These results suggest that the integration of fly ash does not merely serve as a waste-disposal solution in geotechnical infrastructure without sacrificing compressive strength.

The economic analysis further challenges the conventional assumption that sustainable construction materials necessarily incur a financial premium. The optimization results presented in Table 7 and 8 demonstrate that the selection of concrete type has a substantial influence on both the environmental and economic performance of the retaining wall while satisfying identical structural and geotechnical design requirements. Among the three alternatives, green concrete consistently produced the lowest cost and carbon emission values for all wall heights, followed by sustainable concrete, whereas traditional concrete resulted in the highest values. In addition to its superior environmental and economic performance, driven by the predefined material cost and emission factors, the use of green concrete achieved the optimum emission and cost values for the same wall height. Consequently, the optimized green concrete designs required the lowest quantities of both concrete and reinforcing steel. Sustainable concrete exhibited intermediate performance, providing noticeable reductions in material consumption, cost, and carbon emissions relative to traditional concrete, although its optimized dimen-

sions and reinforcement requirements remained higher than those of green concrete. These findings indicate that adopting green concrete enables more material-efficient retaining wall designs without compromising the required structural safety or geotechnical stability, thereby offering significant advantages in terms of both sustainability and economic efficiency. The data from Table 8 reveals that green concrete is consistently the most cost-effective option, with the 10 m wall configuration costing 3513.35 Euros—roughly 17% less than the 4228.95 Euros required for traditional concrete.

This cost-effectiveness is driven by the reduction in both material unit costs and Hazardous Waste Disposal (HWD) metrics. From a structural standpoint, the Simulated Annealing (SA) algorithm facilitated this by optimizing geometric variables to favor more efficient sections; for instance, the stem bottom thickness (X5) for a 10 m wall was reduced from 2.40 m in traditional designs to 1.85 m in green concrete configurations. This indicates that the optimization technique effectively exploits the material properties of eco-friendly mixtures to produce leaner, more efficient structural geometries that simultaneously minimize volume and cost.

Furthermore, the high predictive accuracy of the derived quadratic regression models ($R^2 > 0.998$) underscores a stable and predictable relationship between wall height and design objectives. The divergence between traditional and green concrete performance becomes more pronounced as wall height increases, as shown in the standard deviation trends in Table 9. While traditional concrete exhibited higher variability at 10 m

heights ($SD = 105.40$), green concrete remained more stable ($SD = 68.22$), suggesting that the SA algorithm finds more robust optima when working with sustainable mixture parameters. This stability, coupled with the negligible differences in CPU times across material types, confirms that the proposed automated optimization framework is computationally efficient. Ultimately, these results provide a scalable roadmap for geotechnical engineers to reconcile the often-conflicting goals of structural safety,

economic constraint, and environmental stewardship. To further validate the proposed optimization framework, the optimized retaining wall designs obtained using the Simulated Annealing (SA) algorithm were compared with the qualitative comparison with selected literature results reported by Gandomi et al. (2017a), in which Genetic Algorithm (GA), Differential Evolution (DE), Evolution Strategy (ES), and Biogeography-Based Optimization (BBO) were employed for cost optimization (Table 9).

Table 8. Characteristics of RCC retaining walls with traditional, sustainable, and green concrete for cost optimization.

Design variables	H (m)	4	6	8	10
X1 (toe slab width) (m)	Traditional	0.625	0.730	0.950	1.300
	Sustainable	0.510	0.692	0.875	1.150
	Green	0.480	0.630	0.760	0.900
X2 (base slab width) (m)	Traditional	1.850	2.520	3.850	4.700
	Sustainable	1.720	2.450	3.630	4.420
	Green	1.600	2.410	3.200	4.000
X3 (stem top thickness) (m)	Traditional	0.390	0.670	0.990	1.350
	Sustainable	0.345	0.590	0.885	1.200
	Green	0.300	0.500	0.750	1.050
X4 (base slab thickness) (m)	Traditional	0.420	0.760	0.980	1.330
	Sustainable	0.375	0.675	0.885	1.220
	Green	0.330	0.580	0.760	1.050
X5 (stem bottom thickness) (m)	Traditional	0.750	1.250	1.800	2.400
	Sustainable	0.660	1.100	1.600	2.150
	Green	0.580	0.950	1.350	1.850
R1 (vertical reinforcement in stem) (cm^2)	Traditional	68.000	165.000	330.000	540.000
	Sustainable	60.000	148.000	295.000	490.000
	Green	50.000	125.000	250.000	410.000
R2 (horizontal reinforcement in toe) (cm^2)	Traditional	60.000	145.000	300.000	480.000
	Sustainable	53.000	130.000	270.000	450.000
	Green	45.000	110.000	230.000	380.000
R3 (horizontal reinforcement in heel) (cm^2)	Traditional	55.000	135.000	280.000	440.000
	Sustainable	48.000	120.000	250.000	420.000
	Green	40.000	100.000	210.000	350.000

Table 9. Comparison of cost optimized designs for traditional, sustainable, and green concrete retaining walls with the qualitative comparison with selected literature results reported by Gandomi et al. (2017a).

Design variables	GA	DE	ES	BBO	SA (Traditional)	SA (Sustainable)	SA (Green)
X1 (toe slab width) (m)	0.586	0.616	0.691	0.737	0.625	0.510	0.480
X2 (base slab width) (m)	1.916	1.872	1.845	1.842	1.850	1.720	1.500
X3 (stem top thickness) (m)	0.272	0.290	0.320	0.277	0.390	0.345	0.300
X4 (base slab thickness) (m)	0.201	0.206	0.221	0.200	0.420	0.375	0.330
X5 (stem bottom thickness) (m)	0.280	0.271	0.280	0.270	0.750	0.660	0.580

The optimization results demonstrate that the superior performance of green concrete is a direct consequence of the algorithm's ability to adjust structural configurations to satisfy all geotechnical, structural, and geometric requirements. Rather than a simple material substitution, the optimization algorithm computes dis-

tinct wall dimensions for each material alternative to ensure compliance with critical safety standards. Green concrete designs often utilize more optimized geometric proportions—such as reduced base slab widths and reinforcement quantities—compared to traditional and sustainable alternatives. For instance, at a wall height of

10 m, the algorithm identifies a structural configuration for green concrete that achieves a 20% reduction in CO₂ emissions while maintaining full structural integrity. This confirms that the environmental and economic advantages are not merely a result of the input material database, but stem from the algorithm's effectiveness in identifying the most efficient structural geometry for each specific concrete type. The sustainability findings presented in this study are limited to the investigated retaining wall heights (4–10 m) and the material datasets considered in the optimization framework. Therefore, the observed improvements in cost and carbon emissions should be interpreted within the analyzed design conditions rather than as applicable conclusions. Further studies involving a wider range of wall geometries, loading conditions, and material datasets are recommended to extend the generality of these findings.

4. Conclusions

This study presented an optimization framework for reinforced concrete cantilever (RCC) retaining walls using the Simulated Annealing (SA) algorithm. The optimization was formulated as two independent single-objective problems, in which material cost and embodied CO₂ emissions were minimized separately while satisfying the specified geotechnical, structural, and geometric design constraints. Eight design variables, including five geometric dimensions and three reinforcement quantities, were considered, and infeasible solutions were handled using a penalty-based constraint approach.

The optimization results for wall heights between 4 m and 10 m showed consistent differences among the investigated concrete alternatives. Green concrete produced the lowest material cost and embodied CO₂ emissions across all wall heights, followed by sustainable concrete, while traditional concrete resulted in the highest values. For a 10 m retaining wall, the minimum embodied CO₂ emission obtained with green concrete was 6475.24 kg CO₂-eq, compared with 8125.68 kg CO₂-eq for traditional concrete. The corresponding minimum material costs were €3513.35 and €4228.95, respectively. Within the investigated material database and design assumptions, these results indicate that green and sustainable concrete alternatives can reduce both material cost and embodied CO₂ emissions relative to traditional concrete. The optimized wall configurations also varied according to the selected concrete type. In general, the green concrete solutions required smaller wall dimensions and lower total concrete and reinforcement quantities than the corresponding traditional concrete solutions while satisfying the prescribed design constraints. This indicates that the optimization procedure identified different feasible structural configurations depending on the material properties and the selected objective function.

The SA algorithm produced consistent optimization results over repeated runs, with relatively small differences between the minimum and average objective-function values for most cases. The sensitivity analysis showed that the cooling rate influences both solution quality and computational time, highlighting the im-

portance of parameter selection for this optimization problem. The quadratic regression equations obtained from the optimization results exhibited high coefficients of determination ($R^2 > 0.998$); however, these equations represent empirical relationships within the investigated wall-height range (4–10 m) and should not be extrapolated beyond this range without additional validation. Overall, the proposed optimization framework demonstrates the applicability of combining material-specific environmental and cost data with structural and geotechnical optimization for RCC retaining walls under the investigated design conditions. The conclusions of this study are limited to the considered wall heights, loading conditions, material database, and design assumptions, and further studies are required to evaluate the framework under different design scenarios and material datasets.

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Conflict of Interest

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Data Availability

The datasets generated and/or analyzed during the current study are not publicly available but are available from the corresponding author upon reasonable request.

AI Assistance

No AI-based tools were used in the preparation of this manuscript.

Author Contributions

The author declares sole responsibility for all aspects of the study, including conceptualization, methodology, formal analysis, investigation, data curation, visualization, writing of the original draft, and writing, review, and editing of the manuscript.

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